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Article

The short-run and long-run dynamics of GDP and trade in a seemingly unrelated regression framework

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The short-run and long-run dynamics of GDP and trade in a seemingly unrelated regression framework[1]

Abstract: *While most time series studies consider country specific or restrictive panel models to study the short-run and long-run dynamic relationships of economic variables, we pursue an unrestrictive system framework to explore these relationships. We show that instead of estimating traditional VAR and VECM models in a one-country setting, using a Seemingly Unrelated Regression (SUR) system estimation can help gain efficiency in coefficient estimates and standard errors. We study the long-run and short-run dynamics between GDP and Trade in a reduced form VAR and VECM setting for Canada, the USA and Mexico. We estimate the models for each country separately and compare the estimates with those obtained in a SUR system allowing for cross-country contemporaneous correlations. We find that the results change considerably when the models are estimated in the SUR system, in terms of Granger-causality as well as the long-run adjustment parameters.*

Keywords: *Trade; GDP; Seemingly unrelated regression; VECM.*

Classificação JEL: F14; C32.

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1. Introduction

Economists have tried to estimate the causal relationship between trade and income in many instances, attempting to test whether higher trade leads to higher income and vice versa. This is an old topic and the literature on the relationship between trade and income is quite substantial. We take a fresh look not at the particular research question but rather the settings in which time-series studies have been conducted to estimate the dynamic relationships between trade and income for different countries.

Most of the empirical work involves testing the hypothesis of dynamic relationships between Trade and economic growth. (Jung & Marshall, 1985) is one of the first to attempt to test for Granger-causality between GDP and exports using a sample of 37 developing countries. (Ghartey, 1993) is one of the first papers to consider questions regarding stationarity carefully and estimate a vector autoregressive model. They find that productivity may cause exports in a country where the degree of openness is low and with relatively abundant resources. (Dutt & Ghosh, 1996) perform a test for cointegration and then test for Granger-causality, before which very few papers employed these techniques in economic studies. A relevant work was done by (Zestos & Tao, 2002). They consider one-country based analyses on Canadian and U.S. data of GDP, exports and imports. They find that Canadian GDP and Trade are closely related and there is a bidirectional causality. On the other hand, U.S. exhibits a weak relation between GDP and Trade. Finally, (Giraldo & Cañas, 2016) explore the causal link between trade and economic growth. They use the North American Free Trade Agreement (NAFTA) to estimate the causal relationship between growth and trade flows. They find bidirectional Granger-causality between exports and economic growth; however, they did not find conclusive evidence that trade within a trade bloc is more important for growth than trade with the rest of the world.

In this paper, we study for Granger-causality and long-run relationship between GDP and Trade for three countries: Canada, U.S. and Mexico. We model each country separately in their relevant VAR and VECM frameworks as well as in a system framework to see if cross-country contemporaneous correlations across the country shocks enhance our estimates by providing additional information in the model.

We find bidirectional Granger-causality for Canada in both country-specific & system frameworks. There is also a statistically significant long-run relationship between GDP and Trade for Canada. The long-run relationship between GDP and Trade has been found to be weak for the USA which is consistent with previous literature (Zestos & Tao, 2002). Allowing for a system framework enhances the coefficient estimates for Mexico significantly in the sense that GDP & Trade exhibit bidirectional Granger-causality after allowing for cross-country contemporaneous correlations, while there is Granger non-causality in the country-specific model. Studying the long-run relationships for the three countries reveal three different behavior in the three countries. The magnitude of coefficient estimates in the system framework are also higher & statistically significant compared to those of the country-specific models.

Our results thus suggest that instead of pursuing country-specific time-series models or panel models with the restriction of symmetry in coefficients among countries, an unrestricted model allowing for contemporaneous correlations among countries that might have common shocks in their errors provide more robust estimates based on more generous information set. Similar argument has also been made by (Kónya, 2006). Since, a SUR system allows current period values of GDP & Trade of one country to affect current period values of the same variables for other countries, estimation based on a SUR system would reflect a country's closely related trading partner's effects on its economy in addition to the country specific dynamics.[4]

[4] For example, Exports of motor vehicles and parts fell over 80% in Canada in April 2020, as widespread shutdowns impacted auto production on both sides of the Canada-U.S. border due to Covid-19. (Statistics Canada, Table: 12-10-0121-01) https://www150.statcan.gc.ca/n1/pub/11-626-x/11-626-x2020004-eng.htm
A country specific model would not have the same explanatory power that a SUR system would have being able to incorporate the impact of US shutdowns on Canadian exports.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework adopted and the model specification. Section 3 discusses the time series nature of the data and the specifics of lag selection & presence of unit root. Section 4 presents the findings and discusses the main empirical results. Section 5 concludes the paper. The appendix gives all the relevant tables not reported in the main text.

2. Theoretical framework

2. 1 Reduced form vector autoregressive (VAR) model

Vector Autoregressive model (VAR) (Sims, 1980) is used to understand the relationship between several variables allowing for dynamics and get better forecasts. Let us consider a bivariate VAR(p) model

$$y_{1t} = \alpha_1 + \phi_{11} y_{1,t-1} + \ldots + \phi_{1p} y_{p,t-p} + \varepsilon_{1t}$$

$$y_{2t} = \alpha_2 + \phi_{21} y_{1,t-1} + \ldots + \phi_{2p} y_{p,t-p} + \varepsilon_{2t}$$

With $E(\varepsilon_{it}) = 0, E(\varepsilon_{it}\varepsilon_{js}) = \sigma_{ij}$ for $t = s$ and=0 otherwise; $i, j = 1,2$. This implies ε_{1t} and ε_{2t} are correlated when there is feedback between y_{1t} and y_{2t} . Clearly, y_{1t} and y_{2t} are interrelated endogenous variables, with the current value of each endogenous variable dependent on the lagged values and disturbances. We at first consider country-specific VAR models and then combine the three countries to consider a panel SUR with unrestricted coefficients. In the SUR system the current period values of each dependent variable can affect the current period values of other variables in the system while the lagged values of all variables in the system are considered uncorrelated as they are pre-determined variables.

If we rewrite the system for a bivariate VAR(1) in matrix notation, it becomes the following:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \quad (1)$$

Or, $y_t = \alpha + \phi_1 y_{t-1} + \varepsilon_t; \quad \varepsilon_t \sim (0, \Omega)$

Where, $\Omega = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$

In this paper, we consider the following VAR(p_i) model with a time trend where p_i is the number of lags considered for each country. The time trend plays a role of proxy variable for some omitted variables from the original specification (Kónya, 2006). Additionally, (Ashley & Verbrugge, 2009) advise inclusion of a time trend because the downside risk to omitting a time trend is quite large when the data generating process consists of stationary fluctuations around a linear trend. Since we have two economic variables, GDP and Trade, we may rewrite our bivariate Vector autoregressive model as follows:

$$\text{GDP}_{i,t} = \alpha_{1,i} + \varphi_{11,i} \sum_{j=1}^{p_j} \text{GDP}_{i,t-j} + \varphi_{12,i} \sum_{j=1}^{p_j} \text{Trade}_{i,t-j} + \varepsilon_{i,1t} \quad (2)$$

$$\text{Trade}_{i,t} = \alpha_{2,i} + \varphi_{21,i} \sum_{j=1}^{p_j} \text{GDP}_{i,t-j} + \varphi_{22,i} \sum_{j=1}^{p_j} \text{Trade}_{i,t-j} + \varepsilon_{i,2t} \quad (3)$$

Here i=Canada, the USA and Mexico respectively. If we consider the bivariate VAR (p_i) for three countries simultaneously, we can see that each equation has different predetermined variables. A possible link among the set of equations is contemporaneous correlation. Therefore, these set of equations imply a SUR system instead of a VAR model. After selecting the lag order for each country, we can estimate a Seemingly Unrelated Regression (SUR) directly if we are certain that the series are stationary.

2.2 Granger-causal map

Vector autoregressive (VAR) models are widely used to test Granger causality (Granger, 1969) relationships, which aims to draw inferences on the dynamic impact of one variable on another. For example, a variable y_2 GCs a variable y_1 when available past information of y_2 allows us to predict y_1 better than we can when the past information of y_2 is not used. Granger causality tests the null hypothesis of the zero restrictions on the lagged dependent variables. Wald test is the standard approach to test the zero restrictions on the coefficients of the VAR models. If the variables of interest are stationary, then Wald statistic has an asymptotically chi-square distribution with q degrees of freedom (q is the number of restrictions under the null hypothesis.

This theory is not valid if the variables under study are integrated or cointegrated (Park & Phillips, 1989), (Sims, Stock, & Watson, 1990), and (Toda & Phillips, 1993). One possible solution is to consider a VAR with the variables in their first differences. In that case, the standard asymptotic theory is valid. To determine the order of integration, a pre-test is needed before we proceed to estimate the VAR model. However, the route of pretesting for unit roots, selecting lag order and deterministic terms and then undertaking the main test of interest results in over-rejection of the non-causal null hypothesis i.e., we believe there is causality when it actually is not there. To deal with this issue, one approach is the over-specifying or augmented lag method of (Toda & Yamamoto, 1995) and (Dolado & Lütkepohl, 1996), henceforth-TYDL.

They suggest a modified Wald (MWALD) test in a lag augmented VAR which has standard asymptotic chi-square distribution when a VAR (p_i+dmax) is estimated, where pi the lag order for each country and dmax is the maximal order of integration (Emirmahmutoglu & Kose, 2011).

Since both GDP and Trade are integrated series of order 1 or I (1) (see section 3.1) for Canada, the USA and Mexico, we write the system VAR (p_i+1) according to TYDL as follows:

$$\mathrm{GDP}_{i,t} = \alpha_{1,i} + \sum_{j=1}^{p_j+1} \varphi_{1j,i}\mathrm{GDP}_{i,t-j} + \sum_{j=1}^{p_j+1} \varphi_{1j,i}\mathrm{Trade}_{i,t-j} + \varepsilon_{i,1t} \tag{4}$$

$$\mathrm{Trade}_{i,t} = \alpha_{2,i} + \sum_{j=1}^{p_j+1} \varphi_{2j,i}\mathrm{GDP}_{i,t-j} + \sum_{j=1}^{p_j+1} \varphi_{2j,i}\mathrm{Trade}_{i,t-j} + \varepsilon_{i,2t} \tag{5}$$

Here, i= Canada, the USA and Mexico respectively. TYDL show that inclusion of irrelevant lags helps us to get back the limiting null chi-square distribution when testing restrictions on the parameters of the first p_i-lagged variables. That is, we can continue to test for Granger non-causality in the LVAR (Level VAR), irrespective of the non-stationary characteristics as long as we over specify the lag order by a sufficient amount.

Now, we re-estimate our system VAR with (p_i+1) lags and finally undertake the Granger-causal map.

$$\mathrm{GDP}_{i,t+1} = \alpha_{1,i} + \sum_{j=1}^{p_j+1} \varphi_{1j,i}\mathrm{GDP}_{i,t+1-j} + \sum_{j=1}^{p_j+1} \varphi_{1j,i}\mathrm{Trade}_{i,t+1-j} + \varepsilon_{i,1t+1} \tag{6}$$

$$\text{GDP}_{i,t+1} = \alpha_{2,i} + \sum_{j=1}^{p_j+1} \varphi_{2j,i}\text{GDP}_{i,t+1-j} + \sum_{j=1}^{p_j+1} \varphi_{2j,i}\text{Trade}_{i,t+1-j} + \varepsilon_{i,2t+1} \quad (7)$$

To test for Granger non-causality from, Trade_t to GDP_t , $\text{Trade}_t \nrightarrow \text{GDP}_t$ or GDP_t to Trade_t , $\text{GDP}_t \nrightarrow \text{Trade}_t$, we examine the following null hypothesis for each country using Wald tests:

$H_0 : \varphi_{i1} = \varphi_{i2} = \varphi_{i3} = ... = 0$ vs H_1 : at least one of $\varphi_{ij} \neq 0$ where $j = 1,...p$ and $i = 1,2,3$.

Under the cross-section independence assumption, (Im, Pesaran, & Shin, 2003) show that individual Wald statistics have an identical chi-squared distribution and average Wald statistic converges to a standard normal distribution, when time period and number of countries are large.

2.3 Vector error correction model (VECM) & cointegration

We have discussed that one way to deal with the non-stationarity of GDP and Trade variables is to filter the series by taking first difference. This suggests our system VAR be specified in the I(0) form of each series under study. However, filtering in such a way may remove information, if there exists any long-run relationship between the variables. On the other hand, if the variables are cointegrated then one way is to employ a Vector Error Correction Model (VECM). Let us consider a bivariate VAR(p) where y_t integrated of order 1.

$$y_t = \sum_{j=1}^{p} \Phi_j y_{t-j} + \varepsilon_t$$

In this study y_t represents a $2\text{x}1$ vector of GDP and Trade for each country and both of them are integrated of order 1. This LVAR(p) model in y_t can be equivalently represented in its VECM(p-1) form

$$\Delta y_t = \Pi y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + \varepsilon_t$$

Where $\Pi = -\left(I_m - \sum_{j=1}^{p} \Phi_j \right)$ and $\Gamma_j = -\sum_{i=j+1}^{p} \Phi_i$ for j=1,2, …, (p-1). The matrix Π

provides the information on cointegration. If $y\ I(1)$ then $\det(I_m - \Phi_1 z - ... - \Phi_p z^p)$ for $z = 1$ and Π is singular. To overcome the situation, we consider that the rank (Π) is reduced, i.e. rank $(\Pi) = r > m$, where m is the dimension of y_t . This is known as the cointegrating rank of the system. This enables us to examine three possible cases. If rank $(\Pi) = r = 0$, then $y\ \text{I}(1)$ and there is no cointegrating relationship. In that case VECM(p-1) reduces to a Differenced VAR(p-1) or DVAR(p-1).

$$\Delta y_t = \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + \varepsilon_t$$

If rank $(\Pi) = r = m$, then $y\ \text{I}(0)$ and the series is stationary in levels. In this case suitable representation is the LVAR(p) model.

Finally, if $0 < \text{rank}(\Pi) = \text{r} < \text{m}$, then $y\ \text{I}(1)$ and Π is of reduced rank. There exists 'r' cointegrating vectors or 'r' stationary linear combinations. In this case the matrix Π

is decomposed as $\Pi = \alpha\beta'$, where Π is a mxm, α and β is a mxr matrix. The matrix α contains the error correction or adjustment vectors as they measure the rate of adjustment of the process of y to the disequilibrium error. On the other hand, β is the cointegrating matrix with βy_{t-1} stationary. We may re-write the VECM(p-1) as follows

$$\Delta y_t = \alpha\beta' y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + \varepsilon_t$$

In this paper, we have considered the following VECM(p-1) which assumes $\beta'\delta = 0$ (orthogonal), i.e. no trending in long-run relationships. This is useful when we believe that all series are I(1) with drift but there is no trending in the cointegrating relationship. This follows the Case III in (MacKinnon, Haug, & Michelis, 1999).

$$\Delta y_{i,t} = \Phi_i * \delta_i + \alpha_i(\beta_i y_{i,t-1} - \mu_{0,1}) + \sum_{j=1}^{p_j - 1} \Gamma_{j,i} \Delta y_{i,t-j} + \varepsilon_{i,t} \quad (8)$$

Here, i= Canada, the USA and Mexico respectively. This is a two-step estimation approach. First, we form the estimated error correction vector:

$$z_{i,t-1} = \widehat{\beta_i'} y_{t-1} + \widehat{\mu_{0,i}}$$

This enables us to write the VECM as-

$$\Delta y_{i,t} = \Phi_i * \delta_i + \alpha_i z_{i,t-1} + \sum_{j=1}^{p_j - 1} \Gamma_{j,i} \Delta y_{i,t-j} + \varepsilon_{i,t}$$

This is the usual method to estimate the VECM, as it is linear in parameters given $z_{I,t-1}$.

If there exists any long-run relationship between the variables, estimating a DVAR might be inappropriate. To identify any such relationship between GDP and Trade for each country, we adopt Johansen's test (Johansen, 1991). We perform Johansen's test for each country separately. When there is cointegration, relevant information is ignored by a DVAR(p-1) model contained in the matrix Π. We have a bivariate system for each country, where Π is a (2x2) matrix. When there is cointegration, $\Pi = \alpha\beta'$ where α and β both are (2x1) matrices. If there exists a cointegrating relationship, we shall get as a reduced rank matrix. That is, rank $(\Pi) = \text{rank}(\alpha) = \text{rank} = (\beta) = 1$. This rank is defined as the number of cointegrating vectors. This forms the basis of Johansen's test.

The Johansen's test is the maximum likelihood (ML) estimation approach that aims to determine the rank of Π using a reduced regression technique based on canonical analysis: λ max test and trace test. The λ max test considers the following null and alternative hypothesis:

$$\text{H}_0: \text{ r cointegrating vectors} \quad \text{vs} \quad \text{H}_1: \text{ (r+1) coitegrating vectors}$$

The test statistic is $\lambda_{\max} = -\text{T}\log(1\text{-}\lambda_{r+1})$ which has a nonstandard limiting null distribution. We apply a sequential testing strategy for r = 0, 1 until the null hypothesis is supported for the first time. The trace test considers the null and alternative hypothesis:

$$\text{H}_0: \text{ r cointegrating vectors} \quad \text{vs} \quad \text{H}_1: \text{r} = 2 \text{ cointegrating vectors}$$

The test statistic is trace = $-\text{T}\sum_{i=r+1}^{2} \log(1-\lambda_1)$ which has a nonstandard limiting null distribution. We apply a sequential testing strategy for r = 0, 1 until the null hypothesis is supported for the first time.

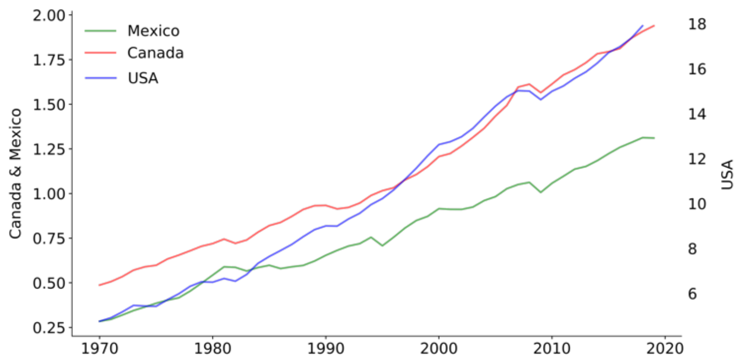
3. Data

In this paper, we consider annual GDP and Trade data in their level forms for three countries: Canada, US and Mexico spanning 50 years (1970-2019).

GDP is constructed by adding the sum of gross value added by all resident producers in the economy and any product taxes and subtracting any subsidies not included in the value of the products. Data are in constant 2010 U.S. dollar (trillions).

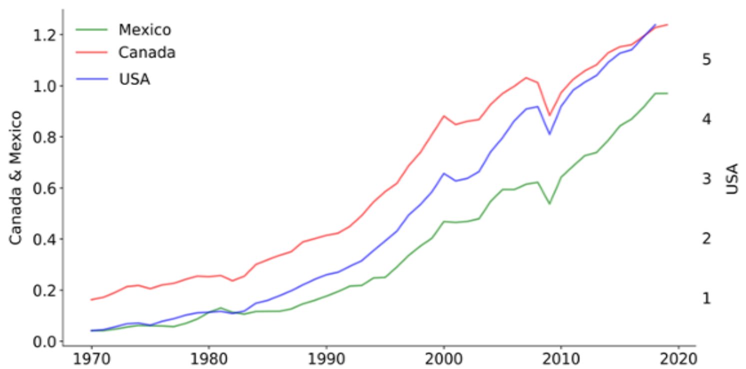
Trade is calculated as the sum of total imports and total exports. Data are in constant 2010 U.S. dollar (trillions). Imports of goods and services represent the value of all goods and other market services received from the rest of the world. Exports of goods and services represent the value of all goods and other market services provided to the rest of the world. Both datasets have been collected from World Development Indicators or The World Bank Databank.

Figure 1. Time series plot of GDP.



Source: Elaborated by the authors.

Figure 2. Time series plot of Trade.



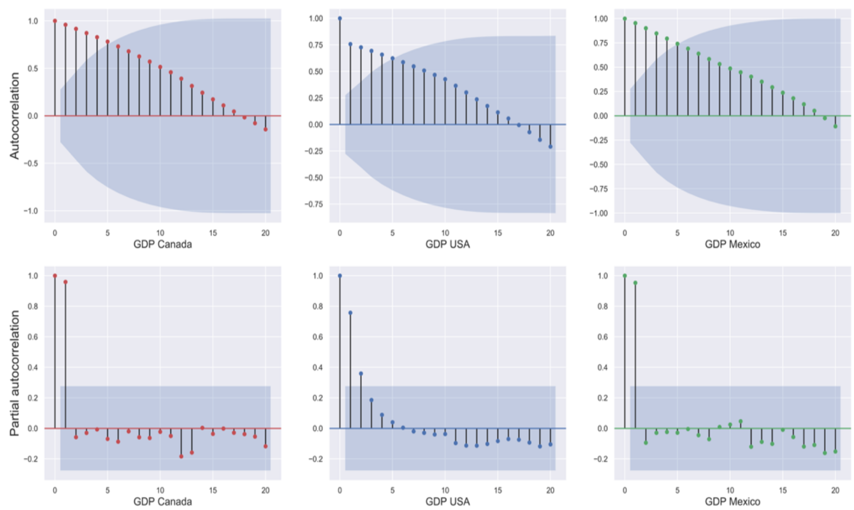
Source: Elaborated by the authors.

Both GDP and Trade exhibit an upward trend for the three countries. There is a sharp dip in both the series in the year 2008-9 during the financial crisis. There might be another structural break in Trade data around the year 2000. The graph also reveals that the data might be non-stationary, which we test formally in the next section.

3.1 Unit root tests

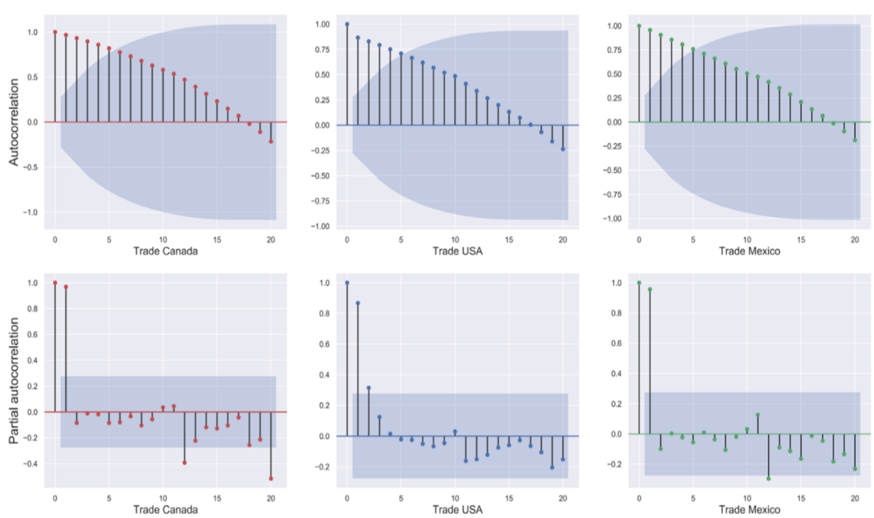
In our study, both GDP and Trade exhibit upward trend and a unit root. After examining the autocorrelation functions (ACF), we say there is strong persistence in both GDP and Trade.

Figure 3. Correlogram of GDP



Source: Elaborated by the authors.

Figure 4. Correlogram of Trade



Source: Elaborated by the authors.

For both GDP and Trade the Autocorrelation functions (ACF) are statistically significant up to 10-12 lags. On the other hand, the Partial Autocorrelation functions (PACF) are statistically significant up to 1 or 2 lags. After a careful examination of the PACFs, we have decided to model an autoregressive process of order 1 for GDP and Trade separately, for Canada, US and Mexico.

Assuming that both GDP and Trade series can be represented by an AR (1) process, we may assume the following DGP (data generating process):

$$\Phi_p(L)(y_t - \mu - \delta t) = \varepsilon_t \quad (9)$$

Where ε_t is assumed to be white noise and $\Phi(L) = \Phi_1(L)$. If this process has a unit root, the appropriate representation (9) is the stochastic trend ARI(p-1,1) model with drift which is as follows:

$$\Delta y_t = \delta^* + \gamma_1 \Delta y_{t-1} + \varepsilon_t \quad (10)$$

When the root is under the unit circle, we may write the deterministic model as follows:

$$y_t = \alpha + \beta t + \varphi_1 y_{t-1} + \varepsilon_t \quad (11)$$

We perform the Augmented Dickey-Fuller (ADF) test (Dickey & Fuller, 1979), Dickey-Fuller with GLS (DF-GLS) detrending (Elliot, Rothenberg, & Stock, 1996) test and finally the KPSS (Kwiatkowski, Phillips, Schmidt, & Shin, 1992) test to examine whether there is a unit root.

For the ADF test the integrating regression is-

$$\Delta y_t = \delta^* + \rho y_{t-1} + \varepsilon_t \quad (12)$$

We then test $H_0 : \rho = 0$ vs $H_1 : \rho < 0$ using the usual t-ratio but a nonstandard limiting null distribution. In other words, the null corresponds to model (10) and the alternative to test to model (11).

(Elliot et al., 1996) propose a simple modification of the ADF test in which the series is detrended prior to running the test regression

$$\Delta y_t^d = \rho y_{t-1}{}^d + \varepsilon_t \quad (13)$$

Moreover, the Elliott, Rothenberg and Stock DF-GLS test has higher power than the ADF test. This test also has the same null hypothesis as the ADF test, which is that the series being tested has a unit root.

Since the ADF test is a low power test, it is usual to undertake a trend stationary test in addition to ADF tests. One such test is the KPSS test. It has the null hypothesis that the series being tested is stationary. In other words, the null corresponds to model (11) and alternative to model (10). The test estimates the null model using least squares and stores the associated residuals, say $\widehat{v_t}$. The next step is to form the partial sum as follows-

$$S_t = \sum_{i=t}^{t} \widehat{v_t}$$

Finally, a Lagrange multiplier (LM) statistic is defined as-

$$\eta = \frac{n^2 \sum_{t=1}^{n} S_t^2}{\widehat{\sigma}_v^2}$$

Table 1. Unit Root Tests

	GDP			Trade		
	Canada	**USA**	**Mexico**	**Canada**	**USA**	**Mexico**
ADF	-1,11	-2,228	-2,325	-2,136	-1,753	-1,307
DF-GLS	-0,88	-1,691	-2,404	-1,61	-0,985	-0,811
KPSS	0.223***	0.171**	0.139*	0.147**	0.228***	0.233***

Note: ADF: Augmented Dickey–Fuller statistics; DF-GLS: Elliot-Rothenberg-Stock DF-GLS statistics; KPSS: Kwiatkowski-Phillips-Schmidt-Shin statistics. *** Significance at 1% level. ** Significance at 5% level. * Significance at 10% level.

Source: Elaborated by the authors.

For the ADF test, the calculated t-ratio for GDP (for all three countries) suggest that we cannot reject the null hypothesis at the 10% nominal level of significance and support that there is a unit root i.e., stochastic trend. Similarly, for Trade, we get t-ratios which imply there is also stochastic trend. We get consistent results for both GDP and Trade having a unit root from the Dickey-Fuller with GLS detrending and the KPSS test. In other words, all the series are integrated of order one or I (1).

3.2 Lag selection

We already know that all of the series are integrated of order one in this study when we apply the ADF, KPSS and DF-GLS tests, allowing for a drift and trend in each series. After setting up a Bivariate VAR for each country, the various information criteria suggest a maximum lag length of 10 for Canada (AIC), 12 for the USA (AIC) and 10 for Mexico (LR). Swartz Criterion (SC) and Hannan Quinn Information Criterion (HQ) suggest 1 lag for Canada, 2 for USA and 1 for Mexico.

Table 2. Lag Length Selection

	Canada	USA	Mexico
AIC	10	12	1
SC	1	2	1
HQ	1	2	1
LR	7	2	10

Source: Elaborated by the authors.

However, to make our VAR model well specified, we have to ensure that there is no serial correlation in the residuals. One possible way to resolve the autocorrelation issue is to increase the number of lags. Hence, we consider country specific VAR models, where we augment the lags each time and apply the Lagrange Multiplier (LM) test to see if there is any serial correlation in the residuals. The null hypothesis of this test is serial independence or no serial correlation at lag h against the alternative of AR(k)/MA(k), for k=1, …, 12.

We find that serial correlation is removed at 5% significance level for almost all the series up to 12 lags when we consider up to 7 lags for Canada, 2 lags for USA and 9 lags for Mexico. Since our data is integrated of order one, we now use the lag-augmented method of testing for Granger Causality following TYDL.

Table 3. VAR Residual Serial Correlation LM Tests

Null Hypothesis: No serial correlation at lag h			
	Canada(p=7)	**USA(p=2)**	**Mexico (p=9)**
Lag	**Prob.**	**Prob.**	**Prob.**
1	0,6542	0,3202	0,2219
2	0,2291	0,5378	0,0815
3	0,1524	0,8246	0,5809
4	0,6364	0,8239	0,7949
5	0,7579	0,585	0,603
6	0,5811	0,9597	0,3598
7	0,2673	0,1532	0,1135
8	0,045	0,359	0,5158
9	0,3163	0,1909	0,1968
10	0,6143	0,3832	0,9268
11	0,0709	0,8303	0,0697
12	0,326	0,3172	0,6927

Note: Each column represents p-values for LM test statistics based on country-specific VAR models with p lags.

Source: Elaborated by the authors

4. Findings & discussion

Table 5 provides us the estimates of the Granger-Causality tests following the TYDL method, based on country-specific VAR models.

Table 5. VAR Granger Causality/Block Exogeneity Wald Tests

CANADA			
Dependent Variable: GDP			
Excluded	Chi-sq	df	Prob
Trade	39,318	7	0
Dependent Variable: Trade			
Excluded	Chi-sq	df	Prob
GDP	36,564	7	0
USA			
Dependent Variable: GDP			
Excluded	Chi-sq	df	Prob
Trade	2,621	2	0,2697
Dependent Variable: Trade			
Excluded	Chi-sq	df	Prob
GDP	4,567	2	0,102
MEXICO			
Dependent Variable: GDP			
Excluded	Chi-sq	df	Prob
Trade	13,546	9	0,1394
Dependent Variable: Trade			
Excluded	Chi-sq	df	Prob
GDP	13,706	9	0,1332

Source: Elaborated by the authors.

We find that there is a bidirectional Granger causality between GDP and Trade for Canada only and Granger non-causality for the USA and Mexico at 5% significance level based on country specific VAR models.[5]

Table 6. System Residual Portmanteau Tests for Autocorrelations

Null Hypothesis: No residual autocorrelation up to lag h

Lag	Prob.
1	0,9768
2	0,9344
3	0,8249
4	0,7968
5	0,9073
6	0,935
7	0,9028
8	0,883
9	0,878
10	0,7585
11	0,6163
12	0,6137

Source: Elaborated by the authors.

Table 6 provides us the System Residual Portmanteau tests for autocorrelation testing the null hypothesis that there is no autocorrelation up to lag h. This test confirms that there is no serial correlation in the residuals in the considered SUR System at 5% significance level. Now we consider testing for Granger noncausality based on the estimates of the VAR in the SUR system.

Table 7. Testing Granger Causality in the SUR system LVAR following TYDL

CANADA			
Dependent Variable: GDP			
Excluded	Chi-sq	df	Prob
Trade	41,538	7	0
Dependent Variable: Trade			
Excluded	Chi-sq	df	Prob
GDP	27,565	7	0,0003
USA			
Dependent Variable: GDP			
Excluded	Chi-sq	df	Prob
Trade	0,127	2	0,9383
Dependent Variable: Trade			
Excluded	Chi-sq	df	Prob
GDP	0,524	2	0,7694
MEXICO			
Dependent Variable: GDP			
Excluded	Chi-sq	df	Prob
Trade	37,086	9	0
Dependent Variable: Trade			
Excluded	Chi-sq	df	Prob
GDP	64,632	9	0

Source: Elaborated by the authors.

[5] (Zestos & Tao, 2002) got similar results in their country specific model for Canada. (McCarville & Nnadozie, 1995) got similar results for Mexico in their country specific model testing for GC between export growth and GDP growth.

Table 7 provides us the estimates of the block exogeneity Wald tests following the TYDL method. Although we observe Granger non-causality between GDP and Trade of Mexico in the country-specific VAR, here we find that there is a bidirectional Granger causality for Mexico after allowing for cross-country contemporaneous correlations. Allowing for the countries to interact in the SUR system, indirectly accounts for trade openness among the three countries. The results make intuitive sense following (Lal, 2017) where he finds unidirectional Granger causality from Foreign Direct Investment to Trade Openness. The USA and Canada produce consistent test results in the sense that the USA still exhibits Granger non-causality and Canada exhibits bidirectional Granger causality for both variables.

Table 8. Share of trade among countries under study

% of Total Trade (1991-2018)	With USA	With Canada	With Mexico
USA	-	17%	12%
Canada	68%	-	3%
Mexico	67%	3%	-
% of Total Trade (2018)			
USA	-	15%	14%
Canada	63%	-	4%
Mexico	61%	3%	-

Source: Elaborated by the authors.

Table 8 shows that from 1991-2018 around 68% of Canada's total trade was undertaken with the USA and 3% with Mexico. For Mexico, around 67% of its total trade happened with the USA and 3% with Canada. For the USA, around 17% of its total trade was done with Canada and 12% with Mexico. This bolsters the idea that there is additional information to be gained in an unrestricted system specification considering the share of each country's total trade occurring among these countries. Our findings are consistent with the results of (Kónya, 2006) who also studied GDP and exports in a panel SUR. (Tekin, 2012) also employs a similar estimation strategy for 18 Least Developed Countries (LDC) to test the relationship between economic growth and foreign direct investment in a SUR system. He makes analogous argument, such as, the United Nation's "Programme of Action" for promoting international development assistance among the LDCs and the similarity in major policy tools among these countries substantiate the need to study them together.

Table 9 (reported in the appendix) shows the test statistics from Johansen's Cointegration test for the three countries under study. The test suggests 1 co-integrating equation for Canada and Mexico and no-cointegration for the USA. We now consider the estimates of the VECM.

Table 10 provides the estimates of the long-run dynamics of VECM from the SUR estimation and country-specific models for comparison. If the cointegrating relationships matter then we should see statistically significant adjustment parameters. For Canada the two adjustment parameters are statistically significant in both country-specific & system framework. It is interesting that the two equations are adjusting almost similarly in the country-specific model but in the system framework Trade is adjusting faster than GDP. Since we did not get any cointegration between GDP and Trade for the USA, we do not consider USA in the analysis of long-run dynamics. On the other hand, though we found long-run relationship for Mexico, the adjustment coefficients in both country-specific and system VECM are not statistically significant at 5% significance level. However, we might say that the adjustment coefficient for GDP is statistically significant at 10 % level of significance in the SUR system. Since we have a constant in our model, it implies that the equilibrium would be zero and hence the adjustment should be negative for the series to come back to equilibrium. However, positive adjustment coefficients may be possible as well, as we can see for Mexico.

Table 10. Estimates of long-run dynamics from VECM

Adjustment coefficient α	Country-specific		SUR System	
	ΔGDP_t	$\Delta Trade_t$	ΔGDP_t	$\Delta Trade_t$
Canada	**-0.193*** **	**-0.197*** **	**-1.322*** **	**-1.550*** **
	-0,039	**-0,055**	**-0,351**	**-0,235**
Mexico	-0,329	0,054	**0.678***	0,082
	-0,185	-0,222	**-0,398**	-0,467

***Significance at 1% level. ** Significance at 5% level. * Significance at 10% level

Source: Elaborated by the authors.

Since both the adjustment coefficients of Mexico are not statistically different from zero at the 5% nominal significance rate, this suggests that there is no co-integrating equation for Mexico. This inference stands in contrast to the findings of Johansen's cointegration test. Some Monte Carlo studies suggest that Johansen's test finds too much cointegration. For example, (Hjalmarsson & Österholm, 2010) investigate the properties of Johansen's maximum eigenvalue and trace tests for cointegration under the empirically relevant situation of near-integrated variables, using Monte Carlo techniques. They find that the probability of reaching an erroneous conclusion regarding the cointegrating rank of the system is generally substantially higher than the nominal size. Maybe we don't really have cointegration even though the test is suggesting that for Mexico. Causality in a VECM comes from two parts. If we have a long-run equilibrium then there is causality from the long-run dynamics. However, we can also get causality from the short-run dynamics. For Mexico we observe bidirectional Granger causality in the system framework. Hence, we can still have causality from the short-run dynamics but not from the long-run. Another interesting observation in the cointegrating vectors is that the magnitudes of all the VECM coefficients are much higher in the system equations than the country-specific models. When we consider the system, we get higher adjustment parameters implying quicker adjustment towards the equilibrium. One important aspect to keep in mind is that causality relations are sensitive to econometric methods, treatment of variables, the time period selected and the number of variables used in the analysis. (Hsiao & Hsiao, 2006).

Table 11. Wald Test of Symmetry in adjustment coefficients

	Test Statistic	df	Prob
$H_0 = \alpha_1 = \alpha_3$	12,471	1	0,0004

	Test Statistic	df	Prob
$H_0 = \alpha_2 = \alpha_4$	8,653	1	0,0033

Source: Elaborated by the authors.

Finally, we conduct Wald tests of symmetry in adjustment coefficient. Table 11 provides us the test estimates. We find that there is no symmetry in the coefficient estimates of the adjustment parameters. This confirms that different countries have different cointegrating vectors and they are not attracting to the same equilibrium space.

5. Conclusion

In this paper, we use a novel approach in studying short-run and long-run dynamics with time-series data by applying system framework on GDP and Trade data of the USA, Canada and Mexico allowing for cross-country contemporaneous correlations. When we look at links between GDP and Trade for individual countries based on country-specific models, we are losing some information and the results can really change if we look at more than one country simultaneously. We get bidirectional Granger causality between GDP and Trade for Canada in both frameworks and for Mexico in system framework only. The USA displayed Granger non-causality and an absence of long-run equilibrium between GDP and Trade. Canada portrays consistent results with statistically significant long-run adjustment parameters while Mexico shows statistically insignificant adjustment coefficients at 5% nominal significance level.

While most time-series studies exploring the links between GDP and Trade consider country-specific models, our results show that the magnitudes of the coefficients along with their statistical significance can change when cross-country contemporaneous correlations are allowed (Kónya, 2006). Another way that most of the literature have gone is these panel approaches that impose restrictions which are too restrictive and perhaps a system with unrestricted coefficients might be a better way to model. Panel studies impose coefficients that are identical across all countries. Our results even just for Canada, the USA and Mexico really highlight that looking at a panel with constant coefficients is really not a good way to go.

This research leaves some interesting avenues to pursue for further study. A structural system compared to reduced form system can be considered. Additionally, other groups of countries such as those in the European Union and those in Trans-Pacific Partnership, that trade heavily among themselves, maintain multilateral trade relationships and are more likely to have common shocks in their errors can be studied to further check the robustness & gains from introducing a cross-country system estimation in time-series studies. Further, structural break unit root tests and the implications of structural break on the results can be explored.

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Appendix

Table 9 A. The Johansen's cointegration test (for Canada)

Hypothesized no. of CE(s)	The trace test		0.05 Critical value	p-value
	Eigen value (λ)	Trace statistic		
$H_0: r=0$	0,455	28,335	15,495	0,0004
$H_1: r=1$	0,051	2,243	3,841	0,1342
Hypothesized no. of CE(s)	The λ max test		0.05 Critical value	p-value
	Eigen value (λ)	Max-Eigen statistic		
$H_0: r=0$	0,455	26,092	14,26	0,0005
$H_1: r=1$	0,051	2,243	3,841	0,1342

Source: Elaborated by the authors.

Table 9 B. The Johansen's cointegration test (for USA)

Hypothesized no. of CE(s)	The trace test		0.05 Critical value	p-value
	Eigen value λ	Trace statistic		
$H_0: r=0$	0,1756	9,117	15,495	0,3546
$H_1: r=1$	0,0009	0,044	3,842	0,8335
Hypothesized no. of CE(s)	The λ max test		0.05 Critical value	p-value
	Eigen value λ	Max-Eigen statistic		
$H_0: r=0$	0,1756	9,073	14,265	0,2801
$H_1: r=1$	0,0009	0,044	3,841	0,8335

Source: Elaborated by the authors.

Table 9 C. The Johansen's cointegration test (for Mexico)

Hypothesized no. of CE(s)	The trace test		0.05 Critical value	p-value
	Eigen value λ	Trace statistic		
$H_0: r=0$	0,3895	21,12	15,495	0,0064
$H_1: r=1$	0,0214	0,887	3,841	0,3462
Hypothesized no. of CE(s)	The λ max test		0.05 Critical value	p-value
	Eigen value λ	Max-Eigen statistic		
$H_0: r=0$	0,3895	20,233	14,265	0,0051
$H_1: r=1$	0,0214	0,887	3,841	0,3462

Source: Elaborated by the authors.