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Stock Market Volatility and Multi-Scale Positive and Negative Bubbles

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Abstract

We study whether booms and busts in the stock market of the United States (US) drives its volatility. Given this, first, we employ the Multi-Scale Log-Periodic Power Law Singularity Confidence Indicator (MS-LPPLS-CI) approach to identify both positive and negative bubbles in the short-, medium, and long-term. We successfully detect major crashes and rallies during the weekly period from January 1973 to December 2020. Second, we utilize a nonparametric causality-in-quantiles approach to analyze the predictive impact of our bubble indicators on daily data-based weekly realized volatility (*RV*). This econometric framework allows us to circumvent potential misspecification due to nonlinearity and instability, rendering the results of weak causal influence derived from a linear framework invalid. The MS-LPPLS-CIs reveal strong evidence of predictability for *RV* over its entire conditional distribution. We observe relatively stronger impacts for the positive bubbles indicators, with our findings being robust to an alternative metric of volatility, namely squared returns, and weekly realized volatilities derived from 5 (*RV5*)- and 10 (RV10)-minutes interval intraday data. Furthermore, we detect evidence of predictability for *RV5* and *RV10* of nine other developed and emerging stock markets. Finally, we also find strong evidence of causal feedbacks from *RV5* and *RV10* on to the MS-LPPLS-CIs of the 10 countries considered. Our findings have significant implications for investors and policymakers.

JEL Classification: C22, G15

Keywords: Multi-Scale Positive and Negative Bubbles; Realized Volatility; Nonparametric Causality-in-Quantiles Test; International Stock Markets

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1. Introduction

As pointed out by Poon and Granger (2003) and Rapach et al., (2008), return volatility is a key component of asset valuation, hedging decisions, and portfolio optimization models. Inaccurate predictions of volatility may lead to mis-pricing in financial markets, over/underhedged business risks and incorrect capital budgeting decisions, with significant repercussions on earnings and cash flows. Naturally, modeling and predicting stock market volatility is of paramount interest not only for investors and corporate decision makers, but also for policy makers who need to assess the implications of financial fundamentals and investor confidence for the financial cycle and macroeconomic fluctuations.

Given the importance of monitoring volatility, we shed light on the link between stock market volatility of the United States (US) and bubbles over the 1st week of January, 1973 to 2nd week of September, 2020. The theoretical grounding our investigating is the insight that negative (positive) returns are generally associated with upward (downward) revisions of volatility. The explanation for this inverse association, originally put forward by Black (1976), is based on the so-called "leverage effect". This effect implies that when asset prices decline, firms become more leveraged because their debt-to-equity ratio rises, which, in turn, lets the leverage of their capital structures rise. The increased leverage deteriorates the financial state of companies and, as a result, the systematic risk of common stocks, and potentially also of banks, increases.[1] A similar effect may arise even when a firm has no or almost no debt because of the presence of a so- called "operating leverage" (fixed costs that cannot be eliminated, at least in the short run, hence when expected revenues fall, profit margins decline as well). Furthermore, models developed by Blanchard and Watson (1982) and Flood and Hodrick (1986), while providing a theoretical background for the variance bounds test used to test for market efficiency or inefficieny, and more recently Rotermann and Wifling (2014), have demonstrated that (excess) volatilities of asset prices (compared with those of market fundamentals) could be attributed to rational and speculative bubbles, as empirically confirmed historically by Brunnermeier and Oehmke (2013).

Against this theoretical background, we employ the Log-Periodic Power Law Singularity (LPPLS) model (Johansen et al., 1999, Johansen et al., 2000, Sornette, 2003) for the detection of both positive (upward accelerating price followed by a crash) and negative (downward accelerating price followed by a rally) bubbles. We then apply the Multi-Scale (MS) LPPLS Confidence Indicators (CIs) from Demirer et al. (2019) to characterize positive and negative bubbles at different time scales, specifically short-, medium-, and long-term. These correspond to estimation windows associated with trading activities over one-month to three-months, three-months to a year, and one-year to two-years, respectively. It is important to emphasize at this stage that the identification of both positive and negative multi-scale bubbles is not possible using other available statistical tests (see Balcilar et al. (2016), Zhang et al. (2016), and Sornette et al. (2018) for detailed reviews). We consider these aspects, i.e., nature and time-scale of bubbles important from two perspectives. First, they allow the possible asymmetric predictive effect of so-called positive- and negative-news, resulting from the positive and negative bubbles, to be assessed. Second, crashes and recoveries at alternative horizons can convey differential information for different market participants, as suggested by the Heterogeneous

[1] Christie (1982) provided a theoretical explanation of leverage effect under a Modigliani and Miller (1958) economy.

Market Hypothesis (HMH; Müller et al., 1997). The HMH states that different types of market agents (that is, investors, speculators and traders) populate asset markets, and that these different types of market agents differ in their sensitivity to information flows at different time horizons. In particular, traders and speculators are likely to react sensitive to short- and medium-term bubbles, whereas investors are possibly going to be more concerned with long-term bubbles.

After obtaining six stock market bubble indicators for the US, we analyze the predictive impact of the MS-LPPLS-CIs on stock returns volatility using the nonparametric causality-in-quantiles test proposed by Jeong et al. (2016). At this stage, it must emphasized that we measure stock market volatility using realized volatility (RV), which, we capture by the sum of daily and intraday squared returns over a week (following Andersen and Bollerslev, 1998). RV is known to provide an accurate, observable, and unconditional metric of volatility (unlike generalized autoregressive conditional heteroscedasticity (GARCH) and stochastic volatility (SV) models), which is otherwise a latent process (McAleer and Medeiros, 2008). The nonparametric causality-in-quantiles framework has the key advantage that it enables us to detect predictability across the entire conditional distribution of RV, resulting from the MS-LPPLS-CIs, while simultaneously controlling for misspecification due to potential nonlinearity and structural breaks in these relationships, for which we provide statistical evidence. In other words, the nonparametric causality-in-quantiles framework simultaneously controls for misspecification due to nonlinearity and regime changes, compared to conditional mean-reliant nonlinear and/or nonparametric causality tests (see, for example Hiemstra and Jones (1994), Diks and Panchenko (2005, 2006), Nishiyama et al. (2011)). Moreover, given the presence of a fat tail in the unconditional distribution of RV, a quantiles-based nonparametric predictive approach is more relevant in our context. The nonparametric causality-in-quantile framework, thereby, is a more elaborate procedure for detecting causality at each point of the conditional distribution of RV, capturing the existence or non-existence of predictability due to MS-LPPLS-CIs at various quantiles of the distribution of RV. This makes the test inherently time-varying in nature. As a more general framework, our method is more likely to identify causality at specific quantiles when conditional mean-based tests may fail. Additionally, because we do not need to determine the number of regimes as in Markov-switching models of causality (Ben Nasr et al. 2015; Balcilar et al. 2018) and can test for predictability at each point of the conditional distribution characterizing specific RV-regimes, our method does not suffer from any misspecification in terms of specifying and testing for the optimal number of regimes.

To the best of our knowledge, this is the first paper to analyze the high-frequency predictive impact of multi-scale positive and negative bubbles (as captured by the MS-LPPLS-CIs) of the US stock market on its (realized) volatility using a nonparametric quantiles-in-causality approach. Having said this, though the focus is on the US, we also consider the predictive impact of MS-LPPLS-CIs on intraday data-based RVs for a set of developed and emerging stock markets, namely Brazil, Canada, China, France, Germany, India, Italy, Japan and the United Kingdom (UK). In this regard, the choice of these countries was driven by data availability, as well as due to the importance of the risks and uncertainties of these countries, captured by RV, in defining the sustainability of the global financial system.

The remainder of the paper is organized as follows: Section 2 outlines the methodologies associated with the detection of bubbles and the nonparametric causality-in-quantiles test.

Section 3 is devoted to the discussion of the data. Section 4 presents the empirical results, and Section 5 concludes the paper.

2. Methodologies

2.1. Estimating the Multi-Scale Log-Periodic Power Law Singularity (LPPLS) Model[2]

In this sub-section, we discuss the econometric framework that we utilize to detect our multiscale positive and negative bubbles indicators. Utilizing the LPPLS model, we adopt the stable and robust calibration scheme developed by Filimonov and Sornette (2013):

$$\ln E[p(t)] = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t)^m - \phi) \tag{1}$$

The parameter t_c represents the critical time (the date of the termination of the bubble). A represents the expected log value of the observed time-series, i.e., the stock price-dividend ratio, at time t_c. B represents the amplitude of the power law acceleration, C represents the relative magnitude of the log-periodic oscillations, and the exponent of the power law growth is given by m. The frequency of the log-periodic oscillations is given by ω and ϕ represents a phase shift parameter.

Like Filimonov and Sornette (2013), equation (1) is reformulated so as to reduce the complexity of the calibration process by eliminating the nonlinear parameter ϕ and expanding the linear parameter C to be $C_1 = C\ \cos\phi$ and $C_2\ =\ C\ \cos\phi$. The new formulation can be written as

$$\ln E[p(t)] = A + B(f) + C_1(g) + C_2(h) \tag{2}$$

where

$$f = (t_c - t)^m$$
$$g = (t_c - t)^m \cos[\omega \ln(t_C - t)]$$
$$h = (t_c - t)^m \sin[\omega \ln(t_c - t)]$$

To estimate the 3 nonlinear parameters: $\{t_c, m, \omega\}$, and 4 linear parameters: $\{A, B, C_1, C_2\}$, we fit equation (2) to the log of the price-dividend ratio. This is done by using the L^2 norm to obtain the following sum of squared residuals:

$$F(t_c, m, \omega, A, B, C_1, C_2) = \sum_{i=1}^{N} \left[\ln p(\tau_i) - A - B(f_i) - C_1(g_i) - C_{2(h_i)}\right]^2 \tag{3}$$

Because the estimation of the 3 nonlinear parameters depends on the four linear parameters, we have the following cost function:

$$F(t_c, m, \omega) = \min_{A,B,C_1,C_2} F(t_c, m, \omega, A, B, C_1, C_2) = F\left(t_c, m, \omega, \hat{A}, \hat{B}, \hat{C}_1, \hat{C}_2\right) \tag{4}$$

The 4 linear parameters are estimated by solving the optimization problem:

[2] The discussion of the MS-LPPLS-CIs approach draws heavily from Demirer et al. (2019), Caraiani et al. (2023), Gupta et al. (2023), and van Eyden et al. (2023).

$$\{\hat{A}, \hat{B}, \hat{C}_1, \hat{C}_2\} = \arg \min_{A,B,C_1,C_2} F(t_c, m, \omega, A, B, C_1, C_2) \tag{5}$$

which can be done analytically by solving the matrix equation given by:

$$\begin{pmatrix} N & \sum f_i & \sum g_i & \sum h_i \\ \sum f_i & \sum f_i^2 & \sum f_i g_i & \sum f_i h_i \\ \sum g_i & \sum f_i g_i & \sum g_i^2 & \sum g_i h_i \\ \sum h_i & \sum f_i h_i & \sum g_i h_i & \sum h_i^2 \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{B} \\ \hat{C}_1 \\ \hat{C}_2 \end{pmatrix} = \begin{pmatrix} \sum \ln p_i \\ \sum \mathrm{f_i} \ln p_i \\ \sum \mathrm{g_i} \ln p_i \\ \sum \mathrm{h_i} \ln p_i \end{pmatrix} \tag{6}$$

Next, the 3 nonlinear parameters can be determined by solving the following nonlinear optimization problem:

$$\{\hat{t}_c, \widehat{m}, \widehat{\omega}\} = \arg \min_{t_c,m,\omega} F(t_c, m, \omega) \tag{7}$$

We use the Sequential Least Squares Programming (SLSQP) search algorithm (Kraft, 1988) to estimate the three nonlinear parameters $\{t_c, m, \omega\}$.

The LPPLS confidence indicator, introduced by Sornette et al. (2015), measures the sensitivity of bubble patterns in the log price-dividend ratio time series. The larger is the LPPLS confidence indicator (CI), the more reliable is the LPPLS bubble pattern, and vice versa. It is calculated by calibrating the LPPLS model to shrinking time windows by shifting the initial observation t_1 forward in time towards the final observation t_2 with a step dt. For each LPPLS model fit, the estimated parameters are filtered against established thresholds and the qualified fits are taken as a fraction of the total number of positive or negative fits. A positive fit has estimated $B < 0$ and a negative fit has estimated $B > 0$.

As in the work of Demirer et al. (2019), we incorporate bubbles of varying multiple time-scales into this analysis and sample the time series in steps of 5 trading days. We create the nested windows $[t_1, t_2]$ and iterate through each window in steps of 2 trading days. In this way, we obtain a weekly resolution, based on which we construct the following indicators:

- Short-term bubble: A number $\in [0,1]$ which denotes the fraction of qualified fits for estimation windows of length $dt := t_2 - t_1 \in [30:90]$ trading days per t_2. This indicator is comprised of $(90 - 30)/2 = 30$ fits.

- Medium-term bubble: A number $\in [0,1]$ which denotes the fraction of qualified fits for estimation windows of length $dt := t_2 - t_1 \in [30:90]$ trading days per t_2. This indicator is comprised of $(300 - 90)/2 = 105$ fits.

- Long-term bubble: A number $\in [0, 1]$ which denotes the fraction of qualified fits for estimation windows of length $dt := t_2 - t_1 \in [30:90]$ trading days per t_2. This indicator is comprised of $(745 - 300)/2 = 223$ fits.

- Filter Conditions: After calibrating the model, the following filter conditions are applied to determine which fits are qualified:

$$m \in [0.01,0.99]$$

$$\omega \in [2,15]$$

$$t_c \in [max(t_2 - 60, t_2 - 0.5(t_2 - t_1)), min(252, t_2 + 0.5(t_2 - t_1))]$$

$$O > 2.5$$

$$D > 0.5$$

where

$$O = \frac{\omega}{2\pi}\ln\left(\frac{t_c - t_1}{t_c - t_2}\right)$$

$$D = \frac{m|B|}{\omega|C|}$$

2.2. Nonparametric Causality-in-Quantiles Test

In this sub-section, we briefly present the methodology for testing nonparametric quantiles-based causality as developed by Jeong et al. (2012).[3] Let y_t denote RV and x_t specific MS-LPPLS-CI. Further, let $Y_{t-1} \equiv (y_{t-1}, \dots, y_{t-p})$, $X_{t-1} \equiv (x_{t-1}, \dots, x_{t-p})$, $Z_t = (X_t, Y_t)$, and $F_{y_t|\cdot}(y_t| \bullet)$ denote the conditional distribution of y_t given $\bullet$. Defining $Q_\theta(Z_{t-1}) \equiv Q_\theta(y_t|Z_{t-1})$ and $Q_\theta(Y_{t-1}) \equiv Q_\theta(y_t|Y_{t-1})$, we have $F_{y_t|Z_{t-1}}\{Q_\theta(Z_{t-1})|Z_{t-1}\} = \theta$ with probability one. The (non-)causality in the θ-th quantile hypotheses to be tested are:

$$H_0: \ P\left\{F_{y_t|Z_{t-1}}\{Q_\theta(Y_{t-1})|Z_{t-1}\} = \theta\right\} = 1 \tag{8}$$
$$H_1: \ P\left\{F_{y_t|Z_{t-1}}\{Q_\theta(Y_{t-1})|Z_{t-1}\} = \theta\right\} < 1 \tag{9}$$

Jeong et al. (2012) show that the feasible kernel-based test statistics has the following format:

$$\hat{J}_T = \frac{1}{T(T-1)h^{2p}} \sum_{t=p+1}^{T} \sum_{s=p+1, s\neq t}^{T} K\left(\frac{Z_{t-1} - Z_{s-1}}{h}\right)\hat{\varepsilon}_t\hat{\varepsilon}_s \tag{10}$$

where $K(\bullet)$ is the kernel function with bandwidth h, T is the sample size, p is the lag order, and $\hat{\varepsilon}_t = \mathbf{1}\{y_t \leq \hat{Q}_\theta(Y_{t-1})\} - \theta$ is the regression error, where $\hat{Q}_\theta(Y_{t-1})$ is an estimate of the θ-th conditional quantile and $\mathbf{1}\{\bullet\}$ is the indicator function. The *Nadarya-Watson* kernel estimator of $\hat{Q}_\theta(Y_{t-1})$ is given by

$$\hat{Q}_\theta(Y_{t-1}) = \frac{\sum_{s=p+1, s\neq t}^{T} L\left(\frac{Y_{t-1} - Y_{s-1}}{h}\right) \mathbf{1}\{y_s \leq y_t\}}{\sum_{s=p+1, s\neq t}^{T} L\left(\frac{Y_{t-1} - Y_{s-1}}{h}\right)} \tag{11}$$

with $L(\bullet)$ denoting the kernel function.

The empirical implementation of causality testing via quantiles entails specifying three key parameters: the bandwidth (h), the lag order (p), and the kernel types for $K(\cdot)$ and $L(\cdot)$. We use a lag order of one based on the Schwarz Information Criterion (SIC) and determine h by the leave-one-out least-squares cross validation. Finally, for $K(\cdot)$ and $L(\cdot)$, we use Gaussian kernels.

3. Data

The positive and negative weekly bubble indicators at short-, medium-, and long-term for the US are derived based on the natural logarithmic values of the daily price-dividend ratio. The

[3] Our presentation relies on expositions of the the nonparametric quantiles-based causality test in several prominent recent papers, for example, Balcilar et al. (2017, 2018a, 2021), Gkillas et al. (2019, 2021), among others.

stock price index and dividend series are obtained from Refinitiv Datastream. It is important to note that, since we use the price-dividend ratio in line with the existing literature, the underlying metric for obtaining the bubble indicators is free of any currency units and is unaffected by exchange rate movements. Each of the six derived multi-scale LPPLS-CI values for the US, as derived from the econometric model discussed in sub-section 2.1, is sampled at a weekly frequency. The corresponding weekly RV of the US is based on the sum of daily squared log-returns in percentages of the aggregate stock index over a week. The RV is then matched with the data on the bubble indicators, giving us a sample period covering the 1st week of (7th) January, 1973 to 2nd week of (13th) September, 2020, i.e., 2489 observations. The MS-LPPLS-CIs of the US and the RV are plotted in Panels (a) and (b) of Figure 1, respectively.

[INSERT FIGURE 1]

The short-, medium-, and long-term indicators are displayed in different colors (green, purple, and red, respectively), while the log price-to-dividend ratio is shown in black in Panel (a) of Figure 1. Higher LPPLS-CI values for a particular scale indicate that the LPPLS signature is present for many of the fitting windows to which the model was calibrated, making it more reliable. From a brief visual inspection of the plots in Figure 1, we find that there are many spikes in the LPPLS-CI values preceding regime shifts in the underlying log price-to-dividend ratio.

As stated in sub-section 2.1, the long-term positive LPPLS-CI (red lines in Figure 1) is comprised of 223 single LPPLS model fits spanning fitting windows of size 300 to 745 observations. This represents nearly 3 years of data. Due to the larger calibration time-period we anticipate that large indicator values will occur less frequently at this scale than they would for smaller scales. We see 4 strong positive long-term LPPLS-CI values. The first is observed in from January, 1973 to December, 1974. This crash came on the heels of the collapse of the Bretton Woods system, and the dollar devaluation from the Smithsonian Agreement. Next, we see a strong positive long-term LPPLS-CI value preceding ‘‘Black Monday’’ in October, 1987. A similar observation can be made during the Asian Financial Crisis of 1997. We also see a clustering of highly positive LPPLS-CI values leading up to the Dot-com bubble burst over March, 2000 to October, 2002, but immediately following the crash, we see strong negative LPPLS-CI values, which in turn, signal rallies.

The medium-term LPPLS-CI (purple lines in Figure 1) uses 105 fits and spans fitting windows of size 90 to 300 observations. This represents a little over one year of data. In general, we observe pronounced LPPLS-CI values (positive and negative) at points where we detect the same for the long-term indicators. In addition, we find that strong positive medium-term LPPLS-CI values were formed before strong long-term LPPLS-CI values leading up to the GFC.

The short-term LPPLS-CI (green lines in Figure 1) uses 30 fits from fitting windows of size 30 to 90 observations. This represents just 1 month. As can be seen from Figure 1, this scale produces the most signals. It can also be inferred from the figure that the smallest crashes/rallies are signalled from this scale, possibly due it picking up idiosyncratic signals. However, we still can see small corrections immediately following a strong short-term LPPLS-CI value. It is also interesting to notice, just as with the medium-term indicators preceding the long-term indicators, the short-term indicators tend to lead the medium-term ones, in the context of the major bubble dates identified by the medium- and long-run indicators discussed above. This adds support to the finding from Demirer et al. (2019) that the maturation of the bubble towards instability is present across several distinct time-scales.

Note that, besides the crises episodes discussed above, these indicators in general also show spikes associated with crashes and recoveries before and around the global financial crisis of 2007 to 2008, the European sovereign debt crisis from 2009 to 2012, the ‘‘Brexit’’ in 2016, and to some extent COVID-19 as well.

When we compare the identified bubble episodes with the plot of *RV* in Panel (b) of Figure 1, we see that the bubble episodes are closely linked with the behavior of RV, in particular during the "Black Monday" episode, the global financial crisis, the European sovereign debt crisis, and COVID-19, with persistent volatility effects observed during the Asian financial crisis to the Dot-com bubble burst.

Overall, our empirical findings support the claim made in the introduction that the LPPLS framework is a flexible tool for detecting positive and negative bubbles across different time-scales, and can also be associated with peaks of *RV*s. But the causal influence needs to be formally tested using the nonparametric causality-in-quantiles method.

Since the computation of the weekly *RV* using daily data involves at most 5 observations, which can lead to imprecise calculations, especially if there are outliers during a week, we also utilized intraday data sampled at 5 and 10-minute intervals to obtain the weekly *RV* of the US, which we call *RV5* and *RV10* respectively. The final intraday *RV* data was secured from the Oxford-Man Institute of Quantitative Finance.[4] While *RV5* and *RV10* data ended in the 2nd week of September, 2020, it only starts on the 4th week of December, 1999, due to intraday data becoming available only relatively recently, and hence covering shorter samples compared to the starting date of the daily data-based *RV* of the US. *RV5* and *RV10* are plotted in Panels (c) and (d) of Figure 1. When compared to Figure 1(b), the peaks of *RV*, *RV5* and *RV10* over the common sample tend to align well with the identified periods of crashes and recoveries in the US stock market. The summary statistics of *RV5* and *RV10* in Table 1 also confirms their non-normality just as *RV* (and the bubble indicators). This provides an initial motivation for our quantiles-based causality framework.

[INSERT TABLE 1]

4. Empirical Results

4.1. Main Findings

In this section we analyse the nature of predictability emanating from the short-, medium-, and long-term positive and negative bubble indicators on daily and intraday data-based *RV*s of the US in particular, and also that of the remaining nine countries, for which we focus only on intraday data-based *RV*s (details of which have been provided below).

We can draw the following observations from the predictive analyses:

(a) For the sake of completeness and comparability with the nonparametric causality-in-quantiles framework, we conduct the linear Granger causality test reported in Table 2.

[4] Note that, this freely available data library has been discontinued since the end of 2022.

As evident, there is no indication of predictability running from the MS-LPPLS-CIs to the daily data-based weekly RV of the US at the conventional 5% level of significance, with weak causality (at the 10% level of significance) detected for RV originating from the long-term positive bubbles indicator;

[INSERT TABLE 2]

(b) Having observed lack of any strong evidence of causality based on the linear specification, we next examine whether the finding of non-causality might be due to model misspecification that assumes a linear predictability relationship. Therefore, in order to explore whether the linear model is misspecified, we first test for the presence of nonlinearity in the relationship between RV and the six MS-LPPLS-CIs. In this regard, we use the Brock et al. (1996, BDS) test on the residuals from the linear model used in the linear Granger causality test, and check whether the null hypothesis of *i.i.d.* residuals at various dimensions (m) can be rejected or not. Table 3 presents the results of the BDS nonlinearity tests. As shown in the table, the BDS test yields overwhelming evidence of nonlinearity, that is, we reject the null hypothesis of linearity (*i.i.d.* residuals) at the highest level of significance, consistently across all 6 predictive cases considered. In sum, the BDS test confirms that the linear model is indeed misspecified due to the existence of uncaptured nonlinearity, and hence, further predictive inference must rely on a nonlinear model, which happens to be our nonparametric causality-in-quantiles approach.

[INSERT TABLE 3]

(c) Next, we address the issue of instability in the linear model and potential misspecification by examining the presence of possible structural breaks in the relationship between RV and indicators of stock market bubbles in the US. For this purpose, we utilize the powerful *UDmax* and *WDmax* tests multiple structural breaks as proposed by of Bai and Perron (2003) on the equations of the linear Granger causality test. Based on the results reported in Table 4, we find that there is widespread evidence of regime changes, especially before, during or after the periods of major bubbles identified and discussed in the data segment, i.e., Section 3. Given that the parameter estimates are indeed unstable over the full sample period, we conclude that our linear Granger causality results are invalid. To achieve accurate causal analysis in our context, we must rely on an econometric model that is inherently time-varying, which we accomplish through our quantiles-based nonlinear setup;

[INSERT TABLE 4]

In light of the presence of nonlinearity and regime changes in the relationship between RV and the six MS-LPPLS-CIs, our linear Granger causality results are clearly unreliable. This provides us with a strong statistical motivation to utilize the nonparametric causality-in-quantiles testing method, which can accommodate such misspecifications. Now, examining the standard normal test statistics, derived from the quantiles-based results, over the range of 0.10 to 0.90, we can draw the following conclusions:

(i) Unlike the linear Granger causality findings, as observed from Table 5, the quantiles-based model detects strong evidence of predictability from all the six MS-LPPLS-CIs over the set of quantiles of RV, being studied, primarily at the 1% level of significance. The exceptions are at the extreme quantiles, i.e., 0.10 and 0.90, of the short-term positive bubbles CI, and for all the scales of the negative bubbles indicators, and at the quantile of 0.20 as well for the negative short-term bubbles CI, whereby causality holds at the 5% level of significance. Overall, MS-LPPLS-CIs are always found to predict the US RV at the conventional significance level of 5% at least. Interestingly, the standard normal test statistics depict an inverted u-shaped structure over the quantiles, with the peak occurring primarily at the quantile of 0.60, barring the cases of the short-term positive and long-term negative LPPLS-CIs, in which cases the highest value is attained at 0.40. This pattern can possibly be explained by the fact that at lower levels of initial market uncertainty, agents are not too worried about future uncertainty, and rely on past volatility primarily, leading to a small predictive content from the LPPLS-CIs. However, the role of the bubble indicators start to rise, and become especially important when uncertainty crosses the normal phase, i.e., the median. Beyond that again, the role of the MS-LPPLS-CIs start declining as agents tend to herd at higher levels of uncertainty (Balcilar et al., 2013);

(ii) Although robust predictive inference is derived based on the nonparametric causality-in-quantiles test, it is also interesting to estimate the sign of the effect of the six MS-LPPLS-CIs on RV at various quantiles, especially to validate the theoretical background of "leverage effects" on which we base our predictive analyses. As per this effect, it is expected that positive bubbles ("good news") will reduce RV, while negative bubbles ("bad news") will increase RV. However, in a nonparametric framework, this is not straightforward, as we need to employ the first-order partial derivatives. Estimation of the partial derivatives for nonparametric models can give rise to complications, because nonparametric methods exhibit slow convergence rates, due to the dimensionality and smoothness of the underlying conditional expectation function. However, one can look at a statistic that summarizes the overall effect or the global curvature (i.e., the global sign and magnitude), but not the entire derivative curve. In this regard, a natural measure of the global curvature is the average derivative (AD) using the conditional pivotal quantile, based on approximation or the coupling approach of Belloni et al. (2019), which allows us to estimate the partial ADs. Based on the ADs reported in Table 6, we find consistent evidence of the "leverage effect" at each quantile, since the positive CIs reduce RV, while the opposite holds true for the negative CIs across the three time-scales;

(iii) Having confirmed the leverage effect, when we compare the values of the test statistics, we find that, in general, the predictive impact is strongest for the long-term, followed by the medium- and short-run under the positive bubbles over the entire conditional distribution of RV. Under the negative bubbles, this pattern holds until below the median, with the short- and medium-run dominating beyond the median and higher quantiles. Moreover, when we compare across the positive and negative bubbles indicators within a particular time-scale, the former always carries relatively bigger causal influence on the entire conditional distribution of RV. Though there is asymmetry in terms of the causal strength of the positive and negative bubbles across the three time-scales, thus vindicating our decision to disaggregate the patterns of the observed bubbles, the long-term positive bubbles CI (closely followed by its medium-run counterpart) carries the highest predictive impact (from quantiles of 0.30 and above). In other words, early signals of possible severe forthcoming deep crashes from accelerating prices, carries the most valuable information for the future path of US stock market volatility.

(iv) As a matter of a first robustness check, we utilize the k-th order nonparametric causality-in-quantiles approach of Balcilar et al. (2018b), which is basically an extension of the conditional mean-based higher-order nonparametric causality test of Nishiyama et al. (2011) into the quantiles-dependent framework of Jeong et al. (2012).[5] More specifically, this test of Balcilar et al. (2018) allows us to detect causal impact of the MS-LPPLS-CIs on both the conditional distribution of returns and squared returns, with the latter capturing volatility. In other words, we can check if our results based on RV carries over to squared returns or not, staying within the realms of a nonparametric quantile model. Focussing on the results for squared returns presented in Panel B of Table 7,[6] we find that the standard normal statistics for the predictive effect of the six bubbles indicators peaks after the conditional median, with the positive MS-LPPLS-CIs, in particular the long- and medium-term scales, carrying stronger causal effect than their negative counterparts. In other words, our main findings for RV, continues to hold for squared returns;

[5] In this case, using log-returns as the dependent variable in equations (8)-(11) outlined in Section 2, provides the test for the causality-in-quantiles in the *first-moment*. As far as the second moment, i.e., squared returns (volatility is concerned), the null and alternative hypotheses are given by: H_0: $P\left\{F_{y_t^k|Z_{t-1}}\{Q_\theta(Y_{t-1})|Z_{t-1}\} = \theta\right\} = 1, \;\; k = 1,2,\ldots,K$, and H_1: $P\left\{F_{y_t^k|Z_{t-1}}\{Q_\theta(Y_{t-1})|Z_{t-1}\} = \theta\right\} < 1, \;\; k = 1,2,\ldots,K$. The causality-in-variance test can then be calculated by replacing y_t in euations (10) and (11) with y_t^2. Testing approach is sequential and failing to reject the test for $k = 1$ does not automatically lead to no-causality in the *second* moment, i.e., one can still construct the test for $k = 2$.

[6] As far as stock returns is concerned, the bubbles indicators capturing the extreme movements of the price-dividend ratio, popularly known as a valuation ratio and widely considered as an important predictor in the US stock returns literature (Rapach and Zhou, 2013; 2022), is shown to carry predictability for the two-ends of the conditional distribution of returns, corresponding to bearish and bullish-phases of the market.

(v) As a second robustness check, we now conduct the predictive analysis by relying on the intraday data-based *RV5* and *RV10* as dependent variables, over the shorter sample period of the 4th week of December, 1999 to the 2nd week of September, 2020. As can be observed from Table 8, the general pattern of stronger predictability around the median, as observed with *RV*, continues to hold, but the extreme quantiles at times are less predictable. Though now, the short-term LPLLS-CI for positive bubbles, also serves as an important predictor along with the other two scales within this category, particularly at lower quantiles. Finally, the relatively stronger evidence of predictability under *RV10* compared to *RV5* based on the six MS-LPPLS-CIs is possibly an indication of relatively more precise estimate of the realized volatility derived under less-frequently sampled intraday data, as the former allows one to circumvent liquidity issues (or the lack thereof), extreme high-frequency noise from no-activity periods observed mainly in comparatively shorter-time windows, and zero prices (Bouri, et al., 2021).

(vi) Though our main focus is on the US, we also are interested in whether our findings for the US in terms of volatility being predicted by the MS-LPPLS-CIs carry over to other developed markets namely, Canada, France, Germany, Italy, Japan and the United Kingdom (UK), i.e., the remaining of the G7 countries, as well as to the emerging economies of Brazil, China, and India. As with the US, the underlying data sources for computing the bubbles indicators (based on the price-dividend ratios) and the *RV5* and *RV10* are Refinitiv Datastream and Oxford-Man Institute of Quantitative Finance, respectively. While all *RV* data ends in the 2nd week of September, 2020, their starting dates differs, with that for Brazil, France, Germany, and India, beginning from the 4th week of December, 1999; China and the UK from the 1st week of January, 2000; Japan from the 1st week of February, 2000; Canada from the 4th week of April, 2002, and; Italy from the 4th week of May, 2009. The bubbles of these 9 countries, unsurprisingly in light of the well-established fact of high-degree of interconnectedness of global stock markets, depict similar timing of strong (positive and negative) MS-LPPLS-CI values compared to that of the US over common samples, lending support to the idea of synchronized boom and bust cycles.[7] As observed from Tables 9 and 10, within the developed countries and across *RV5* and *RV10*, predictability is relatively weak for Germany and Italy. At the same time, for the three emerging markets, India tends to show strong evidence of causality particularly under *RV10*, with volatility in Brazil and China primarily affected by the MS-LPPLS-CIs at lower ends of the conditional distributions of *RV5* and *RV10*. Overall, we find international evidence of stock market volatility being predicted by their own bubbles, but the effect is relatively pronounced for the US. This could be due to bubbles primarily originating in the US, and indirectly affecting the volatility in the other developed and developing stock

[7] The plots of the MS-LPPLS-CIs for the nine economies are available upon request from the authors.

markets, through volatility spillover (see, Marfatia et al. (2017) for a detailed discussion).

[INSERT TABLES 5, 6, 7, 8, 9 AND 10]

4.2. Additional Findings

Our focus is on predicting *RV* based on the MS-LPPLS-CIs, which in turn hinges on the idea of the leverage effect of Black (1976). While Black (1976) calls the negative impact of returns on volatility a "direct causation", the author also defines the idea of "reverse causation", wherein the causal relationship runs from volatility changes to stock returns. More specifically, changes in tastes and technology lead to an increase in the uncertainty about the payoffs from investments. Because of the increase in expected future volatility, stock prices must fall, so that the expected return from the stock rises to induce investors to continue to hold the stock. In other words, it could be possible that *RV* can actually predict the MS-LPPLS-CIs. Furthermore, market liquidity, i.e., the ease with which stocks are traded have been widely associated with US stock market bubbles (see for example, Nneji (2015), Demirer et al. (2019)),[8] and volatility in turn has been shown to drive liquidity risks (Stoll 2000; Ramos and Righi, 2020). Naturally then, *RV* can lower liquidity,[9] and cause the MS-LPPLS-CIs. In this sub-section, this is what we investigate using the nonparametric causality-in-quantiles set-up for the G7 and the BRICS blocs.

We find that predictability due to the country-specific *RV5* and *RV10* for the six respective bubbles indicators to hold quite strongly, i.e., primarily at 1% level of significance in Tables 11 and 12.[10] Specifically speaking, in only 21 instances each out of the 540 cases each considered where *RV5* or *RV10* served as a predictor, do we fail to detect predictability for the MS-LPPLS-CIs, at the highest considered quantile of 0.90.

4. Conclusion

[8] Jarrow et al. (2012) provided a liquidity-based theoretical model for financial asset price bubbles that explained bubble formation and bubble bursting. These authors defined the asset's fundamental price process exogenously and asset price bubbles are endogenously determined by market trading activity. This enabled them to generate a model that explains both bubble formation and bubble bursting. In their model, the quantity impact of trading activity on the fundamental price process, i.e., liquidity risk, is what generated price bubbles.

[9] In fact, when we regressed the data on aggregate, traded and non-traded measures of liquidity for the US, as derived by Pástor and Stambaugh (2003) over the monthly period of 1962:08 to 2022:12 (downloaded from: https://faculty.chicagobooth.edu/-/media/faculty/lubos-pastor/data/liq_data_1962_2022.txt), on corresponding daily data-based monthly *RV* of the S&P500 (with the stock prices obtained from: https://finance.yahoo.com/quote/%5EGSPC/history/), we found that the effect was negative (with coefficients of -4.830, -3.600, -0.803, respectively) and statistically significant at 1% level (for all the three metrics of liquidity).

[10] Our findings are unlike those of Sornette et al. (2018), who examined forty well-known asset price bubbles around the world and, using creative graphical representations to capture robustly the transient dynamics of the volatility, found that the dynamics of the volatility would not have been a useful predictor of the subsequent crashes. Though not directly comparable, our strong evidence is possibly a reflection of the utilization of the sophisticated approaches namely, the MS-LPPLS-CI method to detect positive and negative bubbles, and the nonparametric causality-in-quantiles test to determine predictability due to *RV*.

The primary objective of our paper is to analyze the predictive impact of equity market bubbles of the US on its volatility. In this regard, we first detect positive and negative bubbles at short-, medium- and long-run by using the Multi-Scale LPPLS Confidence Indicator (MS-LPPLS-CI) approach. Our findings reveal the ability of these indicators to detect major crashes and rallies over the weekly period of January, 1973 to September, 2020. In the second-step, we utilize a nonparametric causality-in-quantiles approach to analyse the predictive ability of the MS-LPPLS-CIs on the weekly realized volatility (*RV*), computed from daily data. Our results demonstrate strong evidence of predictability over the entire conditional distribution of *RV* from the six MS-LPPLS-CIs based on the nonparametric causality-in-quantiles method, particularly for the positive bubbles. These findings are in sharp contrast with the evidence of virtually non-existent causality from the linear Granger test, which in turn is due to the existence of nonlinearity and structural breaks that we statistically depict. Our findings of predictability is robust to an alternative metric of volatility namely, squared returns, and weekly realized volatilities derived from 5- and 10-minutes interval intraday data, i.e., *RV5* and *RV10* respectively. Furthermore, we detect evidence of predictability for *RV5* and *RV10* of Brazil, Canada, China, France, Germany, India, Italy, Japan, and the UK, though the impact is comparatively weaker to that of the US, particularly for Germany and Italy. Finally, we also find strong evidence of reverse causality, i.e., causal feedbacks from *RV5* and *RV10* on to the MS-LPPLS-CIs of the 10 countries considered, barring some exceptions at the highest quantile considered.

With MS-LPPLS-CIs carrying predictive content for international equity market volatility, albeit in an asymmetric manner in a nonparametric quantiles-based setting, this information should be of immense value to investor in designing state-specific portfolios, and for policy authorities to monitor financial market uncertainty due to the booms-busts cycles. At the same time, as bubbles are associated with the behaviour of economic activity and has welfare implications (Caraiani et al., 2023), our findings associated with predictability also running from volatility to bubbles should be of paramount importance to policymakers in coming up with appropriate policy responses, particularly when bubbles burst. Finally, academically, our findings imply the violation of the efficient market hypothesis in a nonparametric fashion, with booms and busts in the G7 stock markets being driven by a state variable, when accounting for misspecification due to nonlinearity and structural breaks.

As part of future research, it would be interesting to extend our in-sample predictability of volatility (and bubbles) to a full-fledged out-of-sample forecasting exercise, by including other macroeconomic and behavioural controls (as discussed in detail by Gupta et al. (2023), and van Eyden et al. (2023))

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Figure 1 (a): Multi-Scale Log-Periodic Power Law Singularity Confidence Indicators (MS-LPPLS-CIs)

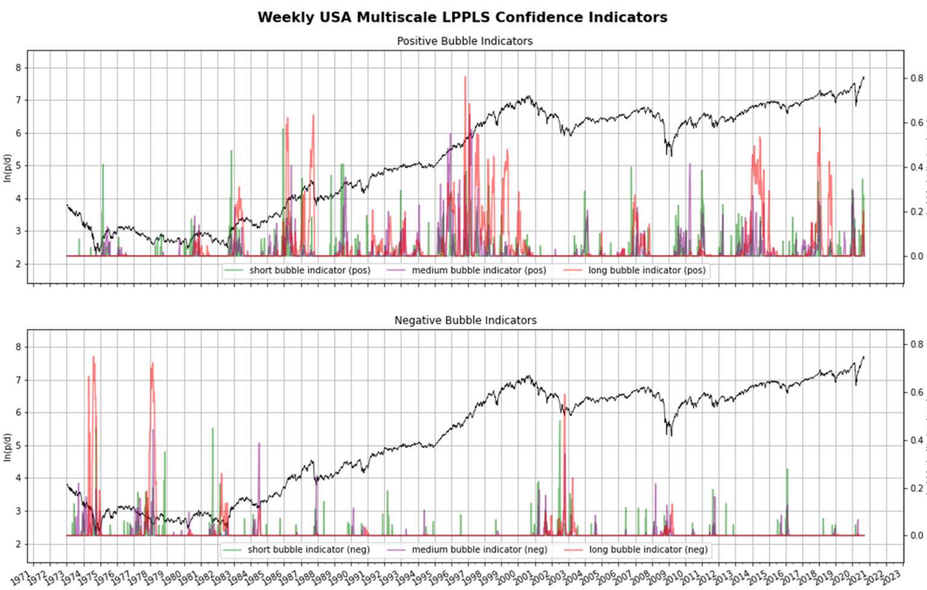

Figure 1(b): Daily Data-Based Weekly Realized Volatility (*RV*) of the US

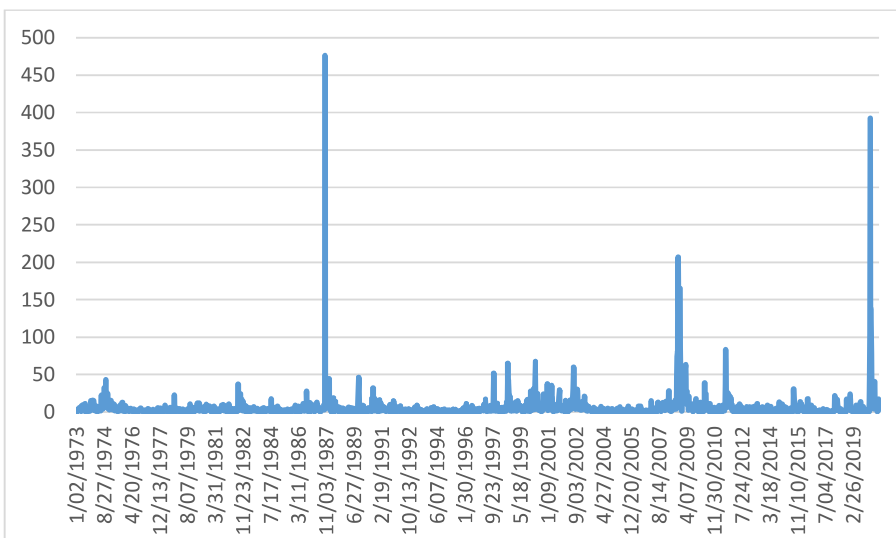

Figure 1(c): 5-Minute Interval Intraday Data-Based Weekly Realized Volatility (*RV5*) of the US

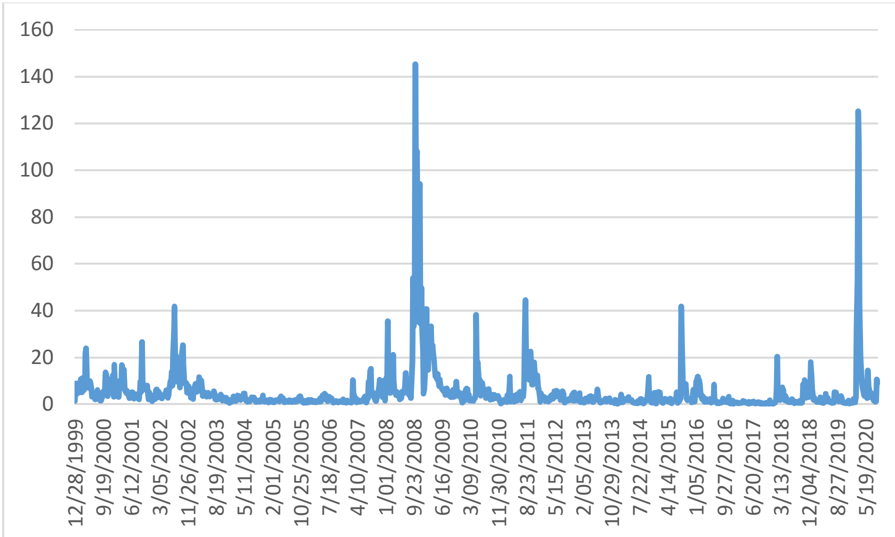

Figure 1(d): 10-Minute Interval Intraday Data-Based Weekly Realized Volatility (*RV10*) of the US

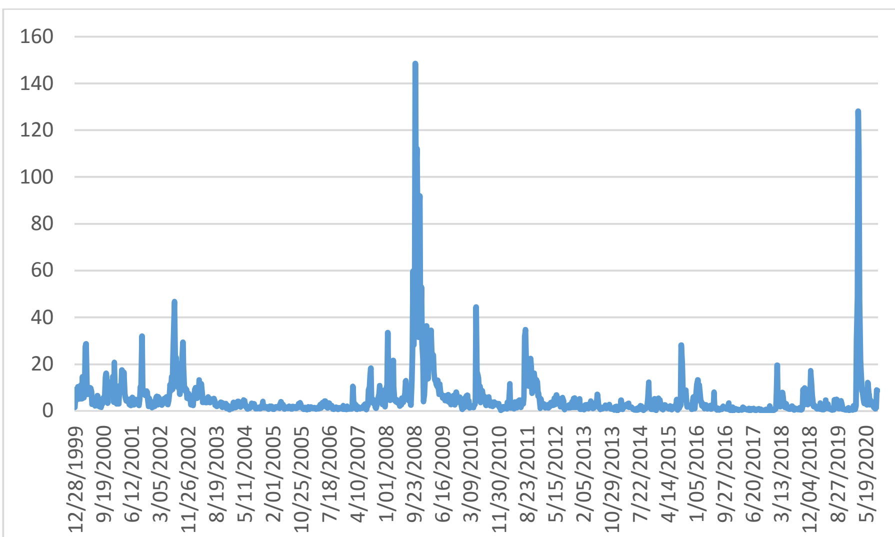

Table 1: Summary Statistics for the US Data

Statistic	*RV*	*RV5*	*RV10*	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
Mean	5.612	5.373	5.456	0.013	0.020	0.041	0.005	0.006	0.013
Median	2.596	2.527	2.556	0.000	0.000	0.000	0.000	0.000	0.000
Maximum	475.829	145.310	148.514	0.571	0.635	0.806	0.480	0.443	0.750
Minimum	0.012	0.135	0.136	0.000	0.000	0.000	0.000	0.000	0.000
Std. Dev.	16.514	10.964	11.071	0.048	0.059	0.102	0.028	0.028	0.074
Skewness	17.108	7.174	7.153	5.388	5.278	3.289	9.814	8.073	7.520
Kurtosis	407.735	69.426	69.694	38.416	38.352	14.455	126.815	89.655	62.689
Jarque-Bera	17109893***	208014.7***	209565.9***	142119.5***	141164.2***	18096.41***	1629808***	805790.3***	392941.5***
Observations	2489	1081	1081	2489	2489	2489	2489	2489	2489

Note: *RV*, and *RV5* and *RV10* are weekly realized volatility based on daily data, and 5 and 10-minute interval intraday data, respectively; Std. Dev. stands for standard deviation; the null hypotheses of the Jarque-Bera test correspond to the null of normality; *** indicates rejection of the null hypothesis at the 1% level of significance.

Table 2: Linear Granger Causality Test Results for the Daily Data-Based Weekly *RV* of the US

	Predictor					
Dependent Variable	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
RV	3.159	1.651	11.739*	9.743	0.446	0.636

Note: See Note to Table 1. Entries correspond to $\chi^2(p)$ test statistic of the null hypothesis of no Granger causality, with the lag-length p (= 3, 3, 6, 6, 2, and 5, respectively) determined by SIC; * indicates rejection of the null hypothesis at the 10% level of significance.

Table 3: Brock et al. (1996) BDS Test of Non-Linearity for the US

MS-LPPLS-CI	m=2	m=3	m=4	m=5	m=6
Positive Short-Term	22.658***	25.309***	27.203***	28.865***	30.933***
Positive Medium-Term	22.93***	25.773***	27.796***	29.537***	31.639***
Positive Long-Term	22.046***	24.516***	26.466***	28.238***	30.343***
Negative Short-Term	22.875***	25.287***	27.249***	29.014***	31.071***
Negative Medium-Term	21.986***	24.682***	26.987***	28.829***	30.965***
Negative Long-Term	23.573***	26.115***	28.17***	29.894***	31.946***

Note: See Notes to Table 1. Entries correspond to the z-statistic of the BDS test with the null of *i.i.d.* residuals across various dimensions (m), with the test applied to the residuals recovered from the *RV* equation with p lags each of *RV* and MS-LPPLS-CIs; *** indicates rejection of the null hypothesis at 1% level of significance.

Table 4: Bai and Perron (2003) Breakpoint Dates for the US

MS-LPPLS-CI	Break Dates
Positive Short-Term	3/25/1980, 5/12/1987, 6/28/1994, 8/21/2001, 11/25/2008
Positive Medium-Term	3/25/1980, 5/12/1987, 6/28/1994, 10/02/2001, 11/25/2008
Positive Long-Term	4/01/1980, 10/13/1987, 11/29/1994, 2/05/2002, 11/25/2008, 3/24/2009
Negative Short-Term	4/01/1980, 5/19/1987, 7/12/1994, 10/09/2001, 11/25/2008
Negative Medium-Term	3/11/1980, 9/10/1985, 5/05/1987, 11/03/1992, 6/28/1994, 7/31/2001, 8/21/2001, 10/14/2008
Negative Long-Term	3/25/1980, 5/12/1987, 6/28/1994, 10/09/2001, 11/25/2008

Note: See Notes to Table 1. Entries correspond to the dates of structural breaks, with the test applied to the RV equation with p lags each of RV and MS-LPPLS-CIs.

Table 5: Causality-in-Quantiles Test Results for *RV* of the US with MS-LPPLS-CIs

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.10	2.426**	2.851***	2.880***	2.208**	2.409**	2.437**
0.20	2.985***	3.949***	3.898***	2.472**	2.803***	3.029***
0.30	3.992***	4.752***	4.973***	3.397***	3.477***	4.026***
0.40	4.469***	5.583***	5.634***	4.425***	4.276***	4.595***
0.50	4.593***	5.763***	5.926***	4.308***	4.142***	4.267***
0.60	4.612***	5.975***	6.130***	4.450***	4.297***	4.371***
0.70	3.954***	5.128***	5.389***	3.757***	3.733***	3.749***
0.80	3.295***	4.018***	4.294***	2.851***	3.063***	2.950***
0.90	2.298**	3.083***	3.442***	2.285**	2.133**	2.122**

Note: *** and ** indicates rejection of the null hypothesis of no Granger causality at the 1% and 5% levels of significance respectively, i.e., critical values of 2.575 and 1.96 for the standard normal test statistic, from MS-LPPLS-CIs to RV for a particular quantile.

Table 6: Average Derivative Estimates for the Effect of MS-LPPLS-CIs on RV

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.10	-0.036	-0.040	-0.004	0.163	0.067	0.056
0.20	-0.088	-0.101	-0.032	0.250	0.075	0.028
0.30	-0.117	-0.084	-0.063	0.372	0.107	0.015
0.40	-0.151	-0.128	-0.052	0.389	0.161	0.165
0.50	-0.207	-0.228	-0.060	0.432	0.432	0.409
0.60	-0.282	-0.320	-0.055	0.659	0.786	0.779
0.70	-0.346	-0.351	-0.063	0.869	2.186	1.041
0.80	-0.495	-0.373	-0.254	0.876	2.864	1.828
0.90	-0.856	-0.924	-0.954	2.283	4.625	2.515

Note: Entries correspond to average derivative (AD) estimates of the sign of the effect of the six MS-LPPLS-CIs on to RV at the considered quantiles at a particular quantile.

Table 7: Causality-in-Quantiles Test Results for Returns and Squared Returns (Volatility) of the US with MS-LPPLS-CIs

Panel A: Returns

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1.187	0.943	1.038	0.928	1.185	1.129
0.2	2.304^{**}	1.898^{*}	1.574	1.668^{*}	1.905^{*}	1.591
0.3	2.235^{**}	2.243^{**}	1.786^{***}	1.728^{*}	1.875^{*}	1.674^{*}
0.4	1.195	1.270	1.434	0.979	0.893	0.842
0.5	1.025	1.224	1.117	0.920	0.715	0.717
0.6	1.806^{*}	1.602	1.438	1.524	1.468	1.222
0.7	1.291	1.518	1.589	1.916^{*}	1.828^{*}	1.444
0.8	1.602	1.747^{*}	1.914^{*}	2.961^{***}	3.222^{***}	2.783^{***}
0.9	1.126	1.583	2.309^{**}	2.145^{**}	2.333^{**}	1.835^{*}

Panel B: Squared Returns

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1.473	1.826^{*}	2.103^{**}	1.791^{*}	2.004^{**}	1.817^{*}
0.2	2.617^{***}	2.760^{***}	3.002^{***}	2.439^{**}	2.774^{***}	2.514^{**}
0.3	2.968^{***}	3.173^{***}	3.978^{***}	2.926^{***}	3.091^{***}	2.830^{***}
0.4	3.639^{***}	4.282^{***}	4.491^{***}	3.157^{***}	3.227^{***}	3.310^{***}
0.5	3.052^{***}	4.539^{***}	4.404^{***}	2.795^{***}	2.816^{***}	2.885^{***}
0.6	3.210^{***}	4.955^{***}	4.414^{***}	2.966^{***}	2.860^{***}	3.064^{***}
0.7	3.475^{***}	5.082^{***}	4.579^{***}	2.668^{***}	2.500^{**}	2.775^{***}
0.8	2.805^{***}	3.851^{***}	3.503^{***}	2.286^{**}	2.298^{**}	2.694^{***}
0.9	2.239^{**}	2.787^{***}	2.352^{**}	1.739^{*}	1.661^{*}	1.877^{*}

Note: ***, ** and * indicates rejection of the null hypothesis of no Granger causality at the 1%, 5% and 10% levels of significance respectively, i.e., critical values of 2.575, 1.96 and 1.645 for the standard normal test statistic, from MS-LPPLS-CIs to returns and squared returns (volatility) for a particular quantile.

Table 8: Causality-in-Quantiles Test Results for *RV5* and *RV10* of the US with MS-LPPLS-CIs

Panel A: RV5

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1.358	1.307	1.192	1.510	1.468	1.481
0.2	2.657***	1.510	1.454	2.680***	2.659***	2.458**
0.3	2.172**	1.597	1.592	1.868*	1.886*	1.682*
0.4	1.666*	1.651*	1.628	1.669*	1.665*	1.457
0.5	3.467***	2.538**	2.777***	3.752***	3.726***	3.547***
0.6	1.917*	1.853*	1.846*	1.830*	1.909*	1.785*
0.7	1.636	2.278**	2.037**	1.633	1.610	1.689*
0.8	1.658*	2.232**	2.074**	1.426	1.446	1.564
0.9	1.535	2.111**	1.612	1.095	1.203	1.301

Panel B: RV10

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1.596	1.595	1.382	1.765*	1.806*	1.761*
0.2	3.139***	2.050**	1.744*	2.438**	2.527**	2.413**
0.3	3.057***	1.886*	1.804*	2.974***	2.961***	2.666***
0.4	2.117**	1.469	1.334	2.192**	2.044**	1.912*
0.5	2.922***	1.958**	2.181**	3.177***	2.906***	2.817**
0.6	2.635***	2.010**	2.336**	2.900***	2.579***	2.630***
0.7	1.692*	1.763*	2.078**	1.748*	1.512	1.673*
0.8	2.438**	2.518**	2.362**	2.395**	2.103**	2.219**
0.9	1.440	1.803*	1.734*	1.376	1.279	1.206

Note: ***, ** and * indicates rejection of the null hypothesis of no Granger causality at the 1%, 5% and 10% levels of significance respectively, i.e., critical values of 2.575, 1.96 and 1.645 for the standard normal test statistic, from MS-LPPLS-CIs to 5-minute and 10-minute interval intraday data-based weekly RV, i.e., *RV5* and *RV10* respectively, for a particular quantile.

Table 9: Causality-in-Quantiles Test Results for *RV5* of Remaining of the G7 and Emerging Countries with MS-LPPLS-CIs

Panel A: Brazil

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1.654*	2.176**	2.379**	2.332**	1.774*	1.902*
0.2	3.924***	5.019***	4.514***	4.618***	3.858***	3.369***
0.3	2.189**	2.827***	2.271**	2.151**	1.709*	1.812*
0.4	1.668*	1.909*	1.799*	1.750*	1.547	1.654*
0.5	1.410	1.321	1.471	1.491	1.074	1.174
0.6	0.970	1.180	1.211	0.899	0.931	0.747
0.7	0.836	1.132	1.181	0.792	1.004	0.906
0.8	1.180	1.510	1.307	1.253	1.261	1.023
0.9	0.531	0.524	0.424	0.553	0.507	0.578

Panel B: Canada

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1.743*	1.394	0.943	1.386	1.511	1.529
0.2	2.445**	2.240**	1.878*	1.837*	1.935*	1.617
0.3	4.160***	3.921***	3.469***	3.322***	3.183***	3.121***
0.4	2.980***	2.616***	2.068**	1.886*	1.703*	1.828*
0.5	2.866***	3.206***	2.560**	1.794*	1.788*	1.806*
0.6	3.562***	4.314***	3.824***	3.136***	3.221***	3.079***
0.7	4.592***	3.784***	4.220***	4.429***	4.285***	4.187***
0.8	3.168***	2.686***	2.903***	2.488**	2.541**	2.385**
0.9	1.239	1.156	1.279	0.870	0.912	0.861

Panel C: China

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	0.646	0.866	0.679	0.748	0.647	0.749
0.2	0.993	1.128	1.122	1.129	1.333	1.281
0.3	2.221**	1.545	1.969**	3.570***	3.354***	3.893***
0.4	3.720***	2.186**	2.661***	5.458***	5.244***	6.045***
0.5	2.106**	1.325	1.787*	3.171***	3.238***	4.392***
0.6	0.929	0.870	0.838	1.145	1.378	1.684*
0.7	1.225	1.160	1.164	1.258	1.398	2.137**
0.8	1.246	1.019	0.917	1.147	1.488	2.514**
0.9	0.707	0.819	0.786	0.649	0.636	0.765

Panel D: France

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1.467	2.343**	1.718*	1.920*	1.910*	1.981**
0.2	2.235**	3.148***	2.154**	2.266**	2.361**	2.354**
0.3	2.977***	3.738***	2.865***	2.913***	2.751***	2.996***
0.4	3.087***	3.533***	2.998***	2.803***	2.526**	2.752***

0.5	3.080^{***}	3.194^{***}	2.595^{***}	2.867^{***}	2.631^{***}	2.768^{***}
0.6	1.817^{*}	2.053^{**}	1.750^{*}	1.771^{*}	1.790^{*}	1.749^{*}
0.7	0.942	1.349	1.344	0.942	0.915	0.891
0.8	0.672	1.386	1.199	0.550	0.615	0.530
0.9	0.548	1.052	0.983	0.651	0.646	0.555

Panel E: Germany

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	0.810	1.084	1.001	0.925	0.851	0.861
0.2	1.413	1.875^{*}	1.975^{**}	1.748^{*}	1.625	1.629
0.3	1.308	1.697^{*}	1.408	1.269	1.321	1.338
0.4	1.711^{*}	1.946^{*}	1.461	1.320	1.295	1.395
0.5	2.135^{**}	1.642	1.798^{*}	1.732^{*}	1.969^{**}	1.923^{*}
0.6	1.894^{*}	1.260	1.655^{*}	1.520	1.693^{*}	1.794^{*}
0.7	1.288	1.071	1.514	1.117	1.144	1.240
0.8	0.909	0.864	0.948	0.802	1.025	0.923
0.9	1.318	1.031	1.234	1.523	1.732^{*}	1.592

Panel F: India

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	2.000^{**}	1.661^{*}	1.402	1.854^{*}	1.894^{*}	1.867^{*}
0.2	1.314	1.669^{*}	1.605	1.215	1.268	1.425
0.3	1.171	1.666^{*}	1.290	1.001	1.100	1.060
0.4	1.610	2.007^{**}	1.795^{*}	1.434	1.440	1.436
0.5	1.724^{*}	2.256^{**}	1.823^{*}	1.510	1.729^{*}	1.390
0.6	2.031^{**}	2.068^{**}	1.880^{*}	1.835^{*}	1.992^{**}	1.690^{*}
0.7	2.121^{**}	1.667^{*}	1.185	1.313	1.720^{*}	1.411
0.8	1.167	1.222	1.236	0.912	0.967	0.870
0.9	0.498	0.779	0.731	0.460	0.574	0.452

Panel G: Italy

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	2.007^{**}	2.038^{**}	1.963^{**}	1.834^{*}	1.824^{*}	1.805^{*}
0.2	1.717^{*}	1.884^{*}	2.261^{**}	1.693^{*}	1.770^{*}	1.683^{*}
0.3	0.960	1.830^{*}	1.826^{*}	0.943	0.997	1.037
0.4	0.894	1.595	1.554	1.060	1.151	0.984
0.5	1.391	1.797^{*}	1.820^{*}	1.347	1.464	1.333
0.6	1.805^{*}	1.937^{*}	2.653^{***}	1.888^{*}	1.832^{*}	1.862^{*}
0.7	2.283^{**}	1.955^{*}	2.695^{***}	2.225^{**}	2.193^{**}	1.849^{*}
0.8	1.871*	1.626	2.356^{**}	2.059^{**}	1.793^{*}	1.705^{*}
0.9	0.555	0.655	0.889	0.636	0.562	0.503

Panel I: Japan

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	4.923^{***}	2.955^{***}	4.126^{***}	4.609^{***}	4.842^{***}	5.010^{***}

0.2	3.906***	3.182***	3.908***	3.876***	3.497***	3.777***
0.3	3.520***	2.638***	3.145***	3.265***	2.955***	3.413***
0.4	4.091***	3.264***	3.883***	3.860***	3.700***	3.752***
0.5	3.813***	3.012***	3.852***	3.920***	3.570***	4.015***
0.6	3.919***	2.879***	4.056***	3.992***	3.747***	4.545***
0.7	1.593	1.124	1.895*	1.787*	1.653*	2.196**
0.8	1.923*	1.469	2.284**	2.101**	2.102**	2.546**
0.9	0.700	0.608	0.790	0.609	0.603	0.833

Panel J: UK

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	0.786	1.234	1.108	0.612	0.585	0.574
0.2	0.834	0.807	0.482	0.558	0.814	0.778
0.3	1.003	1.141	0.890	0.765	0.891	0.858
0.4	1.042	1.219	1.713*	1.294	1.134	1.405
0.5	1.024	1.128	1.121	1.144	1.266	1.269
0.6	0.906	1.249	1.122	1.043	1.185	1.122
0.7	1.195	2.045**	1.469	1.279	1.532	1.384
0.8	0.383	0.877	0.779	0.286	0.440	0.476
0.9	0.369	0.724	0.716	0.301	0.451	0.488

Note: ***, ** and * indicates rejection of the null hypothesis of no Granger causality at the 1%, 5% and 10% levels of significance respectively, i.e., critical values of 2.575, 1.96 and 1.645 for the standard normal test statistic, from MS-LPPLS-CIs to 5-minute interval intraday data-based weekly RV, i.e., *RV5*, for a particular quantile.

Table 10: Causality-in-Quantiles Test Results for *RV10* of Remaining of the G7 and Emerging Countries with MS-LPPLS-CIs

Panel A: Brazil

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1.626	2.119**	2.314**	2.246**	1.737*	1.860*
0.2	3.738***	4.795***	4.311***	4.382***	3.670***	3.228***
0.3	2.262**	2.862***	2.430**	2.240**	1.782*	1.911*
0.4	1.782*	2.010**	2.012**	1.851*	1.633	1.783*
0.5	1.618	1.502	1.705*	1.691*	1.220	1.394
0.6	1.165	1.364	1.436	1.094	1.071	0.934
0.7	1.026	1.349	1.381	1.003	1.206	1.116
0.8	1.418	1.744*	1.522	1.519	1.506	1.259
0.9	0.585	0.583	0.481	0.612	0.581	0.632

Panel B: Canada

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1.602	1.238	0.912	1.234	1.298	1.348
0.2	2.029**	1.720*	1.573	1.522	1.568	1.285
0.3	3.652***	3.147***	3.135***	2.976***	2.750***	2.687***
0.4	2.512**	1.878*	1.572	1.472	1.203	1.298
0.5	2.257**	2.042**	1.855*	1.177	1.120	1.204
0.6	2.655***	3.277***	3.062***	2.292**	2.299**	2.163**
0.7	3.266***	2.894***	3.424***	3.418***	3.174***	3.094***
0.8	2.874***	2.289**	2.697***	2.259**	2.255**	2.098**
0.9	0.925	0.852	1.005	0.645	0.552	0.547

Panel C: China

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	0.290	0.479	0.369	0.458	0.379	0.461
0.2	0.439	0.629	0.607	0.654	0.803	0.863
0.3	1.973**	1.325	1.776*	3.340***	2.943***	3.841***
0.4	3.166***	1.801*	2.297**	5.062***	4.657***	5.769***
0.5	1.771*	1.036	1.471	2.969***	2.841***	4.071***
0.6	0.692	0.633	0.567	0.869	1.116	1.264
0.7	0.859	0.711	0.713	0.865	1.010	1.736*
0.8	0.879	0.599	0.518	0.758	1.149	2.144**
0.9	0.495	0.561	0.428	0.379	0.355	0.412

Panel D: France

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1.590	2.376**	1.699*	1.578	1.696*	1.635
0.2	2.416**	3.596***	2.416**	2.352**	2.308**	2.496**
0.3	3.179***	3.271***	2.813***	3.517***	3.221***	3.510***
0.4	4.640***	4.081***	4.015***	5.067***	4.696***	5.053***

0.5	5.691***	4.932***	4.687***	6.171***	5.899***	5.859***
0.6	6.128***	5.799***	5.164***	6.068***	5.857***	5.742***
0.7	3.030***	2.929***	2.791***	3.124***	3.090***	2.976***
0.8	1.172	1.269	1.299	1.030	1.008	1.092
0.9	0.710	1.068	0.922	0.767	0.726	0.703

Panel E: Germany

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	0.786	1.061	0.990	0.912	0.836	0.846
0.2	1.402	1.853*	1.970**	1.760*	1.634	1.637
0.3	1.267	1.631	1.382	1.251	1.305	1.318
0.4	1.639	1.836*	1.421	1.269	1.250	1.341
0.5	2.066**	1.543	1.759*	1.682*	1.930*	1.873*
0.6	1.843*	1.197	1.635	1.476	1.657*	1.754*
0.7	1.228	1.013	1.475	1.053	1.085	1.183
0.8	0.867	0.804	0.909	0.769	0.996	0.893
0.9	1.311	1.015	1.223	1.519	1.729*	1.590

Panel F: India

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	5.883***	5.297***	5.478***	6.864***	6.329***	6.492***
0.2	8.465***	6.638***	8.917***	10.473***	9.115***	9.951***
0.3	9.827***	7.690***	10.116***	11.576***	10.599***	10.972***
0.4	6.042***	4.657***	7.523***	7.717***	7.069***	7.089***
0.5	4.101***	3.356***	6.340***	6.189***	4.957***	5.464***
0.6	2.903***	2.279**	4.705***	4.257***	3.640***	3.660***
0.7	2.179**	1.503	3.016***	2.987***	2.483**	2.638***
0.8	1.731*	1.601	3.073***	1.963**	1.689*	1.673*
0.9	0.525	0.758	1.102	0.671	0.745	0.592

Panel G: Italy

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	2.018**	2.030**	1.977**	1.847*	1.826*	1.811*
0.2	1.724*	1.902*	2.277**	1.704*	1.777*	1.687*
0.3	0.978	1.873*	1.861*	0.962	1.013	1.052
0.4	0.931	1.640	1.585	1.104	1.194	1.021
0.5	1.435	1.858*	1.851*	1.384	1.498	1.357
0.6	1.876*	1.995**	2.711***	1.958*	1.899*	1.918*
0.7	2.362**	2.026**	2.747***	2.293**	2.258**	1.899*
0.8	1.948*	1.685*	2.404**	2.130**	1.848*	1.768*
0.9	0.592	0.692	0.927	0.670	0.590	0.530

Panel I: Japan

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	2.907***	2.208**	2.493**	2.938***	3.062***	3.357***

0.2	3.318***	3.564***	3.368***	3.402***	3.283***	3.878***
0.3	3.821***	3.496***	4.139***	4.150***	4.133***	4.609***
0.4	4.279***	4.454***	4.974***	4.508***	4.232***	4.504***
0.5	3.981***	3.848***	4.479***	4.124***	4.079***	4.573***
0.6	4.118***	3.862***	4.098***	3.997***	3.879***	4.586***
0.7	4.243***	3.751***	4.290***	4.379***	4.145***	4.948***
0.8	2.547**	2.240**	2.566**	2.601***	2.518**	2.822***
0.9	1.103	1.220	1.216	1.178	1.291	1.386

Panel J: UK

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	0.608	0.398	0.599	0.205	0.356	0.265
0.2	0.954	0.532	0.705	0.504	0.752	0.553
0.3	3.989***	2.357**	1.559	2.645***	3.049***	2.915***
0.4	2.672***	2.021**	1.331	1.644	2.100**	2.113**
0.5	2.265**	2.167**	1.190	2.037**	2.702***	2.654***
0.6	2.361**	2.236**	1.390	2.083**	2.665***	2.307**
0.7	2.990***	2.957***	1.773*	2.731***	3.138***	2.737***
0.8	2.326**	2.046**	1.817*	2.243**	2.725***	2.290**
0.9	0.292	0.575	0.460	0.471	0.551	0.447

Note: ***, ** and * indicates rejection of the null hypothesis of no Granger causality at the 1%, 5% and 10% levels of significance respectively, i.e., critical values of 2.575, 1.96 and 1.645 for the standard normal test statistic, from MS-LPPLS-CIs to 10-minute interval intraday data-based weekly RV, i.e., *RV10*, for a particular quantile.

Table 11: Causality-in-Quantiles Test Results for MS-LPPLS-CIs of the G7 and Emerging Countries with *RV5*

Panel A: Brazil

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1595.232***	1273.264***	1306.405***	1704.368***	1601.119***	1568.784***
0.2	915.485***	726.019***	752.391***	984.991***	919.462***	906.100***
0.3	586.962***	474.928***	486.458***	637.085***	593.556***	592.640***
0.4	381.272***	304.775***	321.103***	418.798***	390.598***	395.071***
0.5	240.137***	187.918***	208.431***	268.492***	254.236***	253.909***
0.6	140.497***	105.690***	129.641***	162.635***	151.957***	152.900***
0.7	70.283***	51.206***	71.104***	87.075***	78.131***	83.937***
0.8	25.506***	14.164***	28.758***	33.424***	33.631***	33.971***
0.9	1.816*	0.415	3.765***	5.014***	4.568***	4.307***

Panel B: Canada

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1391.357***	1159.883***	1250.142***	1575.210***	1460.266***	1312.219***
0.2	798.465***	661.897***	728.772***	918.600***	839.695***	760.663***
0.3	512.810***	420.759***	473.434***	602.157***	545.112***	498.120***
0.4	334.208***	275.499***	312.430***	403.497***	363.120***	334.474***
0.5	210.763***	171.754***	202.143***	264.963***	239.876***	220.658***
0.6	123.371***	97.134***	125.633***	163.919***	145.989***	138.379***
0.7	60.999***	47.446***	66.881***	89.705***	76.867***	77.727***
0.8	19.510***	17.927***	26.504***	37.254***	31.974***	34.475***
0.9	0.472	1.019	3.124***	6.719***	4.723***	6.779***

Panel C: China

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1168.197***	1081.854***	1455.492***	1669.657***	1467.823***	1221.38***
0.2	668.705***	617.207***	854.567***	963.763***	837.407***	694.908***
0.3	434.130***	399.104***	552.494***	622.281***	535.636***	440.722***
0.4	289.138***	264.403***	372.761***	408.062***	355.175***	293.057***
0.5	189.064***	171.551***	236.745***	259.495***	228.904***	178.290***
0.6	116.570***	105.242***	141.700***	153.903***	133.686***	102.733***
0.7	63.566***	56.686***	77.381***	78.521***	66.449***	52.978***
0.8	26.193***	22.736***	32.109***	30.160***	21.852***	17.885***
0.9	3.834***	2.779***	5.313***	3.331***	1.016	0.276

Panel D: France

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1191.542***	1119.353***	1144.205***	1732.120***	1083.287***	1655.705***
0.2	678.077***	649.955***	671.303***	1010.123***	621.543***	980.813***
0.3	437.410***	419.905***	421.840***	663.559***	406.903***	645.853***
0.4	288.977***	271.963***	274.801***	446.379***	274.761***	436.199***

0.5	186.859***	164.172***	167.697***	294.962***	183.476***	285.989***
0.6	113.280***	90.386***	100.102***	184.358***	117.008***	176.044***
0.7	59.989***	42.780***	45.512***	102.799***	67.789***	96.261***
0.8	23.192***	10.744***	11.148***	44.589***	32.069***	44.884***
0.9	2.667***	0.000	0.093	8.598***	8.593***	9.208***

Panel E: Germany

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1051.426***	1155.599***	1217.288***	1587.715***	1545.034***	1625.106***
0.2	594.509***	646.670***	695.485***	923.920***	891.749***	962.262***
0.3	380.486***	405.023***	458.233***	607.794***	586.057***	628.113***
0.4	248.733***	258.876***	296.420***	410.481***	389.401***	427.439***
0.5	158.422***	159.483***	184.122***	273.135***	251.396***	278.904***
0.6	93.794***	86.579***	108.478***	172.740***	150.486***	171.629***
0.7	47.572***	37.344***	54.956***	98.415***	81.439***	96.432***
0.8	16.601***	10.986***	19.572***	44.770***	30.069***	40.387***
0.9	1.420	1.642	0.827	10.246***	3.559***	6.796***

Panel F: India

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1456.842***	1221.198***	1269.403***	1590.493***	1385.405***	1752.100***
0.2	833.313***	688.524***	731.678***	923.324***	795.623***	1033.244***
0.3	535.635***	438.568***	474.584***	606.573***	516.265***	682.576***
0.4	350.379***	286.961***	305.333***	409.251***	342.839***	464.451***
0.5	222.550***	176.016***	188.557***	272.081***	222.892***	307.166***
0.6	130.718***	96.908***	106.639***	171.904***	136.104***	194.494***
0.7	65.059***	50.997***	52.429***	97.786***	73.336***	110.043***
0.8	21.812***	15.059***	15.443***	44.329***	29.868***	49.157***
0.9	1.056	0.371	0.579	9.931***	4.006***	11.049***

Panel G: Italy

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	302.625***	609.494***	772.226***	849.097***	898.951***	925.103***
0.2	169.541***	351.754***	443.398***	492.438***	521.866***	550.731***
0.3	108.408***	224.020***	284.323***	324.290***	341.228***	363.946***
0.4	71.148***	146.499***	184.672***	219.933***	228.126***	247.230***
0.5	45.734***	87.208***	117.901***	147.485***	149.752***	162.322***
0.6	27.561***	50.576***	68.894***	94.517***	92.664***	100.354***
0.7	14.516***	24.013***	38.198***	55.147***	51.856***	55.913***
0.8	5.670***	5.248***	13.006***	26.402***	22.035***	26.396***
0.9	1.167	0.003	0.476	7.212***	4.028***	6.465***

Panel I: Japan

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1298.585***	1302.447***	1569.971***	1742.774***	1376.430***	1323.173***

0.2	741.051***	741.459***	934.208***	1016.759***	781.427***	772.755***
0.3	479.682***	476.637***	618.182***	668.421***	496.689***	506.421***
0.4	318.374***	312.943***	407.319***	450.172***	317.645***	337.850***
0.5	207.242***	201.489***	263.226***	297.998***	201.249***	216.804***
0.6	126.941***	120.045***	157.816***	186.794***	114.587***	128.076***
0.7	68.499***	67.150***	89.263***	104.688***	52.442***	68.018***
0.8	27.641***	25.453***	35.973***	45.964***	17.771***	26.683***
0.9	3.897***	2.954***	5.577***	9.465***	0.534	3.269***

Panel J: UK

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1378.798***	922.482***	1239.507***	1794.491***	1567.415***	1413.07***
0.2	789.928***	516.435***	736.230***	1046.046***	910.801***	824.408***
0.3	510.592***	327.062***	480.134***	684.778***	597.436***	541.663***
0.4	337.151***	210.964***	308.863***	457.833***	401.757***	361.047***
0.5	217.384***	131.877***	192.853***	299.596***	266.397***	227.470***
0.6	130.975***	75.824***	115.364***	184.301***	166.873***	140.145***
0.7	68.494***	36.509***	61.127***	100.253***	93.997***	75.488***
0.8	25.668***	11.247***	22.361***	41.543***	41.341***	29.784***
0.9	2.621***	1.916*	1.466	7.584***	8.526***	5.226***

Panel K: US

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1544.305***	1094.961***	1053.767***	1776.648***	1402.675***	1560.310***
0.2	879.853***	607.617***	644.128***	1039.641***	813.075***	908.450***
0.3	558.358***	376.307***	405.388***	685.928***	535.365***	599.687***
0.4	357.386***	240.483***	255.743***	464.114***	363.061***	407.510***
0.5	224.540***	140.983***	153.143***	309.227***	243.454***	273.875***
0.6	130.157***	73.049***	84.629***	195.767***	156.013***	175.994***
0.7	63.638***	31.754***	36.864***	111.655***	91.033***	103.584***
0.8	22.417***	8.603***	7.960***	50.908***	43.646***	50.659***
0.9	0.873	1.336	0.396	11.712***	12.167***	14.437***

Note: *** and * indicates rejection of the null hypothesis of no Granger causality at the 1% and 10% levels of significance respectively, i.e., critical values of 2.575 and 1.645 for the standard normal test statistic, from 5-minutee interval intraday data-based weekly RV, i.e., *RV5*, to MS-LPPLS-CIs to for a particular quantile.

Table 12: Causality-in-Quantiles Test Results for MS-LPPLS-CIs of the G7 and Emerging Countries with *RV10*

Panel A: Brazil

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1595.463***	1264.244***	1302.717***	1704.368***	1601.128***	1563.820***
0.2	915.568***	720.203***	750.764***	984.991***	919.446***	902.874***
0.3	587.000***	470.965***	485.783***	637.085***	593.687***	590.190***
0.4	381.295***	301.971***	320.916***	418.798***	390.740***	393.268***
0.5	240.154***	186.008***	208.833***	268.492***	254.355***	252.652***
0.6	140.512***	104.451***	130.045***	162.635***	152.042***	152.058***
0.7	70.296***	50.449***	71.489***	87.075***	78.184***	83.440***
0.8	25.518***	13.842***	29.091***	33.424***	33.625***	33.690***
0.9	1.820*	0.409	4.014***	5.014***	4.563***	4.220***

Panel B: Canada

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1389.472***	1158.282***	1244.179***	1572.456***	1460.266***	1301.217***
0.2	797.367***	660.927***	724.973***	916.791***	839.695***	753.879***
0.3	512.089***	420.142***	470.873***	600.844***	545.112***	493.551***
0.4	333.723***	275.114***	310.692***	402.516***	363.120***	331.365***
0.5	210.442***	171.541***	201.012***	264.230***	239.876***	218.601***
0.6	123.170***	97.041***	124.845***	163.383***	145.989***	137.066***
0.7	60.888***	47.459***	66.447***	89.335***	76.867***	76.994***
0.8	19.463***	17.962***	26.315***	37.028***	31.974***	34.131***
0.9	0.468	1.035	3.097***	6.623***	4.723***	6.711***

Panel C: China

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1122.776***	1037.507***	1454.736***	1669.724***	1470.362***	1221.193***
0.2	640.971***	590.418***	853.741***	963.806***	839.254***	694.866***
0.3	415.620***	381.386***	552.044***	622.309***	537.013***	440.711***
0.4	276.644***	252.573***	372.676***	408.079***	356.123***	293.105***
0.5	180.856***	163.884***	236.530***	259.504***	229.638***	178.245***
0.6	111.521***	100.629***	141.545***	153.907***	134.273***	102.722***
0.7	60.835***	54.294***	77.304***	78.522***	66.784***	52.980***
0.8	25.091***	21.863***	32.056***	30.159***	22.028***	17.886***
0.9	3.674***	2.783***	5.295***	3.330***	1.053	0.259

Panel D: France

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1192.766***	1119.353***	1144.030***	1732.119***	1085.653***	1655.54***
0.2	678.700***	649.955***	671.125***	1010.065***	622.774***	980.427***
0.3	437.746***	419.905***	421.695***	663.440***	407.556***	645.538***
0.4	289.137***	271.963***	274.674***	446.219***	275.050***	436.009***

0.5	186.900***	164.172***	167.605***	294.778***	183.523***	285.914***
0.6	113.242***	90.386***	100.033***	184.167***	116.889***	175.990***
0.7	59.903***	42.780***	45.469***	102.618***	67.572***	96.258***
0.8	23.092***	10.744***	11.129***	44.436***	31.808***	44.885***
0.9	2.606***	0.000	0.096	8.504***	8.365***	9.224***

Panel E: Germany

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1053.347***	1156.331***	1216.354***	1586.814***	1544.251***	1625.403***
0.2	595.745***	647.216***	694.847***	923.217***	890.936***	962.413***
0.3	381.418***	405.424***	457.676***	607.165***	585.340***	628.188***
0.4	249.477***	259.165***	295.968***	409.908***	388.987***	427.274***
0.5	159.029***	159.683***	183.750***	272.617***	251.090***	278.783***
0.6	94.291***	86.695***	108.178***	172.282***	150.340***	171.594***
0.7	47.971***	37.399***	54.743***	98.029***	81.282***	96.333***
0.8	16.904***	11.002***	19.442***	44.467***	29.980***	40.379***
0.9	1.594	1.621	0.807	10.056***	3.531***	6.816***

Panel F: India

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1461.178***	1222.783***	1270.474***	1605.031***	1402.572***	1754.482***
0.2	836.010***	689.409***	732.375***	932.610***	806.291***	1034.842***
0.3	537.399***	439.046***	475.049***	612.952***	523.480***	683.691***
0.4	351.513***	287.183***	305.614***	413.667***	347.769***	465.233***
0.5	223.236***	176.091***	188.751***	275.065***	226.178***	307.778***
0.6	131.085***	96.920***	106.762***	173.812***	138.172***	194.913***
0.7	65.213***	50.946***	52.477***	98.888***	74.493***	110.309***
0.8	21.852***	15.010***	15.448***	44.847***	30.363***	49.304***
0.9	1.087	0.389	0.580	10.071***	4.074***	11.102***

Panel G: Italy

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	305.180***	609.494***	772.226***	854.158***	902.278***	926.135***
0.2	170.882***	351.754***	443.398***	495.463***	523.857***	551.429***
0.3	109.226***	224.020***	284.323***	326.326***	342.533***	364.429***
0.4	71.658***	146.499***	184.672***	221.339***	228.990***	247.557***
0.5	46.041***	87.208***	117.901***	148.447***	150.332***	162.540***
0.6	27.726***	50.576***	68.894***	95.149***	93.016***	100.486***
0.7	14.582***	24.013***	38.198***	55.529***	52.045***	56.007***
0.8	5.667***	5.248***	13.006***	26.600***	22.048***	26.430***
0.9	1.122	0.003	0.476	7.282***	4.004***	6.488***

Panel I: Japan

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1340.873***	1328.925***	1569.971***	1745.074***	1379.431***	1323.735***

0.2	767.373***	758.114***	934.208***	1018.207***	783.286***	773.084***
0.3	497.489***	487.803***	618.182***	669.271***	497.768***	506.649***
0.4	330.553***	320.405***	407.319***	450.578***	318.225***	338.010***
0.5	215.371***	206.311***	263.226***	298.078***	201.666***	216.906***
0.6	132.054***	122.880***	157.816***	186.648***	114.842***	128.134***
0.7	71.361***	68.774***	89.263***	104.408***	52.562***	68.075***
0.8	28.876***	26.012***	35.973***	45.645***	17.788***	26.720***
0.9	4.076***	2.921***	5.577***	9.243***	0.445	3.270***

Panel J: UK

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1409.025***	958.292***	1239.507***	1794.765***	1588.474***	1413.07***
0.2	808.176***	537.223***	736.230***	1046.366***	923.852***	824.408***
0.3	522.669***	340.304***	480.134***	685.062***	606.114***	541.663***
0.4	345.224***	219.404***	308.863***	458.068***	407.507***	361.047***
0.5	222.620***	136.976***	192.853***	299.782***	270.072***	227.470***
0.6	134.136***	78.551***	115.364***	184.441***	168.978***	140.145***
0.7	70.151***	37.595***	61.127***	100.352***	94.982***	75.488***
0.8	26.299***	11.304***	22.361***	41.603***	41.564***	29.784***
0.9	2.684***	1.704*	1.466	7.617***	8.437***	5.226***

Panel K: US

Quantile	Positive Short-Term	Positive Medium-Term	Positive Long-Term	Negative Short-Term	Negative Medium-Term	Negative Long-Term
0.1	1544.305***	1091.596***	1055.262***	1775.725***	1397.622***	1557.969***
0.2	879.853***	605.439***	644.996***	1039.189***	810.378***	907.319***
0.3	558.358***	374.815***	406.041***	685.740***	533.681***	599.164***
0.4	357.386***	239.481***	256.024***	464.093***	361.964***	407.362***
0.5	224.540***	140.340***	153.359***	309.314***	242.745***	273.974***
0.6	130.157***	72.685***	84.700***	195.919***	155.572***	176.226***
0.7	63.638***	31.593***	36.910***	111.835***	90.780***	104.012***
0.8	22.417***	8.592***	7.976***	51.082***	43.518***	50.810***
0.9	0.873	1.372	0.396	11.849***	12.114***	14.605***

Note: *** and * indicates rejection of the null hypothesis of no Granger causality at the 1% and 10% levels of significance respectively, i.e., critical values of 2.575 and 1.645 for the standard normal test statistic, from 10-minutee interval intraday data-based weekly RV, i.e., *RV10*, to MS-LPPLS-CIs to for a particular quantile.