

Haugen, Kjetil

Article

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Economics and Business Letters

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Reference: Haugen, Kjetil (2023). The doping dilemma is not the only dilemma in sport. In: Economics and Business Letters 12 (1), S. 40 - 48.
<https://reunido.uniovi.es/index.php/EBL/article/download/18930/15577/57021>.
doi:10.17811/ebl.12.1.2023.40-48.

This Version is available at:

<http://hdl.handle.net/11159/15828>

Kontakt/Contact

ZBW – Leibniz-Informationszentrum Wirtschaft/Leibniz Information Centre for Economics
Düsternbrooker Weg 120
24105 Kiel (Germany)
E-Mail: [rights\[at\]zbw.eu](mailto:rights[at]zbw.eu)
<https://www.zbw.eu/>

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The doping dilemma is not the only dilemma in sport

Kjetil Haugen*

Faculty of Business Administration and Social Sciences, Molde University College,
Specialized University in Logistics, Molde, Norway

Received: 13 September 2022

Revised: 19 October 2022

Accepted: 26 October 2022

Abstract

This article investigates training in sports and argues (and demonstrates) that training (game-theoretically) works exactly as doping do. That is, the Nash equilibrium is, under reasonable assumptions, a Prisoner's Dilemma outcome. Furthermore, several other performance improving categories within sport are examined, and proven to behave similarly. Finally, a link to general competitive economic activity is examined and proven NOT (necessarily) to have similar characteristics. The article also discusses (initially) the Prisoner's Dilemma game and its lack of a sensible definition. Such a definition is proposed, although this proposition is not considered the main finding in the paper.

Keywords: game theory; prisoner's dilemma; economics of doping

JEL Classification Codes: L83, Z20, Z28

1. Introduction

1.1. Background

The Prisoner's Dilemma is among the best-known games ever. First introduced by Tucker (1983)¹, the concept has been discussed in numerous research articles. See for instance Axelrod (1980), Rapoport et.al. (1965), Krebs et. Al (1982), for some noteworthy contributions.

1.2. The Prisoner's Dilemma

The Prisoner's Dilemma (PD) game is defined in Investopedia (2021) as (quote) "*a game in which two individuals acting in their own self-interests do not produce the optimal outcome.*" This somewhat paradoxical outcome is probably the reason why the game has become so popular within many scientific disciplines – it can explain why things can go wrong. The fact

* Corresponding author. E-mail: Kjetil.Haugen@himolde.no.

Citation: Haugen, K. (2023) The doping dilemma is not the only dilemma in sport, *Economics and Business Letters*, 12(1), 40-48.

DOI: 10.17811/eb1.12.1.2023.40-48

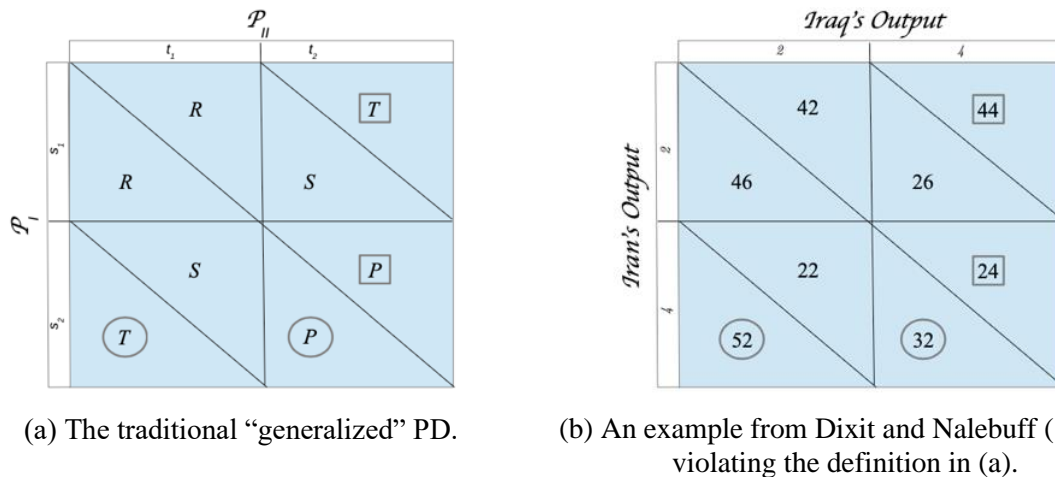
¹ This is the same Tucker as in the "Kuhn-Tucker" or later Karush-Kuhn-Tucker conditions Kuhn and Tucker (1951), Karush (1939). In fact, the story is slightly more complex. Tucker's real contribution seems to be the construction of the story around the game, the one with Prisoners in separate cells and the option of penalty discounts, while Flood and Dresher came up with the original numerical example, see Wikipedia (2021).

that competition not always produce the “right answer” is of course contrasting for economists with strong belief in free markets, as the PD tells us that free-market competition not always produce desired results.

More formally, the PD game, with the added assumptions of a two-player, two-strategy situation, is often defined – See for instance Yao and Darwen (1995), Kolokoltsov and Malafeev (2010), Jiawei and Kendall (2010) or Wikipedia (2021), by the imperfect information game (a) to the left in Figure 1 and the added constraints:

$$T > R > P > S \quad (1)$$

Figure 1. The traditional definition of a PD game is unsatisfactory.



As can be observed by this game, and the added constraints in inequalities (1), the Nash equilibrium (NE), $\{s_2, t_2\}$ is a unique pure strategy NE, and as $P < R$, it satisfies the textual definition above, see Investopedia (2021). However, the (by the way very instructional) example (b) from Dixit and Nalebuff (1991) also satisfies the definition in Investopedia (2021), but it does not fit into the “generalized” framework of the definition in figure 1 (a)³.

The consequence should be obvious. To the best of this authors knowledge, the “generalized” PD-definition in figure 1 (a) is the only mathematical definition of a PD game existing in research literature. Clearly, as figure 1 indicates, it is not adequate. Consequentially, a different, more appropriate definition is needed.

It seems sensible to start with a completely general game⁴ as indicated in Figure 2.

To secure that $\{s_2, t_2\}$ is a NE, the two inequalities (2) are necessary:

$$a_{22} > a_{12} \text{ and } b_{22} > b_{12} \quad (2)$$

Furthermore, to secure uniqueness – to avoid the “Chicken situation”⁵ – the following must hold:

$$\text{Either } a_{11} > a_{21} \text{ or } b_{11} < b_{12} \quad (3)$$

² This example contains a simplified version of OPEC. Here with only IRAN and IRAQ as members and players in a two-player simultaneous game. Both players choose quantity competition through choosing high (4 units) or low (2 units) production amounts. A demand curve is given producing price outcomes for the total production amounts $\{4, 6, 8\}$ as $\{25, 15, 10\}$. Additional extraction costs of \$2 for IRAN and \$4 for IRAQ produces the profit numbers in Figure 1. (b).

³ This is easily observed by noting that figure 1 (b) contains eight different numbers, while figure 1 (a) only allows four.

⁴ We restrict the situation, without loss of generality, to a two-player, two-strategy game of imperfect information

⁵ The “Chicken situation” refers to a game of separation with 3 Nash equilibria, two in pure and one in mixed strategies. Refer for instance to Rapoport and Chamma (1966).

Figure 2. A general two-player, two-strategy game.

		\mathcal{P}_{II}	
		t_1	t_2
\mathcal{P}_I	s_1	a_{11} b_{11}	a_{12} b_{12}
	s_2	a_{21} b_{21}	a_{22} b_{22}

Then, (2) and (3) secure that $\{s_2, t_2\}$ is a unique NE. Furthermore, it is necessary to secure that $\{s_1, t_1\}$ is a Pareto improvement compared to $\{s_2, t_2\}$. There are two possibilities.

$$\text{Either } a_{22} < a_{11} \text{ and } b_{22} < b_{11} \text{ or } a_{22} + b_{22} < a_{11} + b_{11} \quad (4)$$

The left conditions in (4) might perhaps be named a *strong PD*, as both players individually are better off in the $\{s_1, t_1\}$ than in the $\{s_2, t_2\}$ NE. The right part in (4) involves necessities for side-payments, and may hence be referred to as a *weak PD*.

1.3. Economics of Doping

The term *Economics of Doping* was probably introduced by Berentsen (2002) in the article with the same name in 2000. The term has become common, although a better name maybe could be *Game Theoretic Approaches to Doping in Sport*. This happens to be the name of the excellent survey on the matter published by Breivik (2013). Breivik was in fact also the real architect behind this fascinating research area, as he was the first to identify doping as a PD-situation, see Breivik (1987).

Using the notation of Haugen (2004), the doping-PD is easily explained. Think about two equally good athletes (clones) with one simple decision to make; *take drugs* or *not*. If an assumption of a prize a given to the winner of a certain contest between the two athletes, as well as a cost c involved in taking drugs (side-effects, bad conscience etc.) and a probability r of being revealed as a doper, the gaming situation is as follows: If both athletes say no to drugs, they will (expectedly) end up with $\frac{1}{2}a$ each. However, if both choose to take drugs, they will still split the a evenly⁶, but also incur expected costs of rc . If only one of the athletes takes the drug, and a very simplifying assumption on an effective drug is added, the drug taker will win (the whole a) while the clean athlete loses and ends up with nothing. The situation is illustrated in Figure 3.

In Figure 3, under the assumption of $a > 2rc$, the best replies (ellipse for AGENT 1, and rectangle for AGENT 2 in figure 3 indicate the classical result; everybody cheats, or the unique pure strategy $\{D, D\}$ NE. This model is basically just a mathematical formalization of Breivik's pioneering work from 1987.

Surely, the simplicity of assumptions underlying the model described in figure 3 calls for more research. Breivik (2013) calls this (quote) "*Economists enter the scene*". Now, more focus on (quote), "*how different forms of deterrence such as detection possibilities, penalties and exclusion influenced the decision to dope or not to dope*", is introduced.

⁶ They are still clones, and the drug will presumably have the same effect on both

Figure 3. The performance-enhancing drug game – from Haugen (2004).

		<i>AGENT</i> 2	
		D	ND
<i>AGENT</i> 1	D	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border: 1px solid black; border-radius: 50%; padding: 10px;">$\frac{1}{2}a - rc$</div> <div style="border: 1px solid black; padding: 10px;">$\frac{1}{2}a - rc$</div> </div>	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border: 1px solid black; border-radius: 50%; padding: 10px;">$a - rc$</div> <div>0</div> </div>
	ND	<div style="display: flex; justify-content: space-around; align-items: center;"> <div>0</div> <div style="border: 1px solid black; padding: 10px;">$a - rc$</div> </div>	<div style="display: flex; justify-content: space-around; align-items: center;"> <div>$\frac{1}{2}a$</div> <div style="border: 1px solid black; border-radius: 50%; padding: 10px;">$\frac{1}{2}a$</div> </div>

Breivik (2013) discusses research by Berentsen (2002), Berentsen and Lengwiler (2003), Berentsen et. al. (2008), Haugen (2004), Haugen et. al. (2013), Haugen and Petroczi (2012), Eber (2008), Stowe and Gilpatrick (2010) and Ryvkin (2013) as relevant examples of this line of research. The interested reader is referred to Breivik's survey, Breivik (2013) or the original articles referenced above, as this content is of minor relevance here.

The rest of the article is organized as follows: In section 2, a simplified training game is formulated and analyzed, showing the extreme similarity with doping games. In section 3 it is argued that almost all sporting activity involving legal (or illegal) performance-enhancement activity constitute dilemmas. In section 4 an argument is made on the peculiarity of sports in this case, and hence why other economic activity not necessarily give similar dilemmas. In section 5, the case of technological evolution is particularly examined, due to its role in general economic theory. Here, it is argued that the individualization of the economy that sports to some extent represents may be a driver for technological change. Section 6 concludes.

2. The training dilemma

Now, a training game between two (top-level) athletes is investigated. The following assumptions are necessary:

- A.1 Two (top-level) athletes are competing against each other in some upcoming contest.
- A.2 The actual decisions for the players of the game are choice of training. As simplified as possible, the strategic choices are either to train, or not train, and the training method available is the same for both athletes. The choice to perform training involves some cost, C .
- A.3 Again, maximally simplified; the two athletes are assumed to be perfectly equally good (clones).
- A.4 They compete for some utility, a , received by the contest winner. The loser receives nothing.
- A.5 Training works perfectly here, so if one of the athletes chooses to train while the other does not, she wins the contest with certainty.
- A.6 The athletes cannot communicate their choice to each other before the contest starts.

- A.7 Both athletes maximize expected utility.

Surely, most assumptions A.1 to A.7 are not very realistic. Most real sports events involve more than two agents. In the real world, both training amounts as well as methods involve far more combination possibilities than the simplistic assumption of training/no training. Although sports prize-functions typically are quite progressive, the extreme winner-takes-all assumption here is nor very realistic. Clones do not exist, neither do perfect training. However, the point here is not realism in the assumptions, but rather a formulation at a similar precision level as the doping game in figure 3.

Now, given the assumptions – A.1 - A.7, the pay-off contents of a two player (A.1) simultaneous imperfect information game (A.6) are easily explained. (A.3) and (A.4) lead to splitting of the prize as equal quality among the athletes leads to a 50% probability of win or loss for both players. Hence expected (positive) utility (A.7) is $\frac{1}{2} \cdot a + \frac{1}{2} \cdot 0 = \frac{1}{2}a$ to both players for the {No Training, No Training}-situation. If both players train, they receive $\frac{1}{2}a$, but certain costs C (A.2) must be subtracted. In the two remaining situations the athlete who chooses the training decision ends up winning (the whole prize) a (A.5).

Then, all pay-offs in Figure 4 are explained. In order to explain the best replies and ultimately the NE, inequality (5) must be satisfied:

$$a > 2C^7 \quad (5)$$

Figure 4⁸. The training game.

		ATHLETE 2	
		Training	No Training
ATHLETE 1	Training	$\frac{1}{2}a - C$ $\frac{1}{2}a - C$	0 $a - C$
	No Training	$a - C$ 0	$\frac{1}{2}a$ $\frac{1}{2}a$

Reasons for why it is reasonable to assume that (5) is satisfied in the doping situation are well explained in Haugen (2004). The main point relates to the fact that the economics of sports is characterized by 'very few earning very much and very many earning almost nothing'. So, if

⁷ Rearranging equation (5) may prove helpful: $a > 2C \Rightarrow \frac{1}{2}a > C$, which interprets nicely to Expected return bigger than costs. This expression can also be interpreted as a game participation constraint.

⁸ It is straightforward to realize that the game of Figure 4 is simply a special case ($r = 1$ and $c = C$) of the doping game in Figure 3.

we limit our analysis to the very top athletes, it is more than reasonable to assume that (5) is satisfied also in the training case.

Now, if Figures 3 and 4 are compared, an obvious similarity is identified. In fact, the only essential difference is that the doping model in Figure 3 contains the term rc , as opposed to the training model in Figure 4 which contains C at the same spot. Surely, this is obviously just a cosmetic difference. So, this simple example has shown, that training “behaves” just as doping for athletes involved in professional sport.

3. Nothing but dilemmas in sport

The similarity between sports training and doping may induce a suspicion that performance enhancement in sports may induce PD games in general. In this section, a short discussion on such a generality is conducted. The setting in performance enhancement is characterized by an “investment” or resource usage (C) in order improve performance to improve or maximize the probability of winning the contest (a) – see e.g. Sloane (1971), Akerlof (1976) and Sloane (2015). Surely, drugs or training is simply two examples on such situations. Think about the following alternative performance enhancement options:

- Hire a new (and more expensive) medical assistant or physical therapist.
- Induce more R & D to improve training methods.
- Start a new project establishing better performance-enhancing diets.
- Change your preferred lifestyle to improve performance.
- Hire a new, more expensive, and better trainer.
- Develop new running shoes, that makes it easier to run faster.

All examples above are characterized by relatively modest “investments” – C , and at the top level, very big a 's. Hence, a game formulation like the one leading to figure 4 will work. Consequently, all these situations will lead to PD games. Some of these may of course be of a more positive nature than doping, but they are still PD's. As sports, at least from the athlete's perspective basically is related to the topic, performance-enhancement, it seems obvious that the section heading holds – sports is filled with dilemmas. Or if one like to state it differently: The competition between athletes forces them, repeatedly, to engage in non-preferred activities to continue being a part of the game.

4. The main difference between sports and the rest of the economy

In view of the previous section, it is tempting to transfer the acquired knowledge from the sports economic scene into the general economic scene. After all, competitive economics is about improving “product performance”, and the “investment/return” description above is surely quite similar for all economic activity. However, there is a major difference. A difference observable in inequality (5). This inequality states that $a > 2C$. If it is not satisfied, i.e. if $a < 2C$, no dilemmas emerge. On the positive side, there is no doping in sports, but on the negative side, nor any training or technological evolution will take place in sports.

In a so-called well-functioning competitive economy, prices should be close to marginal costs, and profits should be small. In fact, $a > 2C$ means (in a finance perspective) that returns must be larger than 100%. Such profits are, maybe apart from the oil and drug cartel businesses, rare. Therefore, it is easy to realize that this is a peculiar phenomenon in sports⁹.

⁹ Surely, similar phenomena are not restricted to sports only. They are to be expected in other artistic occupations, like music, literature, film, and visual art.

5. The case of technological change

The case of technological evolution is especially interesting. Many economists have argued that technological evolution is imperative for economic growth – see e.g. Carlaw and Lipsey (2003), Mowery and Rosenberg (1989) and Nelson (2005). The role of the PD game in this setting is however more sparsely treated. One noteworthy exception is the work of Benjamini and Gafni (1986). Here, the authors identify the link between technological evolution and the PD game within the framework of a case from medical technology.

In a perfectly competitive economy, incentives for technological evolution are meagre – recall the previous arguments of $a < 2C$ and the absence of dilemmas forcing technological evolution. If we look at today's economic trends, see for instance ElFay (2021), with Bloggers, Influencer's, Youtuber's, TikToker's or Instagramer's, it seems safe to predict a more individualized economy in some near future. In older days, talent, skill, training, and ascetic life was necessary to earn the big prizes in sports. Nowadays, it seems to be enough with simple creativity, a cellphone camera, and an internet connection to succeed in the new individualized economy. Such a development will create Dilemmas (of course as long as $a > 2C$ which does not seem unlikely as followers on social media seem to be more or less perfectly correlated to wages.) and these dilemmas will help technological evolution. Hence in some near future, the paradox of lack of incentives for technological evolution may be softened if the future will bring further individualization in the economy.

6. Conclusions

The fact that a simple training game works similarly as a doping game is the main finding in this article. Furthermore, simple argumentation leads to the conclusion that almost all athlete activity also involves PD-situations. This argument relies on the satisfaction of the inequality $a > 2C$, which typically holds on the sports scene. This is not necessarily the case in more competitive markets, where profits are lower and typically $a < 2C$. The special role of technological evolution, which also constitutes a PD-situation, is also discussed. Here, the somewhat interesting conclusion is that in a modern individualized economy, Prisoner's dilemmas may drive technological evolution and possible economic growth better than in the "old" competitive service and manufacturing economy. Maybe sports economic theory may have more to offer than data as claimed by Bar-Eli et. al. (2020).

The proposed redefinition of the Prisoner's Dilemma game may also have merit, although it is – from the author's point of view – not the main finding.

The obvious duality of Prisoner's Dilemma games is perhaps slightly under-communicated in this article. The OPEC example in Figure 1b is a dilemma for OPEC realizing they could increase profits by colluding on lowering their production to obtain higher prices and profits. It is however not a dilemma for consumers (society) who can enjoy continued low petroleum prices. If OPEC decides to collude – which they of course did back in 1970 – they remove their own dilemma, but produce a big one for society, as consumers must pay a price significantly above the competitive market price. Hence, Prisoner Dilemma games are typically not negative for all economic agents.

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