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A Bayesian Estimation of DSGE Model for the Nigerian Economy

Mutiu Gbade Rasaki[1]

Abstract: This paper develops and estimates a small open economy dynamic stochastic general equilibrium (DSGE) model for the Nigerian economy using the Bayesian technique. We include a number of frictions, rigidities, and shocks in our model. The results show a considerable evidence of price stickiness in Nigeria. Furthermore, the results suggest that the forward-looking component dominates price setting behaviour in Nigeria. Moreover, the findings indicate that external shocks such as external debt, exchange rate and foreign inflation shocks largely influence output fluctuations in Nigeria while inflation is driven by money supply, productivity, nominal exchange rate and domestic interest rate shocks. Lastly, the findings indicate that the monetary authority responds strongly to real exchange rate shocks.

Keywords: Frictions; Price stickiness; External shocks; Output fluctuations; Forward-looking

JEL Classification: C11; E31; E32

1. Introduction

This paper formulates and estimates a small open economy monetary dynamic stochastic dynamic general equilibrium (DSGE) model for the Nigerian economy. Due to its micro foundation and theoretical consistency, the DSGE model has become a popular quantitative tool for analysing macroeconomic fluctuations, evaluating macroeconomic policies and economic forecasting, especially for monetary authorities and policymakers in the developed and emerging economies.[2] In line with the existing literature on DSGE modeling (e.g. Smets & Wouters, 2003; Castelnuovo, 2012), our model features frictions that are required to account for persistence that is observed in the Nigerian data. Our model also incorporates a number of shocks that are considered quantitatively relevant for the Nigerian economy.

Despite the increasing adoption of DSGE model for macroeconomic policy analysis in both the developed and emerging economies, there have been very few studies focusing on Nigeria. A notable exception is the work by Olofin et al. (2014) who estimate a small-scale macroeconometric model to evaluate the monetary authority trade-off between stable exchange rate and lower lending rate. Their model, however, only estimates reduced-form parameters and does not consider variables influencing macroeconomic fluctuations in Nigeria. Their model does not include foreign variables. Giving the dependence of Nigerian firms on imported inputs and rising external debts, prices of foreign inputs and foreign interest rate shocks may impact economic fluctuations in Nigeria. Our model also includes oil price shocks giving the importance of oil revenues in Nigeria.

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[2] See (Adolfson et al. 2007; Rasaki & Malikane, 2015).

The gap that this study seeks to fill is to develop and estimate a monetary dynamic stochastic general equilibrium (DSGE) model for the Nigerian economy. Given its theoretical consistency and identification efficiency, Bayesian technique is employed to estimate the model. Existing studies on Nigeria have neither employed monetary DSGE model nor estimate their model with Bayesian technique. For example, Olofin et al. (2014) estimate their model with ordinary least square and maximum likelihood. Olekah and Oyaromade (2007) use a VAR approach to estimate a DSGE model for Nigeria.

The significance of estimating A DSGE model for Nigeria is that policymakers can understand the dynamics of the economy. Specifically, it would assist monetary authority in formulating policies and forecasting policy implications. It would also provide a theory-based assessment of how shocks are transmitted throughout the economy, and how nominal and real frictions propagate these shocks.[1] As also pointed out by Sbordone et al. (2010), understanding structural shocks influencing macroeconomic fluctuations would assist monetary authorities to undertake appropriate policy reactions.

Our study contributes to the existing studies in a number of ways. Firstly, we develop and estimate a monetary DSGE model for the Nigerian economy. Secondly, we incorporate a number of shocks and frictions that are considered relevant for the Nigerian economy. Thirdly, following Andres et al. (2006), we adopt a non-separable money in the utility function. This allows the monetary aggregate to play active role in the economy. Lastly, the model is estimated with Bayesian technique.

The rest of the paper is organised as follows. Section 2 describes the model. Section 3 discusses the data and the estimated results. Section 4 concludes and makes policy recommendations.

2. Literature Review

A large number on the estimation of DSGE model has focused on the developed economies. For example, Smets and Wouters (2003) estimate a non-monetary DSGE model for the Euro area. Using the Bayesian estimation, the findings indicate that there is a high degree of price and wage stickiness in the Euro area. Similarly, Smets and Wouters (2007) estimate a DSGE model for the U.S. The findings suggests among others that demand shocks account for output fluctuations in the short run while supply shocks and productivity shocks influence output in the medium and long run. Moreover, Nimark (2009) estimate a DSGE model for Australia using Bayesian technique. The results suggest that variations in domestic output and inflation are largely caused by external shocks.

Furthermore, Jaaskela and Nimark (2011) estimate a New Keynesian economic model for Australia. They conclude that both domestic and external shocks account for business cycle fluctuations in Australia. Moreover, Curdia and Finocchiaro (2005) estimate a DSGE model for a Sweden model using Bayesian method technique. The findings suggest that monetary policy shocks and preference shocks are important sources of volatility in Sweden. Few Studies have also estimated DSGE model for emerging economies. For instance, Ionita (2017) estimate a DSGE model emerging economies. The findings suggest that exogenous shocks such as government spending shocks and monetary policy shocks influence economic fluctuations in emerging economies.

[1] See (Dib, 2003).

3. The Model

3.1. Households

The model is a monetary DSGE model taken from Rasaki and Malikane (2015). It is an extension of Andrés et al. (2006) and Castelnuovo (2012). In the utility function, money enters in a non-separable way. The model assumes that the households allocate their real holdings between domestic and foreign currencies. This is in line with findings by Elkhafif (2002) and Adom et al. (2008) for African countries. We assume this allocation is in a fixed percentage, thus $S_t M_t^* = \varrho M_t$ where S_t represents the nominal exchange rate, M_t^* represents the foreign nominal money and M_t is domestic nominal money. The representative household's preference is:

$$U_t = \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma}\left(\frac{C_t}{C_{t-1}^h}\right)^{1-\sigma}\left[\left(1+\frac{\varrho}{S_t}\right)\frac{M_t}{P_t}\right]^{\phi} - \frac{N_t^{1+\psi}}{1+\psi}\ldots \quad (1)$$

where C_t is the consumption, $\frac{M_t}{P_t}$ is the real balances, and N_t is the labour hour. The parameters β^t, h, σ, ϕ, and φ represent the discount factor, habit formation, relative risk aversion coefficient, money-interest rate elasticity, and the inverse of the Frisch labour supply elasticity respectively.

The household can hold his wealth in the form of foreign and domestic currency and domestic and foreign bonds. The budget constraint is given as:

$$C_t + \left(1+\frac{\varrho}{S_t}\right)\frac{M_t}{P_t} + \frac{B_t}{P_t} - \frac{S_t D_t^*}{P_t} = \frac{W_t N_t}{P_t} + \frac{B_{t-t}}{P_t}(1+r_{t-1}) + \left(1+\frac{\varrho}{S_t}\right)\frac{M_{t-1}}{P_t} - \frac{S_t D_{t-1}^*}{P_t}\left(1+ \quad r_t^f(q_t d_t^*)\right) \quad (2)$$

where D_t^* is the foreign debt, d_t^* is the ratio of external debt to GDP, W_t is the nominal wage rate, r_t is the domestic nominal interest rate, r_t^f is the foreign interest rate,q_t is the oil price. Eq. (2) shows that foreign debt service payment is related to movement in the oil prices and existing debt to GDP ratio; this is in line with the findings by Senhadji (2003).

The first order conditions are given as:

$$\frac{1}{C_{t-1}^h}\left(\frac{C_t}{C_{t-1}^h}\right)^{-\sigma}\left[\left(1+\frac{\varrho}{S_t}\right)\frac{M_t}{P_t}\right]^{\phi} = \lambda \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$\frac{\phi}{1-\sigma}\left(\frac{C_t}{C_{t-1}^h}\right)^{1-\sigma}\left(1+\frac{\varrho}{S_t}\right)^{\phi}\left(\frac{M_t}{P_t}\right)^{\phi-1}\frac{1}{P_t} = \frac{\lambda_t}{P_t}\left(1+\frac{\varrho}{S_t}\right) - \frac{\beta\lambda_{t+1}}{P_{t+1}}\left(1+\frac{\varrho}{S_t}\right)\ldots(4)$$

$$\frac{W_t}{P_t} = \frac{N_t^{\psi}}{\lambda_t}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$\beta\lambda_{t+1}(1+r_t)\frac{P_t}{P_{t+1}} = \lambda_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6)$$

$$\beta\lambda_{t+1}\frac{S_{t+1}}{P_{t+1}}\left(1+r_t^d\right) = \lambda\frac{S_t}{P_t}\ldots\ldots\ldots\ldots\ldots(7)$$

We combine Eqs.(3 &4) and linearize to derive our consumption Euler equation given as:

$$\hat{c}_t = \frac{h(\sigma-1)}{\sigma_c}\hat{c}_{t-1} + \frac{\sigma}{\sigma_c}E_t\hat{c}_{t+1} + \frac{\phi}{\sigma_c}\widehat{m}_t - \frac{\phi}{\sigma_c}E_t\widehat{m}_{t+1} - \frac{\varrho\phi}{s_0\sigma_c}\hat{s}_t + \frac{\varrho\phi}{s_0\sigma_c}E_t\hat{s}_{t+1}$$

$$-\frac{1}{\sigma_c}(\hat{r}_t - E_t\hat{\pi}_{t+1})\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Where $\sigma_c = \sigma + \sigma h(\sigma - 1)$ and $\widehat{\ }$ connotes percentage deviation from the steady state and. Eq.(8) indicates that consumption depends on the past and expected future consumption, real interest rate, and real balances. Eq.(8) also shows that a depreciation of the exchange rate reduces household wealth and reduces aggregate consumption. This is the contractionary effect of exchange rate depreciation identified in the literature.[1]

To derive the IS curve in terms of output, we write the macro-balance equation as:

$$\hat{y}_t = \gamma_c\hat{c}_t + \gamma_x\hat{x}_t - \gamma_z\hat{z}_t + \varepsilon_{g,t}\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

where $\hat{y}_t$, $\hat{x}_t$, and $\hat{z}_t$ are the percentage deviations of output, exports and imports respectively. The parameter γ_j represents the steady state ratio of variable j to output and $\varepsilon_{g,t}$ is a demand shock. Following McCallum and Nelson (2000), the net export function is written as:

$$\widehat{nx}_t = \gamma_{yf}\hat{y}_t^f - \gamma_y\hat{y}_t + \gamma_r\widehat{rer}_t\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

where $\hat{y}_t^f$ is the foreign output, γ_{yf} is the elasticity of net export to foreign output, γ_y is the elasticity of net export to domestic output and γ_r, is the sum of elasticity of substitution in production for home and abroad. The real exchange rate function is $\widehat{rer}_t = \hat{s}_t + \hat{p}_t^* - \hat{p}_t$. We substitute Eq.(10) into Eq.(9) to yield:

$$\hat{c}_t = \frac{1}{\gamma_c}(1+\gamma_y)\hat{y}_t - \frac{\gamma_r}{\gamma_c}\widehat{rer}_t - \frac{\gamma_{yf}}{\gamma_c}\hat{y}_t\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

We substitute Eq.(11) into Eq.(9) to yield the IS equation:

$$\hat{y}_t = \frac{h(\sigma-1)}{\sigma_c}\hat{y}_{t-1} + \frac{\sigma}{\sigma_c}E_t\hat{y}_{t+1} - \frac{\gamma_c}{\sigma_c(1+\gamma_y)}(r_t - E_t\hat{\pi}_{t+1}) + \frac{\phi\gamma_c}{\sigma_c(1+\gamma_y)}\widehat{m}_t - \frac{\phi\gamma_c}{\sigma_c(1+\gamma_y)}E_t\widehat{m}_{t+1} - \frac{\phi\varrho\gamma_c}{s_0\sigma_c(1+\gamma_y)}\hat{s}_t + \frac{\phi\varrho\gamma_c}{s_0\sigma_c(1+\gamma_y)}E_t\hat{s}_{t+1} - \frac{h(\sigma-1)\gamma_r}{\sigma_c(1+\gamma_y)}\widehat{rer}_{t-1} + \frac{\gamma_r}{(1+\gamma_y)}\widehat{rer}_t - \frac{\sigma\gamma_r}{\sigma_c(1+\gamma_y)}E_t\widehat{rer}_{t+1} - \frac{h(\sigma-1)\gamma_{yf}}{\sigma_c(1+\gamma_y)}\hat{y}_{t-1}^f + \frac{\gamma_{yf}}{(1+\gamma_y)}\hat{y}_t^f - \frac{\sigma\gamma_{yf}}{\sigma_c(1+\gamma_y)}E_t\hat{y}_{t+1}^f + \varepsilon_t^y\ldots\ldots\ldots\ldots(12)$$

ε_t^y is assumed to follow a first-order autoregressive written as $\varepsilon_t^y = \rho_a\varepsilon_{t-1}^y + \mu_t^y$. Eq. (12) shows that output depends positively on the real exchange rate and foreign output. Our dynamic IS equation differs for McCallum and Nelson (2004) as it features output as a function of lags and leads of the real exchange rate and foreign output.

3.2. Firms

Similar to Batini et al. (2005) and Malikane and Mokoka (2014), we assume that the firms exhibit non-linear input requirement in the production function such that $X_{i,t} = Y_t^{\delta i}$ where $X_{i,t}$ represents the amount of non-labour input i required in production, Y_t is output, and $\delta_i > 0$ is the input requirement coefficient. As defined in Smets and Wouters (2002), labour and non-labour inputs are complementary. The production function is:

$$Y_t = A_tN_t^\eta\left[\prod_{i=1}^n Y_t^{\theta_i\delta_i}\right]\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

[1] See (Edwards,1986).

where A_t denotes state of technology, N_t is the level of employment and θ_i is the elasticity of output with respect to input i and 0<∞<1 . The equation in reduced form is written as:

$$Y_t = A'_t N_t^{\alpha} \quad \ldots\ldots(14)$$

The productivity shock is of a first order autoregressive process written as: $A_t = \rho_A A_{t-1} + \varepsilon_t^p$. The total real cost is:

$$TC_t = \frac{W_t Y_t^{\frac{1}{\alpha}}}{A_t^{\frac{1}{\alpha}} P_t} + \frac{P_{it}}{P_t} Y_t^{\delta_i} \ldots\ldots(15)$$

where P is the aggregate price level, P_{it} is the price of foreign intermediate input i and W_t denotes the nominal wages. If p_{it} is the real price of non-labour input, the real marginal cost can be written as:

$$MC_t = \frac{W_t Y_t^{\frac{1-\alpha}{\alpha}}}{\alpha \quad A_t^{\frac{1}{\alpha}} P_t} + \sum_{i=1}^{n} \delta_i \, p_{it} Y_t^{\delta_i - 1} \quad \ldots\ldots(16)$$

where MC_t represents the marginal cost and $\frac{W_t N_t}{P_t Y_t}$ represents the labour share in output.

After linearization and substitution, the real marginal cost is :

$$\widehat{mc}_t = \upsilon_a \hat{y}_t - \upsilon_b \hat{y}_{t-1} - \upsilon_c \hat{m}_t + \upsilon_d \hat{s}_t - \upsilon_e \widehat{rer}_t + \upsilon_f \widehat{rer}_{t-1} - \upsilon_g \hat{y}_t^f + \upsilon_h \hat{y}_{t-1}^f + \upsilon_i \hat{p}_{it} - \upsilon_j \hat{a}_t \ldots\ldots(17)$$

Following Galí and Gertler (1999, 2005), we use a hybrid New Keynesian Phillips curve of the following form:

$$\hat{\pi}_t = \gamma_f E_t \hat{\pi}_{t+1} + \gamma_b \hat{\pi}_{t-1} + \lambda \widehat{mc}_t + \varepsilon_t^{infl} \quad \ldots\ldots(18)$$

where the inflation disturbance is assumed to follow an AR(1) process: $\varepsilon_t^{infl} = \rho_f \varepsilon_{t-1}^{infl} + \mu_t^f$ and where:

$$\gamma_f = \beta\theta\left[\theta + \omega\left(1 - \theta(1-\beta)\right)\right]^{-1}; \quad \lambda = (1-\theta)(1-\beta\theta)(1-\omega)\xi$$

$$\gamma_b = \omega\left[\theta + \omega\left(1 - \theta(1-\beta)\right)\right]^{-1}; \;\; \xi = \frac{(1-\alpha)}{1+\alpha(1-\varepsilon)} \beta\theta\left[\theta + \omega\left(1 - \theta(1-\beta)\right)\right]^{-1}$$

The parameters θ is the price stickiness, ω is the price indexation and ε is the goods' elasticity of substitution.

3.3. Exchange Rate and External Debt

Findings have shown that external debt depends on oil price fluctuations in oil exporting countries.[1] Similarly, studies have also shown that exchange rate fluctuations depend on oil price movements.[2] In line with this, we assume $\hat{r}_t^d = \hat{r}_t^f - \omega_q \hat{q}_t + \omega_d \hat{d}_t^*$. This assumes that a positive shock to the foreign risk-free interest rate increases the country's interest rate spread and the cost of borrowing. This is similar to

[1] See (Swaray, 2005).
[2] See (Bodart et al., 2012).

the findings by Uribe & Yue (2006). Positive shocks to oil prices, however, reduce the spread and cost of borrowing for commodity exporting countries.[1]

We combine Eqs.(5) and (6) to yield:

$$s_{t+1}(1 + r_t^d) = s_t(1 + r_t) \ldots\ldots\ldots\ldots\ldots\ldots(19)$$

We linearise Eq. (19) and substitute $r_t^d = r_t^f - \omega_q \hat{q}_t + \omega_d \hat{d}_t^*$ The UIP can be expressed as follows:

$$\hat{s}_t = E_t \hat{s}_{t+1} - (\hat{r}_t - \hat{r}_t^f) - \omega_q \hat{q}_t + \omega_d \hat{d}_t^* + \varepsilon_t^{er} \ldots\ldots\ldots\ldots\ldots\ldots(20)$$

where $\hat{s}_t$ represents the nominal exchange rate, $\hat{r}_t$represents the domestic interest rate, $\hat{r}_t^f$ is the foreign interest rate, $(\hat{r}_t - \hat{r}_t^f)$ is the risk premium, $\hat{q}_t$ is the oil price and $\hat{d}_t^*$ is external debt to GDP. The innovation is assumed to follow an AR(1) process with an IID-Normal error term: $\varepsilon_t^{er} = \rho_d \varepsilon_{t-1}^{er} + \mu_t^d$. The parameter ω_q is oil price-exchange rate elasticity and ω_d is the external debt-exchange rate elasticity. Eq.(20) suggests an inverse relation between exchange rate fluctuations and oil prices. This indicates that increase in oil prices lead to fall (appreciation) the exchange rate.[2] Similar to Devereux and Lane (2003), Eq.(20) suggests a positive link between exchange rate and external debt to GDP ratio. This shows that high level of external debt to GDP increases (depreciates) the exchange rates. Similar to García and González (2013), the equation also reveals a negative link between risk premium and nominal exchange rate.

Current account deficit and a rising foreign debt service payments increase the external debt position of developing countries. Hence, the ratio of external debt to GDP changes according to the following equation:

$$\frac{\Delta D_t^*}{P_t Y_t} = \frac{Z_t - X_t}{Y_t} + (1 + r_t^d) d_{t-1}^* \ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

where d_t^* is the external debt to GDP ratio, $\frac{Z_t - X_t}{Y_t}$ is the net import to output ratio and r_t^d represents the interest rate on foreign debt. The growth in debt can then be written a

$$\Delta d_t^* = (z_t - x_t) + r_{t-1}^d (1 + \Delta y_t) d_{t-1}^* - \Delta y_t d_t^* \ldots\ldots\ldots\ldots\ldots\ldots(22)$$

Eq.(22) shows the change in external debt overtime. External debt increases as net import and foreign interest rate rise. For instance, an increase in net import i.e when import is greater than export leads to a rise in the level of debt. Also, a rise in interest rate not only results in an increase in the debt service payment but also increases the level of debt. Change in debt is negatively related to change in output, indicating that when output expands, debt declines.

Linearizing Eq.(22) and substituting Eq. (10), we derive the debt equation. Assuming $r_t^d = \hat{r}_t^f - \omega_q \hat{q}_t + \omega_d \hat{d}_t^* \}$, the debt equation is thus written as:

$$\hat{d}_t^* = \beta_a \hat{d}_{t-1}^* + \beta_b \hat{r}_t^f - \beta_c \hat{y}_t - \beta_d \Delta \hat{y}_t - \beta_e \hat{q}_t - \beta_f \widehat{rer}_t - \beta_g \hat{y}_t^f + \varepsilon_t^e \ldots\ldots\ldots\ldots(23)$$

[1] See (Senhadji, 2003).
[2] See (Bodart et al., 2012).

Eq. (23) describes the external debt evolution. It shows an inverse relation between external debt and oil prices. Furthermore, external debt to GDP depends negatively on domestic output and positively on foreign output, foreign interest rate and the real exchange rate.

3.4. Monetary Policy

We equate(4) and (6) to derive the money market equation. This is written as:

$$\frac{\phi}{1-\sigma}\left(\frac{C_t}{C_{t-1}^h}\right)^{1-\sigma}\left(1+\frac{\varrho}{S_t}\right)^{\phi}\left(\frac{M_t}{P_t}\right)^{\phi-1}\frac{1}{P_t}=\frac{\lambda_t}{P_t}\left(1+\frac{\varrho}{S_t}\right)-\frac{\lambda_t}{P_t(1+r_t)}\left(1+\frac{\varrho}{S_t}\right) \ldots\ldots.(24)$$

We linearize Eq.(24) and substitute $\hat{c}_t=\frac{1}{\gamma_c}(1+\gamma_y)\hat{y}_t-\frac{\gamma_r}{\gamma_c}\widehat{rer}_t-\frac{\gamma_{yf}}{\gamma_c}\hat{y}_t^f$ and $x_t-z_t=\gamma_r\widehat{rer}_t-\gamma_y\hat{y}_t+\gamma_{yf}\hat{y}_t^f$. This yields the money market equation written as:

$$r_t=\eta_a\hat{y}_t-\eta_b\widehat{m}_t+\eta_c\hat{s}_t-\varrho E_t\hat{s}_{t+1}-\eta_d\widehat{rer}_t-\eta_e\hat{y}_t^f+\varepsilon_t^b \ldots\ldots\ldots(25)$$

Eq. (25) is the money market equation indicating that interest rate is a positive function of real output and negative function of real balances. The interest rate also depends positively on current exchange rate and negatively on expected future exchange rate. This is in line with findings by Sánchez (2007). Our money market equation differs by indicating that the domestic interest rate is inversely related to the real exchange rate and foreign output.

The monetary authority policy reaction function is approximated with monetary aggregate targeting. Given the influence of oil price shocks on monetary aggregate in oil exporting countries[1], we introduce oil price in the monetary policy reaction function. Our specification is similar to one in Muhanji and Ojah (2011). However, in a way different from Muhanji and Ojah (2011), we also include the real exchange rate in the reaction function. The Taylor-type rule is given as:

$$\widehat{m}_t=\rho_m\widehat{m}_{t-1}+(1-\rho_m)\left(\rho_\pi\hat{\pi}_t+\rho_y\hat{y}_t+\rho_{rer}\widehat{rer}_t+\rho_q\hat{q}_t\right)+\varepsilon_t^m \ldots\ldots..(26)$$

where $\widehat{m}_t$, $\hat{\pi}_t$, $\hat{y}_t$, $\widehat{rer}_t$, and $\hat{q}_t$ represent the monetary aggregate gap, inflation gap, output gap, real exchange rate gap, and oil price gap respectively. The monetary disturbance is an AR(1) process: $\varepsilon_t^m=\rho_c\varepsilon_{t-1}^r+\mu_t^c$ .The policy smoothing rate is represented by ρ_m; policy response to inflation gap by ρ_π; policy response to output gap by ρ_y; policy response to real exchange rate shocks by ρ_{rer}; and policy response to oil price shocks by ρ_q. The model structural shock processes are represented by the following vector:

$$\hat{\xi}_t=\rho_\xi\hat{\xi}_{t-1}+\varepsilon_{\xi,t};\quad \varepsilon_{\xi,t}\sim N\left(0,\sigma_\xi^2\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(27)$$

4. Data and Estimation

4.1. Data Source

Data were obtained from the Central Bank of Nigeria (CBN), IFS, and World Bank databases. We use quarterly time series data on twelve macroeconomic variables for the period 1990:1-2008:4. The variables

[1] See (Raju & Melo, 2003).

include the consumer price index (CPI), interest rate, industrial output, real money balances, external debt to GDP, oil price, nominal exchange rate, real exchange rate, foreign interest rate, foreign output, foreign inflation and foreign inputs price. The foreign interest rate, foreign output, foreign inflation, and foreign input prices are proxied, LIBOR, US real GDP, US consumer price index and US producer price index for manufactured goods. The data were taken from Federal Reserve Bank of St. Louis. We deflate the nominal oil price by the US CPI to derive the real oil price. Due to the non-reliable quarterly data for the GDP, we employ industrial output.

4.2. Prior Distribution of the Parameters

Following the standard convention in the Bayesian literature[1], we form prior distributions and minimize the posterior distributions of the model parameters. In line with Smets and Wouters (2003), the persistence of the AR(1) processes is assumed to be beta distributed with mean 0.5 and standard deviation 0.2. Along the same line, the standard errors of the shocks are assumed to be inverse-gamma distributed with a mean of 0.1 and two degrees of freedom.

As in Smets and Wouters (2003, 2007), the habit parameter h is a beta distribution with mean 0.7 and standard deviation 0.1. Following Castelnuovo (2012), the price stickiness θ is a beta distribution with mean 0.65 and standard error 0.1 while the price indexation ω is a beta distribution with mean 0.5 and standard error 0.15. Also, the interest elasticity of money holding is assumed to be a normal distribution with mean 0.80 and standard error 0.1. The parameter for currency substitution is taken from Elkhafif (2002) and is assumed to be a beta distribution with mean 0.32 and standard error 0.14.

The Central bank of Nigeria (CBN) policy reaction function follows the Taylor's rule. The long run policy reaction to output and inflation are assumed to be a normal distribution with mean 0.12 and 1.5 and standard error 0.05 and 0.25 respectively. The monetary smoothing parameter is a beta distribution with a mean 0.75 and standard error 0.1. Lastly, the policy reaction function to oil price shocks and real exchange rate is a beta distribution with mean 0.5 and standard error 0.1 each. Some parameters are calibrated for the study. The calibration comes from McCallum and Nelson (2000) and Castelnuovo (2012). The model calibration is summarized in Table 1:

Table 1. Model calibration

Param	β	ε	γ_c	γ_y	γ_{ls}	γ_r	γ_{yf}	γ_m	ψ
Value	0.99	6.00	0.58	0.66	0.5	0.66	0.25	0.12	2.00

4.3. Posterior Estimates of the Parameter

Table 2 and 3 present the prior distributions and posterior distributions of the parameters and shocks with their mode, mean and standard deviation. Starting from the estimates of the behavioural parameters (Table 2), the mean of the posterior distribution is relatively close to the mean of the prior assumption. The external habit formation h is estimated to be about 70 percent of past consumption. The parameter for foreign currency holding is estimated to be 0.33. This is similar to estimate by Elkhafif (2002) for South Africa. Moreover, the parameter of price stickiness is estimated to be 0.71 indicating that prices are fixed for roughly three quarters on average. The coefficient of price indexation ω is estimated to be 0.38. This

[1] See (Smets & Wouters, 2003; Araújo, 2015).

suggests price setters are significantly forward looking. This indicates that inflation dynamics in Nigeria is dominated by forward looking behaviour. This is consistent with the findings by Olofin et al. (2014) for Nigerian economy.

The estimates for the CBN reaction function are in line with the principle proposed by Taylor rule. The mean of the long run reaction coefficient to inflation is estimated to be 1.5. This indicates an aggressive long-run reaction of the CBN to inflation. There is a high degree of policy smoothing by the monetary authority, as the coefficient on lagged monetary aggregate is estimated to be 0.68. We find that the monetary authority is moderately sensitive to output gap. Moreover, the coefficient on real exchange rate indicates that the monetary authority react strongly to exchange rate deviation. This is similar to the findings by García and González (2013) for commodity exporting countries. Lastly, the estimate show that monetary policy does not appear to react strongly to oil price shocks.

Table 2. Prior and posterior estimate of the structural parameter

			Prior	Posterior	Posterior
Param.	Definition	Distr.	$Mean$ $(Std.error)$	$Mode$ $(Std.error)$	$Mean$ $(5\%, 95\%)$
h	External habit formation	Beta	0.70 (0.10)	0.69 (0.00)	0.70 (0.69,0.70)
ϕ	Int. rate elast. of money demand	Normal	0.20 (0.05)	0.00 (0.02)	0.00 (0.00,0.00)
σ	Relative risk aversion	Normal	1.50 (0.38)	1.02 (0.01)	1.01 (1.00,1.01)
α	Capital-output share	Beta	0.33 (0.10)	0.33 (0.00)	0.33 (0.32,0.33)
ϱ	Foreign currency holding	Beta	0.30 (0.02)	0.33 (0.00)	0.33 (0.32,0.33)
θ	Price stickiness	Beta	0.65 (0.10)	0.71 (0.00)	0.71 (0.70,0.71)
ω	Price indexation	Beta	0.50 (0.15)	0.42 (0.01)	0.38 0.36,0.40)
ω_d	External debt-exchange rate elast.	Beta	0.20 (0.15)	0.00 (0.01)	0.00 (0.00,0.01)
ω_q	Oil price-exchange rate elasticity	Beta	0.50 (0.15)	0.40 (0.01)	0.40 (0.40,0.41)
ρ_y	Policy response to output	Normal	0.12 (0.05)	0.09 (0.00)	0.08 (0.07,0.08)
ρ_π	Policy response to inflation	Normal	1.50 (0.13)	1.49 (0.00)	1.50 (1.49,1.52)
ρ_{mag}	Policy rate smoothing	Beta	0.75 (0.10)	0.70 (0.00)	0.68 (0.67,0.69)
ρ_{rer}	Policy response to real exch. rate	Beta	0.50 (0.10)	0.79 (0.01)	0.77 (0.76,0.78)
ρ_q	Policy response to oil price shocks	Beta	0.50 (0.10)	0.24 (0.00)	0.25 (0.25,0.26)

Table 3 presents the distributions of structural shocks. The standard errors of the structural shocks show that most parameters are significantly different from zero. Foreign interest rate and foreign input price are the most persistent with an AR(1) coefficient of 1.00 each. This is followed by the real exchange rate, productivity, domestic interest rate, and the nominal exchange rate shocks with an AR(1) coefficient of 0.85, 0.84, 0.83, and 0.72 respectively. The high persistence of these shocks implies that in the long horizon, variance of the real variables will be explained by these shocks. Foreign output shocks have lowest persistence with mean 0.15.

Table 3. Prior and posterior estimate of the structural shocks

Param.	Definition	Distr.	Prior *Mean* (*Std. error*)	Posterior *Mode* (*Std. error*)	Posterior *Mean* (5%, 95%)
ρ_a	Domestic interest rate shocks	Beta	0.50 (0.20)	0.79 (0.02)	0.83 (0.82,0.85)
ρ_b	Money supply shocks	Beat	0.50 (0.20)	0.65 (0.01)	0.66 (0.64,0.00)
ρ_c	Nominal exchange rate shocks	Beta	0.50 (0.20)	0.73 (0.00)	0.72 (0.71,0.73)
ρ_d	External debt shocks	Beta	0.50 (0.20)	0.64 (0.00)	0.63 (0.63,0.64)
ρ_e	Productivity shocks	Beta	0.50 (0.20)	0.83 (0.02)	0.84 (0.83,0.84)
ρ_f	Oil price shocks	Beta	0.50 (0.20)	0.30 (0.01)	0.30 (0.29,0.30)
ρ_g	Foreign interest rate shocks	Beta	0.50 (0.20)	1.00 (0.01)	1.00 0.99,1.00)
ρ_h	Foreign output shocks	Beta	0.50 (0.20)	0.17 (0.02)	0.15 (0.14,0.16)
ρ_i	Foreign input price shocks	Beta	0.50 (0.20)	0.99 (0.01)	1.00 (0.99,1.00)
ρ_j	Foreign inflation shocks	Beta	0.50 (0.20)	0.63 (0.01)	0.60 (0.57,0.62)
ρ_k	Real exchange rate shocks	Beta	0.50 (0.20)	0.86 (0.01)	0.85 (0.83,0.86)

4.4. Forecast Error Variance Decomposition

Table 4 presents the forecast error variance decomposition of output and inflation at various periods. The 4th quarter shows that output variations are driven by foreign inflation, foreign or external debt, domestic interest rate, nominal and real exchange rate shocks. In the 8th quarter, fluctuations in output are influenced by external debt, nominal exchange rate, foreign inflation, money supply and domestic interest rate shocks. External debt, money supply, nominal exchange rate, foreign inflation, and the domestic interest rate explain variations in output in the 16th quarter.

The variance decomposition of inflation indicates that productivity, money supply, nominal exchange rate, foreign inflation, foreign interest rate, and domestic interest rate shocks are the drivers of inflation in the 4th quarter. In the 8th quarter, the variation in inflation is explained by money supply, productivity, foreign inflation, oil price, and the domestic interest rate shocks. In the 16th quarter, inflation is driven by money supply, productivity, domestic interest rate, foreign input price and nominal exchange rate shocks.

Table 4. Variance decomposition of output and inflation

Variable	Quar.	r_t	m_t	s_t	d_t^*	a_t	q_t	r_t^f	y_t^f	p_{it}	π_t^f	rer_t
Output	4	8.61	3.05	6.95	8.79	1.96	3.64	0.54	1.39	2.84	9.41	4.73
Output	8	6.47	6.81	8.71	22.22	1.70	3.59	0.99	1.60	2.31	6.94	3.44
Output	16	6.90	10.12	8.60	21.12	1.58	4.50	1.15	2.35	2.11	6.97	4.08
Inflation	4	4.29	8.78	6.15	1.20	67.03	1.76	5.40	0.53	0.53	1.96	1.74
Inflation	8	4.29	35.46	4.55	2.48	28.90	5.61	3.20	3.50	2.97	6.73	1.12
Inflation	16	6.93	35.80	5.54	4.26	21.04	4.28	2.96	3.14	6.50	5.51	3.04

Where r_t is the domestic interest rate; m_t is the money supply; s_t is the NER; d_t^* is the ratio of external debt to the GDP; a_t is the productivity shock; q_t is the oil price shock; r_t^f is the foreign interest rate shock; y_t^f is the foreign output shock; p_{it} is foreign input price and π_t^f is foreign inflation.

4.5. Impulse Response Function

Figure 1 shows the reaction of output to different shocks. Positive domestic interest rate shocks lead to a fall in output while positive shocks to money supply increase output.[1] Following positive exchange rate shocks (depreciation), output declines suggesting that depreciation is contractionary.[2] Shocks to external debt lead to a decline in output indicating debt overhang.[3] Expectedly, output increases following positive productivity shocks. Similarly, oil price shocks increase output. Shocks to foreign output, foreign input price, and foreign inflation lead to a rise in output suggesting a trade channel.[4]

Figure 2 illustrates the reaction of inflation to various shocks. A positive shock to the domestic interest rate leads to a fall in inflation rate. An increase in money supply initially results in a decline in output but out later rises. Similarly, positive shocks to nominal exchange rate, external debt, oil price, foreign interest rate, and foreign output initially on impact reduces inflation rate but rapidly recovers to increase the inflation rate. Positive productivity shocks reduce the inflation rate while positive foreign input price shocks increase the rate of inflation. Positive shocks to foreign inflation the real exchange rate (depreciation) lead to a rise in inflation.

[1] See (Mallick & Sousa, 2012).
[2] See (Bahmani-Oskooee & Miteza, 2006).
[3] See (Sen et al., 2007).
[4] See (Canova, 2005).

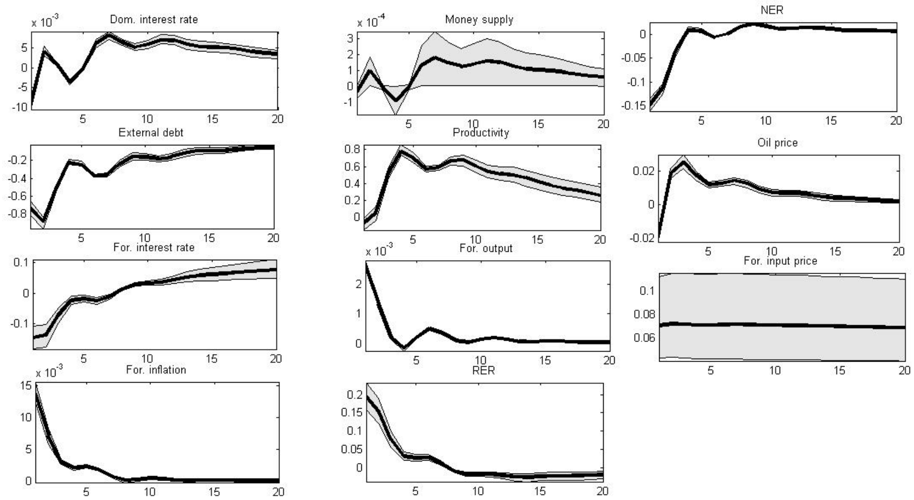

Figure 1. Impulse response functions of output to different shocks

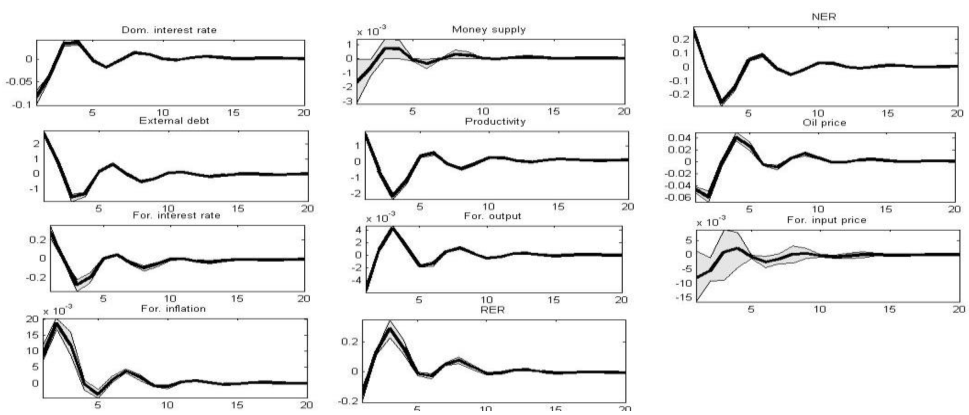

Figure 2. Impulse response functions of inflation to different shocks

4.6. Historical Decomposition of Output and Inflation

Fig. 3 illustrates the historical decomposition of output in Nigeria. We start from fig. 3 and mention the shocks in order of importance. External debt and productivity shocks appear to be the most vital shocks influencing output. Other important shocks that influence output include money supply, domestic interest rate, and nominal exchange rate. Shocks such as foreign input price, foreign output, foreign inflation, and oil price shocks have mild impact on variations in output. The limited influence of movements in oil price shocks on output is similar to the findings by Iwayemi and Fowowe (2011).

Fig. 4, on the other hand, shows the decomposition of inflation in Nigeria. The most important shocks driving inflation in Nigeria are productivity, money supply, oil price and nominal exchange rate shocks. Other shocks that drive inflation include domestic interest rate, foreign output, foreign input price, and foreign inflation shocks.

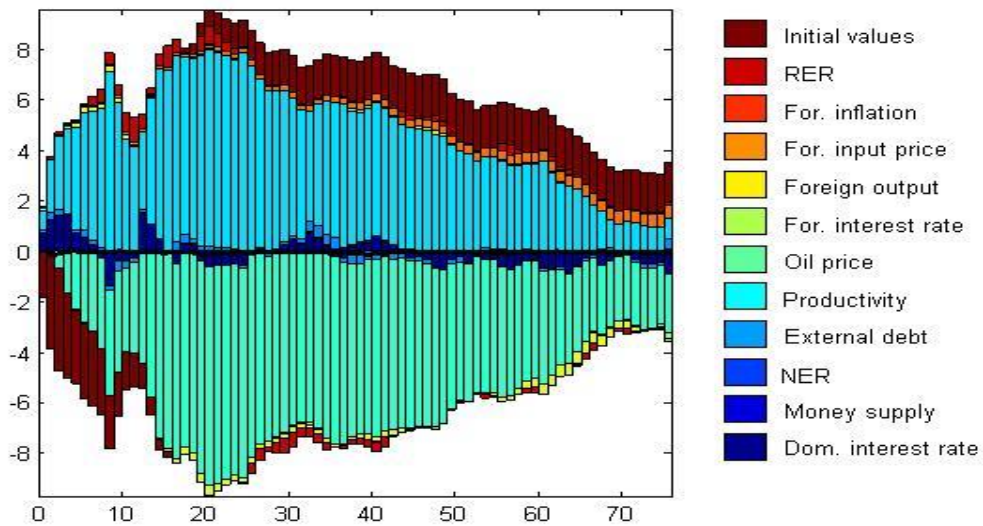

Figure 3. Historical Decomposition of Output

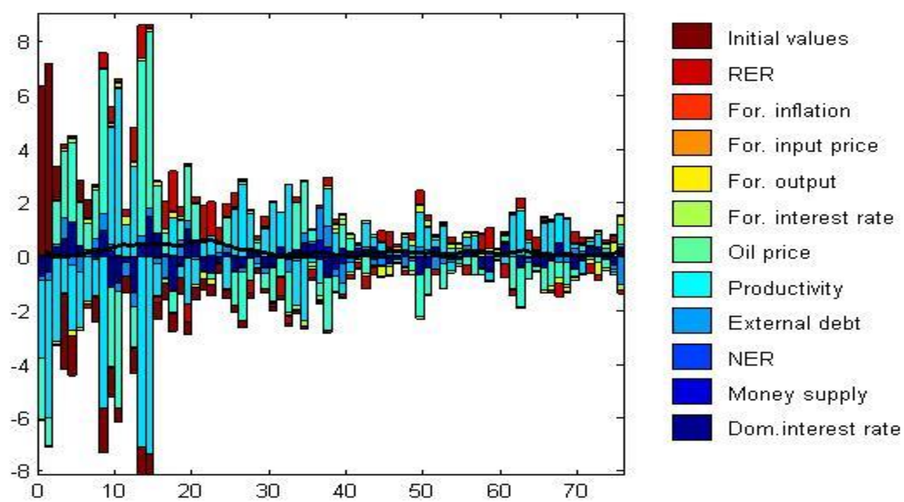

Figure 4. Historical Decomposition of Inflation

5. Conclusion and Policy Recommendation

In this paper, we formulate and estimate a small-open economy monetary DSGE model for the Nigerian economy. Our model shares the essential features of the class of monetary DSGE model such as Smets and Wouters (2003, 2007) such as sticky but forward-looking price setting and habit formation in consumption. The model includes a number of rigidities and shocks. The model parameters are estimated with Bayesian technique.

The findings suggest that there is a substantial degree of price stickiness in Nigeria. Hence, prices move slowly in response to changes in the expected marginal costs. To some extent, price depends on past inflation that introduces a backward component. Yet, the forward-looking component dominates in price setting equation. This is line with findings by Olofin et at. (2014) for Nigerian economy. In addition, there seems to be a high degree of habit formation in consumption. Moreover, we find evidence of foreign currency holding in Nigeria.

The results for structural shocks suggest that external shocks such as external debt, nominal exchange rate and foreign inflation coupled with internal shocks such as money supply and domestic interest rate account for significant output variations in Nigeria. Moreover, the estimates indicate that inflation in Nigeria is driven by money supply, productivity, domestic interest rate, foreign input prices, and nominal exchange rate shocks. Furthermore, our results suggest that monetary policy has been conducted in the manner of Taylor rule. Also, the results indicate that the monetary authority respond strongly to real exchange rate deviations. This implies that the monetary authority attempts to offset the effects of fluctuations in oil prices on the real exchange rate as it is common in commodity exporting countries.[1]

The policy implication of the results is that the CBN should incorporate model with frictions and rigidities in their monetary policy analysis. Moreover, monetary policy should be forward looking and respond to current shocks. In addition, the monetary authority should also adopt appropriate exchange rate policy to offset the effects of exchange rate shocks on the economy. This could be in the form of leaning against the wind to smooth exchange rate fluctuations. Purely market-determined exchange rate system may not be optimal for the Nigerian economy.

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[1] See (García & González, 2013).

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