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Article

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The Complex Typology of the Relationship between GDP and Its Sources[1]

*Jana KOTĚŠOVCOVÁ – Jiří MIHOLA – Petr WAWROSZ**

Abstract

The article describes a complex typology of relations between GDP and its sources: Total factor productivity (TFP) and Total Input Factor (TIF). We analyse how possible changes in TFP and TIF affect GDP development. We give each situation a specific name that clearly explains it. Based on the analysis, the so-called dynamic parameters of intensity and extensity are introduced. The parameters quantify the share of the change in intensive and extensive factors in GDP change. The article further compares our typology with previous ones and discusses the advantages and disadvantages of the chosen parameters.

Keywords: *GDP, economic development, total input factor, total factor productivity, dynamic parameters of intensity and extensity, economic growth*

JEL Classification: O11, O33, O41

Introduction

One of the key topics of economic theory and practice is the question of how to extend the production of goods, and thus satisfy higher needs. It is evident that production can be increased in two different ways: either extensively or intensively. The solely extensive method involves increasing the quantity of inputs with zero technical progress (zero intensive factors). Here, in accordance with Solow (1957), we regard technical progress as meaning not only new product and technological innovations but also better organisation of work, better knowledge, better allocation of resources, optimisation of international exchange, etc. Technical change thus includes all factors that allow the existing

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[1] The paper was created while solving the student project *Zdokonalení penzijního systému jako intenzifikační faktor ekonomiky* which uses the purpose-built support for specific university research of the University of Finance and Administration.

quantity of inputs to produce more outputs. [2] Solely intensive production growth is the state where the quantity of output increases with no change in the quantity of inputs.[3] In reality, solely extensive or solely intensive developments are almost impossible. A rising quantity of inputs involves changes in their organisation, productivity, etc. This means that input growth is connected with the presence of intensive factors. From this perspective, it is worth trying to quantify the shares of extensive and intensive factors. In addition, production growth is not the only real situation. During a recession, production declines or stagnates. Here it is also useful to examine how the individual factors (extensive or intensive) contribute to the given development. An interesting (albeit probably hypothetical) situation may occur where the quantity of inputs decreases, but the economy maintains the same production quantity. In this case, it is evident that the extensive decline must be counterbalanced by intensive growth.

Attempts to clearly quantify extensive and intensive factors were made in the 1950s and 1960s and resulted in a growth accounting equation, which, however, involves numerous limitations (some details are mentioned in the Section 3). Hence the article presents an alternative method[4] for measuring the impact of intensive and extensive factors on economic development. However, it is not enough just to quantify intensive and extensive factors using the appropriate mathematical instruments; it has become clear that it is also worth analysing all the possible relationships between the development of extensive and intensive factors on the one hand and the development of GDP (output) on the other. An appropriate terminology (nomenclature) must be subsequently assigned to the individual situations. Therefore, the article initially (in Section 1) deals with basic relationships among GDP, total factor productivity and total input factors. We assign appropriate names to the basic developments and generalise the task to apply also to mixed developments (Section 2). Then we discuss the dynamic parameters of intensity and extensity (Section 3), which measure the impact of changes in intensive and extensive factors on GDP change. We show how the values of parameters correspond to our typology and we briefly discuss

[2] Solow (1957, p. 312) states: “It will be seen that I am using the phrase ‘technical change’ as a short-hand expression for any kind of shift in the production function. Thus slowdowns, speed-ups, improvements in the education of the labour force, and all sorts of things will appear as ‘technical change’.”

[3] The terms ’extensive’ and ’intensive’ are normally used in literature in this meaning. See for example, Jurečka et al. (2010) in Czech or The Economist (2013) in English.

[4] The article is based on previously published works, such as Mihola (2007a; 2007b), Hájek and Mihola (2009), Cyhelský, Mihola and Wawrosz (2012), Mihola and Wawrosz (2013; 2014; 2015). The article offers for the first time a comprehensive overview of the relationship between GDP on the one hand and inputs and technological progress on the other (see Sections 1 and 2). The dynamic intensity and extensity parameters introduced in the previous publications are newly aligned with system of the relationships between GDP and its sources. We further newly discuss advantages and disadvantages of the parameters.

the advantages of the parameters in comparison with growth accounting. For balance, we also specify some disadvantages of the parameters. Section 4 compares our nomenclature to that proposed by M. Toms. The conclusion summarises the key findings.

1. Measuring Development Intensity Based on a Complex Development Typology

The initial relationship of our analysis consists of the expression of gross domestic product (GDP, *Y*) as a product of the total factor productivity *TFP* times the total input factor *TIF*:

$$Y = TFP \,.\, TIF \tag{1}$$

The formula comes from the basic relationship that sees output as the product of inputs and their efficiency (see e.g. Froeb and Ward, 2015, for details). GDP growth, decline or stagnation may be attributable to a change in only one of these variables, with the other variables unchanged, or both variables having an effect. In that event, the effects may also counteract each other, which may even result in a full compensation of the impact of their changes, if one variable rises and the other falls in such a way that the GDP does not change. A change in *TIF* is related to a change in the amounts of inputs, i.e. to a quantitative or extensive change, and a change in *TFP* is related to a qualitative or intensive change.

As we deem it useful to separate the analysis of the development of intensity and extensity using dynamic parameters from the analysis of the substitution of individual sub-factors,[5] we will initially define[6] the total input factor *TIF* as a weighted geometric aggregation of two primary factors of production:[7] labour *L* and capital[8] *K*, which is a Cobb-Douglas type aggregation.[9]

$$TIF = K^{\alpha} \,.\, L^{(1-\alpha)} \tag{2}$$

To be able to quantify the impact of a change of *TIF*, i.e. *TPF*, on a GDP change, expression (1) needs to be dynamised:[10]

[5] In our case, this particularly includes the substitution of technology by labour, with this being predominant in the historical context.

[6] See Mihola and Wawrosz (2014, p. 587) expression (4) or Cyhelský, Mihola and Wawrosz (2012, p. 38) expression (26).

[7] For a comprehensive study of a multiplicative production function with factors of labour, capital and technical progress, see, for example, Barro and Sala-I-Martin (1999, p. 29), which includes the Cobb-Douglas production function $Y = A \,.\, K^{\alpha} \,.\, L^{(1-\alpha)}$.

[8] The domains of definition for all quantities used result from the domains of definition for labour and capital $L > 0$ and $K > 0$.

[9] This function was originally published in the American Economic Review, see Cobb and Douglas (1928, pp. 139 to 165).

$$I(Y) = I(TFP) \,.\, I(TIF) \tag{3}$$

It may be useful to express the relationships between changes in extensive and intensive factors and GDP changes graphically, using a chart with coordinates $I(TIF)$ on the x axis and $I(TFP)$ on the y axis – see Figure 1. This figure also includes the GDP index isoquants, i.e. $I(Y)$ – these represent all change values of $I(TFP)$ and $I(TIF)$ that lead to the same value $I(Y)$ – the values of $I(Y)$ in the figure are 0.5; 1; 2 and 3).[11] These isoquants can be expressed as follows:

$$I(TFP) = I(Y) \,/\, I(TIF) \tag{4}$$

Expression (4), i.e. Figure 1, shows that the isoquants of constant GDP development $I(Y)$ are equilateral hyperbolas with variable curvature and with constant elasticity equal to 1. The GDP stagnation hyperbola, which intersects the coordinate origin [1; 1], has special importance. All isoquants above it represent GDP growth,[12] while all below it represent GDP decline.

Figure 1 can also express the basic types of relationships between the development of extensive and intensive factors on the one hand and the GDP development on the other:

1. Pure developments, which lie on the coordinate axes of Chart 1. The GDP only grows or declines because of one of the considered factors, either purely extensively or purely intensively. The other remains unchanged, i.e. $I(TIF) = 1$ (for a purely intensive change) or $I(TFP) = 1$ (for a purely extensive change).

2. Balanced developments, where both factors considered have the same effect, i.e. $I(TIF) = I(TFP)$. These developments are expressed in quadrants I and III on a straight line at an angle of 45 degrees.

3. Compensatory developments, where both factors fully compensate each other into GDP stagnation, i.e. $I(Y) = 1$, and thus $I(TFP) = 1/I(TIF)$. These developments lie on the hyperbolic isoquant of GDP stagnation (see above).

If we elaborate on the individual developments, the following is true:

- Pure growth and pure decline can be distinguished for pure developments. For purely extensive development, where $I(TFP) = 1$, purely extensive growth ($I(TIF) > 1$) is indicated by the positive ray of the x axis, while purely extensive

[10] The change index (coefficient) of any variable A is defined as $I(A) = A_n/A_{n\text{-}1}$.

[11] The scope of indices for both factors ($I(TFP)$ and $I(TIF)$) is selected in Figure 1 within the range of (0; 2), i.e. from the decline in GDP towards zero growth (stagnation) and to growth to a doubling. Further text is based on Figure 1, although it is also possible to work with the representation using linear isoquants, which can be obtained when logarithmic coordinates are used.

[12] For example, the isoquant with value 2 in Figure 1 denotes all combinations $I(TIF)$ and $I(TFP)$ that double the output.

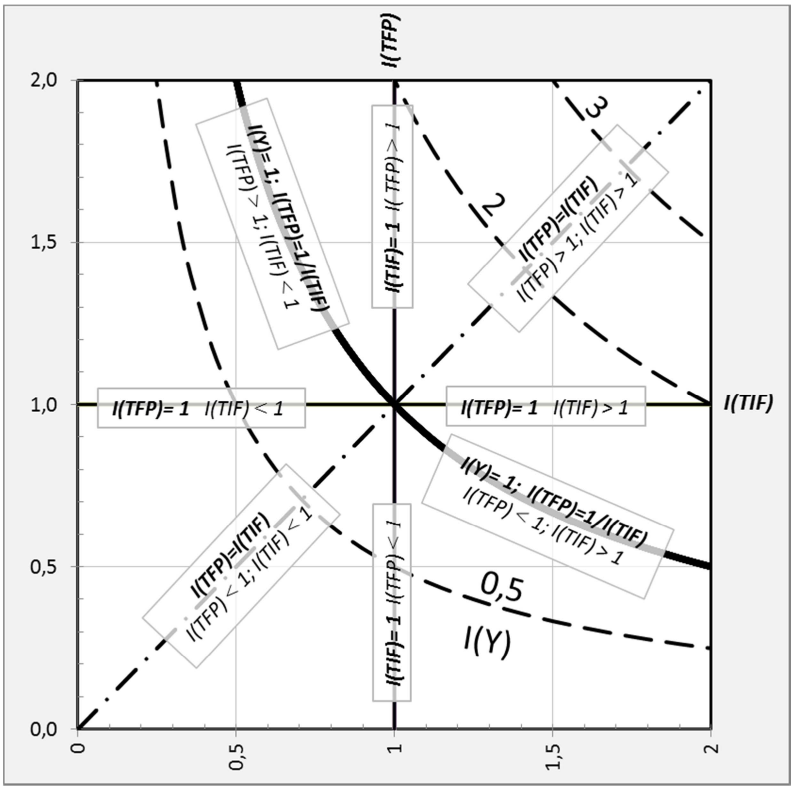
decline ($I(TIF) < 1$) is indicated by the negative ray of the x axis. Then, for purely intensive development ($I(TIF) = 1$), by analogy: purely intensive growth ($I(TFP) > 1$) is indicated by the positive ray of the y axis, while purely intensive decline ($I(TFP) < 1$) is indicated by the negative ray of the y axis.

- For balanced developments ($I(TFP) = I(TIF)$), intensive-extensive growth ($I(Y) > 1$) is indicated by the positive section of the straight line at an angle of 45 degrees, intersecting the origin of the coordinate axes (i.e. the section in quadrant I), while intensive-extensive decline ($I(Y) < 1$) is indicated by the negative section of the straight line at an angle of 45 degrees, intersecting the origin of the coordinate axes (i.e. the section in quadrant III).

- For compensatory development ($I(Y) = 1$, thus $I(TFP) = 1/I(TIF)$), we can distinguish intensive-extensive compensation – the upper half of the stagnation hyperbola, where $I(TFP) > 1$ and $I(TIF) < 1$, or extensive-intensive compensation – the lower half of the stagnation hyperbola, where $I(TFP) < 1$ and $I(TIF) > 1$.

Figure 1

Space Showing and Describing the Basic Types of Development *I*(*Y*), *I*(*TIF*) and *I*(*TFP*)



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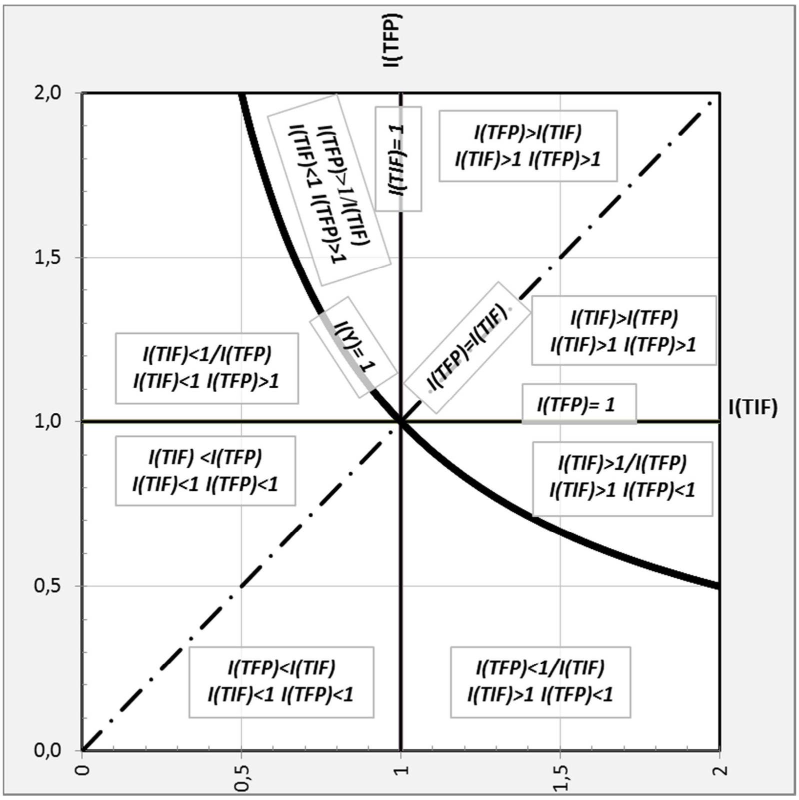
2. Comprehensive Analysis of Relationships among TIF, TPF and Y

As stated in the introduction, the basic types of development are in reality almost impossible. It is not very likely that a) GDP would grow or decline purely intensively or purely extensively; b) that both factors (*I*(*TIF*) and *I*(*TFP*)) would have the same effect on GDP growth or decline; and c) there would be a situation where the GDP does not change at all (*I*(*Y*) = 1) because of the compensatory effects of both factors.

Therefore, attention needs to be paid to mixed types of development. This includes all other situations. Graphically, in the figures we use, such situations lie outside the coordinate axes, the 45-degree straight lines in quadrants I and III, intersecting the origin of the coordinate axes, and outside the stagnation hyperbolic isoquant. This applies to 8 spaces, which can always be characterised by three inequations, which also determine whether GDP grows or does not grow, i.e. *I*(*Y*) > 1 or *I*(*Y*) < 1.

F i g u r e 2

Representation of Basic and Combined Types of Developments



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The first of the three inequations determines the relationship between $I(TFP)$ and $I(TIF)$ or (for compensatory developments) between one quantity and an inverted value of the other. The second inequation determines whether the *TIF* rises or falls, i.e. $I(TIF) > 1$ or $I(TIF) < 1$. The third inequation determines whether the *TFP* rises or falls, i.e. $I(TFP) > 1$ or $I(TFP) < 1$. Thus, for example, the space in which $I(Y) > 1$ and at the same time $I(TFP) > 1/I(TIF)$, with $I(TFP) > 1$ and $I(TIF) < 1$, represents mixed developments shown in the place between the positive direction of the y axis and the upper section of the stagnation hyperbola. With these developments, the GDP grows although the *TIF* declines. This means that the rise in *TFP* not only compensates the decline in *TIF* but also causes the GDP to grow. The relationships between $I(TFP)$ and $I(TIF)$ for all basic and combined types of developments are shown in Figure 2 and Table 1.

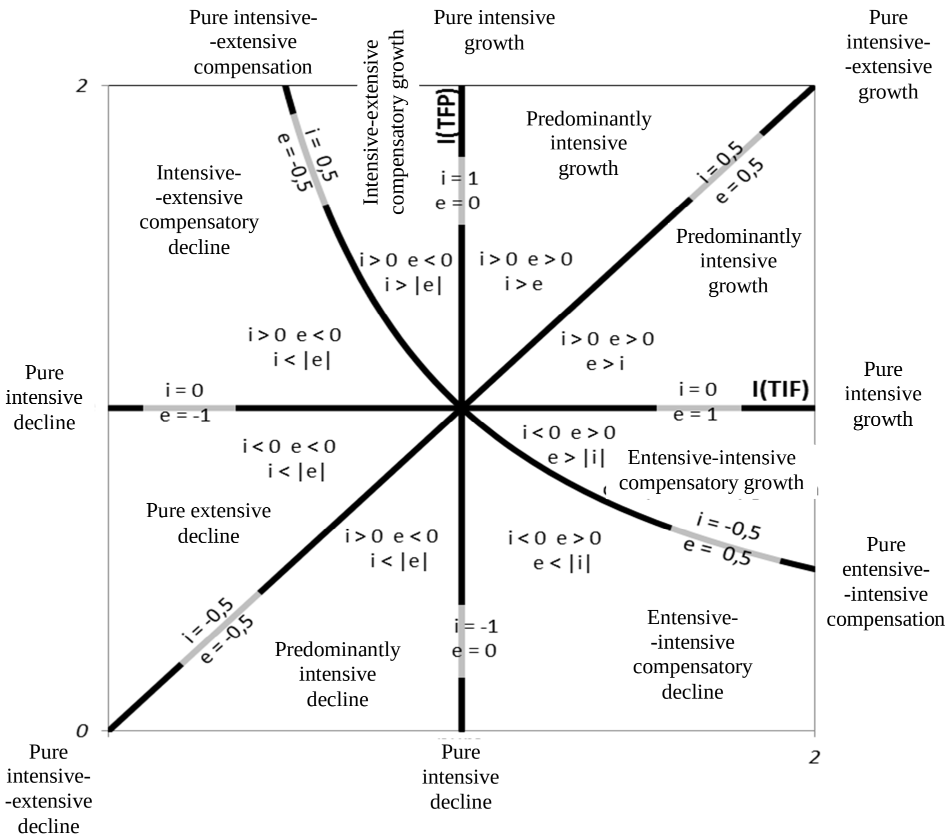
It would be useful to assign names to the individual spaces shown in Figure 2 that, if possible, most accurately express what happens in reality. Therefore, we present a comprehensive nomenclature for all basic and combined developments. We follow the principles below, applied consistently:

- The nomenclature must cover all types of development.
- If the GDP grows, we use the word *growth*; if it declines, we use the word *decline*; if it remains unchanged, we use the words *pure compensation*.
- In the case of basic developments we use the word *pure* (e.g. pure extensive growth, pure intensive-extensive compensation).
- When referring to matched developments, i.e. those where both factors drive growth or both factors drive decline in GDP, but they do so unequally, we use the word *predominantly*, and use the name of the predominant factor. This means that predominantly intensive growth indicates a situation where both factors ($I(TIF)$ and $I(TFP)$) drive growth, but the impact of intensive factors is greater than that of extensive factors. Likewise, predominantly extensive decline depicts a situation where both factors ($I(TIF)$ and $I(TFP)$) drive decline, but the impact of extensive factors is greater than that of intensive factors.
- When referring to opposite developments where one factor drives growth and the other drives decline in GDP, we use the word *compensation* or *compensatory*.
- If the names of combined compensatory developments or pure compensatory developments concurrently include the words *intensive* and *extensive*, the first word used is the one which drives growth, followed by the one that drives decline. For instance, the term intensive-extensive compensatory growth refers to a situation where intensive factors grow so rapidly that they partly compensate a decline in extensive factors, thus making the GDP grow in the end – see the situation above, where $I(Y) > 1$ and at the same time $I(TFP) > 1/I(TIF)$, with $I(TIF) < 1$ and at the same time $I(TFP) > 1$. Similarly, an intensive-extensive

compensatory decline indicates a situation where intensive factors grow while extensive factors decline, making the GDP decline in the end. In this logic, a pure intensive-extensive compensation indicates a situation where intensive factors drive growth and extensive factors concurrently drive decline, making the GDP stagnate in the end. The names of all of the basic and combined developments are specified in Figure 3.

F i g u r e 3

Complex Typology of All Developments of I(TFP), I(TIF) and I(Y)



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3. Typology and Dynamic Parameters of Intensity and Extensity, Their Advantages and Disadvantages

Our typology describes how a change in *TFP* or *TIF* contributes to GDP change for all possible changes in GDP, *TPF* and *TIF*. If we want to calculate the exact impact of any change in *TPF* or *TIF* on GDP change, it is possible to use the so-called dynamic parameters of intensity and extensity. The first expresses the share in the impact of a change in intensive factors ($I(TFP)$) for all the above-mentioned types of development:

$$i = \frac{\ln I(TFP)}{\left|\ln I\left(TFP\right)\right| + \left|\ln I(TIF)\right|} \tag{5}$$

By analogy, the dynamic parameter of extensity expresses the share of the impact of extensive factors ($I(TIF)$) for all types of development:

$$e = \frac{\ln I(TIF)}{\left|\ln I\left(TFP\right)\right| + \left|\ln I(TIF)\right|} \tag{6}$$

Details of the parameters are described for example in Mihola and Wawrosz (2015). The parameters are generally designed to ensure that their values are consistent with the presented typology of GDP, *TFP* and *TIF* developments. Specifically, if a change in any factor contributes to production growth, the relevant parameter should be positive (e.g. if a change in intensive factors contributes to growth, the dynamic parameter of intensity is positive), whereas if it leads to a decline in output, the parameter value is negative. If the given factor remains unchanged, the relevant parameter is equal to zero. The sum of the absolute values of both parameters gives 1 in each situation.

$$|i| + |e| = 1 \tag{7}$$

The assignment of both parameters' values to individual developments[13] is specified in Figure 3, while more detailed characteristics of individual situations are specified in Table 1, in which names according to Figure 3 are also assigned to the situations.

Our paper discusses the same problem as growth accounting, i.e. it strives to quantify the impacts of individual factors. The key methodical difference in our methodology is that dynamic parameters are based on a simple multiplicative link (1) and at the next level on the weighted multiplicative link of labour and capital indices (2), whereas the primary approach to growth accounting is based on the additive weighted aggregation of labour and capital.[14]

$$Y = MPP_k \,.\, K + MPP_l \,.\, L \tag{8}$$

where MPP_k is the marginal product of capital and MPP_l is the marginal product of labour.

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[13] Our overview does not include a very specific situation of total stagnation, where the GDP stagnation $I(Y) = 1$ is due to the stagnation of both factors $I(TFP) = I(TIF) = 1$.

[14] See, for example Solow (1956; 1957), Swan (1956), Kendrick (1961), Denison (1962), Jorgenson and Griliches (1967), Barro (1999), Vacková (2012).

T a b l e 1

Overview of Individual Types of Development *I*(*TIF*) and *I*(*TFP*) and Values of Dynamic Parameters of Intensity and Extensity

	Change of extensive factors (*I*(*TIF*))	**Change of intensive factors (*I*(*TFP*))**	**Change of output (*I*(*Y*))**	**Values of intensity (*i*) and extensity (*e*)**	**Type of development**
1.	growth, ($I(TIF) > 1$)	unchanged, ($I(TFP) = 1$)	growth, ($I(Y) > 1$)	$e = 1$; $i = 0$	pure extensive growth
2.	unchanged, ($I(TIF) = 1$)	growth, ($I(TFP) > 1$)	growth, ($I(Y) > 1$)	$e = 0$; $i = 1$	pure intensive growth
3.	same growth as intensive ones, ($I(TIF) > 1$, $I(TIF) = I(TFP)$)	same growth as extensive ones, ($I(TFP) > 1$, $I(TFP) = I(TIF)$)	growth, ($I(Y) > 1$)	$e = 0.5$; $i = 0.5$	pure intensive-extensive growth
4.	faster growth than intensive ones, ($I(TIF) > 1$), $I(TIF) > I(TFP)$)	slower growth than extensive ones, ($I(TFP) > 1$, $I(TFP) < I(TIF)$)	growth, ($I(Y) > 1$)	$e > 0$; $i > 0$; $e > i$	predominantly extensive growth
5.	slower growth than intensive ones, ($I(TIF) > 1$), $I(TIF) < I(TFP)$)	faster growth than extensive ones, ($I(TFP) > 1$, $I(TFP) > I(TIF)$)	growth, ($I(Y) > 1$)	$e > 0$; $i > 0$;$i > e$	predominantly intensive growth
6.	greater than inverted value of intensive ones, ($I(TIF) > 1$), $I(TIF) > 1/I(TFP)$)	greater than inverted value of extensive ones, ($I(TFP) < 1$, $I(TFP) > 1/I(TIF)$)	growth, ($I(Y) > 1$)	$e > 0$; $i < 0$; $e > \|i\|$	extensive-intensive compensatory growth
7.	greater than inverted value of intensive ones, ($I(TIF) < 1$), $I(TIF) > 1/I(TFP)$)	greater than inverted value of extensive ones, ($I(TFP) > 1$, $I(TFP) > 1/I(TIF)$)	growth, ($I(Y) > 1$)	$e < 0$; $i > 0$; $i > \|e\|$	intensive-extensive compensatory growth
8.	equal to inverted value of intensive ones, ($I(TIF) > 1$), $I(TIF) = 1/I(TFP)$)	equal to inverted value of extensive ones, ($I(TFP) < 1$, $I(TFP) = 1/I(TIF)$)	no change (stagnation), ($I(Y) = 1$)	$e = 0.5$; $i = –0.5$	pure extensive-intensive compensation
9.	equal to inverted value of intensive ones, ($I(TIF) < 1$), $I(TIF) = 1/I(TFP)$)	equal to inverted value of extensive ones, ($I(TFP) > 1$, $I(TFP) = 1/I(TIF)$)	no change (stagnation), ($I(Y) = 1$)	$e = –0.5$; $i = 0.5$	pure intensive-extensive compensation
10.	less than inverted value of intensive ones, ($I(TIF) < 1$), $I(TIF) < 1/I(TFP)$)	less than inverted value of extensive ones, ($I(TFP) > 1$, $I(TFP) < 1/I(TIF)$)	decline, ($I(Y) < 1$)	$e < 0$; $i > 0$; $i < \|e\|$	intensive-extensive compensatory decline
11.	less than inverted value of intensive ones, ($I(TIF) > 1$), $I(TIF) < 1/I(TFP)$)	less than inverted value of extensive ones, ($I(TFP) < 1$, $I(TFP) < 1/I(TIF)$)	decline, ($I(Y) < 1$)	$e > 0$; $i < 0$; $e < \|i\|$	extensive-intensive compensatory decline
12.	faster decline than intensive ones, ($I(TIF) < 1$), $I(TIF) < I(TFP)$)	slower decline than extensive ones, ($I(TFP) < 1$, $I(TFP) > I(TIF)$)	decline, ($I(Y) < 1$)	$e < 0$; $i < 0$; $\|e\| > \|i\|$	predominantly extensive decline
13.	slower decline than intensive ones, ($I(TIF) < 1$), $I(TIF) > I(TFP)$)	faster decline than extensive ones, ($I(TFP) < 1$), $I(TFP) < I(TIF)$)	decline, ($I(Y) < 1$)	$e < 0$; $i < 0$; $\|i\| > \|e\|$	predominantly intensive decline
14.	same decline as intensive ones, ($I(TIF) < 1$), $I(TIF) = I(TFP)$)	same decline as extensive ones, ($I(TFP) < 1$), $I(TFP) = I(TIF)$)	decline, ($I(Y) < 1$)	$e = –0.5$; $i = –0.5$	pure intensive-extensive decline
15.	declining, ($I(TIF) < 1$),	unchanged, ($I(TFP) = 1$)	decline, ($I(Y) < 1$)	$e = –1$; $i = 0$	pure extensive decline
16.	unchanged, ($I(TIF) = 1$)	declining, ($I(TFP) < 1$)	decline, ($I(Y) < 1$)	$e = 0$; $i = –1$	pure intensive decline

Source: Created by the authors.

The methodology of growth accounting is not reasonably able to quantify situations when either GDP or labour or capital decline. Generally, it does not work with complex typologies of all types of developments. If analysts using growth accounting want to express the share in the impact of intensive factors i_f, they divide the TFP growth rate by the rate of GDP growth[15] $G(Y)$. For the share of the impact of intensive factors, we will thus obtain the expression

$$i_f = \frac{\mathrm{G}(TFP)}{\mathrm{G}(Y)} \tag{9}$$

A disadvantage of expression (9) is that its values are not always meaningful for all possible types of development. Expressed in percentages, expression (9) delivers values of $-\infty$ to $+\infty$. With our presented methodology (i.e. the dynamic parameter of intensity), the values according to expression (9) are only consistent for purely intensive growth and purely extensive growth or approximately for very small positive growth rates within 1%. Much the same results would also be obtained for quadrant I, where growth takes place because of both considered factors. In quadrants other than I [0 to 90°], the values of expression (9) and of the dynamic parameter of intensity differ greatly. For purely intensive-extensive growth or decline (i.e. where both *TFP* and *TIF* equally grow or decline), expression (9) delivers values different from 50%, with the deviation rising with the rising rate (see Appendix Table A1). In addition, where one factor drives growth and the other equally drives decline, expression (9) delivers different results for the individual rates of growth or decline (see Appendix Table A2). Dynamic parameters of intensity and extensity describe the impact of TIF and TFP change more clearly than growth accounting. Therefore the parameters can be an appropriate addition or alternative to growth accounting.

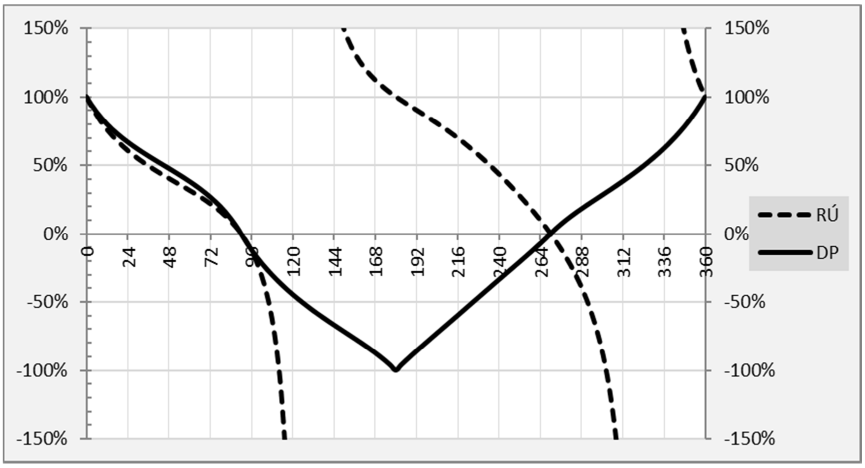
Some possible disadvantages of dynamic parameters must be emphasised for balance. First, their values depend on input data. Our approach uses as the input data only the growth rates of labour, capital and GDP. The analysis looks at labour and capital as a homogeneous factor and it does not consider other features such as education, skill, quality of capital goods and so on. However, identical value of labour or capital change can result in different GDP development. Let us imagine a situation when one country experiences growth of the educated labour force (e. g. people with university education) and a second experiences growth of unskilled persons. Although the labour growth rates are equal in both countries, the GDP growth rates differ. Similarly, if one country introduces modern technically progressive capital goods and another increases the number of obsolete ones, the change in their GDPs will be probably different even when

[15] This method is used for example in Helísek (2002) and Baran (2013).

both changes in capital are identical. It is reasonable to expect that a change containing a qualitative character will result in a higher *TFP* change and thus a higher value of the dynamic intensity parameter.

Figure 4

Comparison of Expressions of Development Intensity Using Expressions (5) and (9)



Note: RÚ = growth accounting values according to expression (11), DP = values of dynamic parameter of intensity according to expression (5).[16]

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Furthermore, the interpretation of the values of the dynamic intensity or extensity parameters can be misleading, especially in the case of sudden demand or supply shocks. The yearly values of both parameters are usually affected by a shock. For instance, in the case of a negative demand shock, output decreases, but the amount of inputs does not usually decrease at the same rate. The input decline is usually lower or inputs can even stagnate or grow, especially at the beginning of the shock when their development is not affected by the shock. The dynamic parameter of intensity is negative in such a case. But the country does not experience real technological regression. It is reasonable for firms not to reduce the amount of inputs at the same rate as output decline. If the negative shock is temporary, it makes sense to maintain the inputs and avoid costs connected with input reduction and subsequent input increase. In contrast, when the demand shock ends, inputs and output usually grow but the input change is lower than the output change. The value of the dynamic intensity parameter is positive but this does not indicate real technological progress. Firms have only started to use the inputs that had not been reduced during the shock.

[16] Quadrant assignments: 0 to 90: quadrant I (top right), 90 to 180: quadrant II (top left), 180 to 270: quadrant III (bottom left), 270 to 360: quadrant IV (bottom right).

Negative supply shock due to sudden price increases of inputs (e.g. oil) can cause misinterpretation, too. With this shock, the change in inputs is usually higher than the change in output. The value of inputs usually grows; the value of the output grows smaller, stagnates or even declines. The result is a negative value for the dynamic intensity parameter which, however, again does not mean technological regression. The economy is not able to respond to the shock appropriately in the short run. Generally, yearly values of dynamic intensity and extensity parameters express what happens on the aggregate level. Their negative values can be seen as a sign of economic problems, but the essence of the problem must be further investigated. It is not possible to conclude without other research that yearly negative values indicate real technological regression or real decline of inputs. A yearly negative value of the dynamic extensive parameters can be further caused by a change in the depreciation methodology or by the fact that new capital goods cost less than the removed ones.

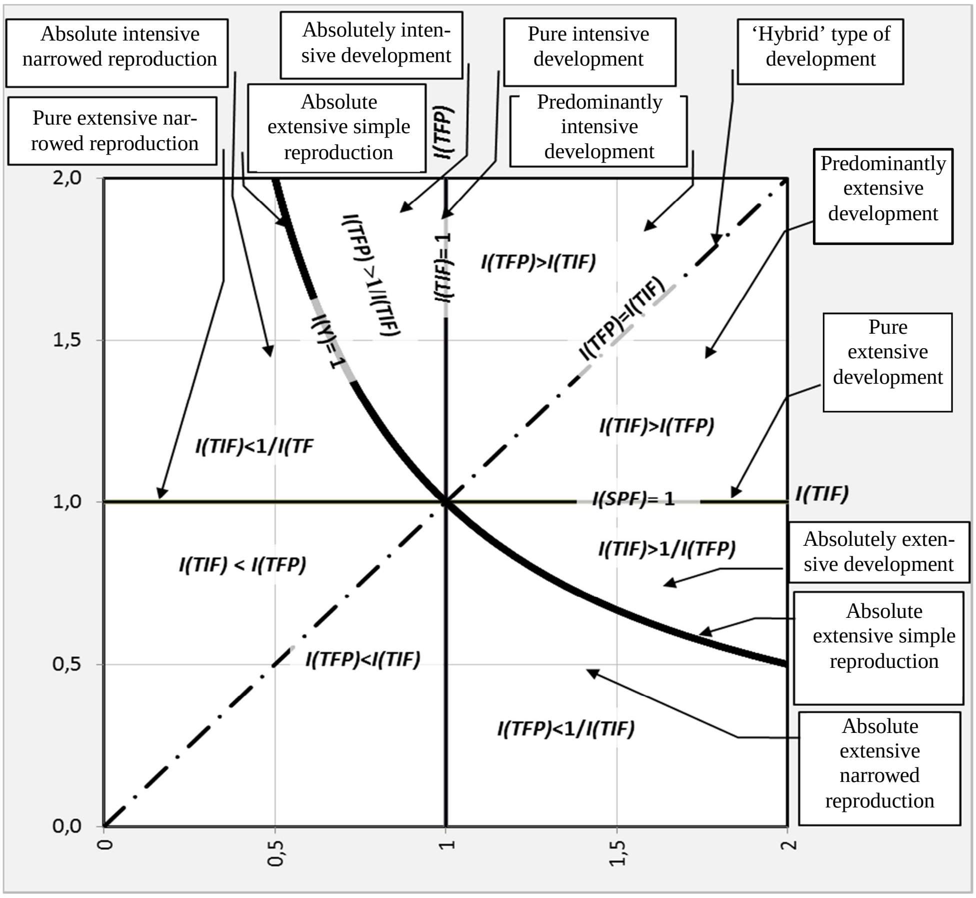
Extraordinary yearly positive values of both parameters must also be carefully analysed, as they often describe a situation where an economy is improving from previous negative development. The positive values thus balance what happened in the past. The value of the parameters can also be misleading if all values ($I(TIF)$, $I(TFP)$ $I(Y)$) are close to 1, so describing a slight change. The small difference in their values in such situation can cause an extraordinary value for any dynamic parameter – e.g. the value of i is 98% and the value of e is 2%. A major technological change seems to be happening, but it does not ensue. The long-run values of both parameters over, for example, a 10-year period describe technological progress or regression more precisely. Long-run development is not affected by temporary shocks, as it contains higher aggregate values of $I(TIF)$, $I(TFP)$ and $I(Y)$ and it is possible to analyse whether GDP development is really based on intensive or extensive factors

4. Comparison of Our Proposed Typology to that of M. Toms

How original is our typology? Extensive or intensive development is used as a standard term, but is usually restricted to growth rather than to decline or compensation. A relatively complex typology of developments was presented by Miroslav Toms, who summarised his historic digression on this topic in Toms (1988, pp. 74 – 83), where he proposed a typology included in Figure 5. This typology is easily comparable to ours for the additional reason that the author uses a similar system of relationships to define the individual types of development. Apart from the fact that Toms uses *development* rather than *growth*, we find that we use the same designations in only four instances.

F i g u r e 5

Typology of Developments Proposed by Miroslav Toms



Source: Toms (1988), created by the authors.

These include *purely intensive* or *purely extensive development* and *predominantly intensive* or *predominantly extensive development*. Toms' typology differs from ours in the following aspects:

- Toms does not consistently use the terms *growth* and *decline*, and prefers *narrowed* and *simple reproduction* instead. We believe that this is related to capacity expansions of the economy rather than directly to GDP growth.
- Toms uses the term *simple reproduction* where we use the term *pure compensation*.
- He refers to instances of *compensatory* developments as *absolute*, which somewhat non-systematically indicates that one of the factors is predominating.
- He refers to growth with the same impact of both factors as "*hybrid*", which is a fairly apposite albeit not quite consistent designation.

• Toms' typology is not complete, because it fails to designate two basic and two combined developments related to GDP decline when the *TFP* falls.

Toms' typology documents that economic science has dealt with this topic in the past, although through one author only.[17] Toms' work was inspirational for our solution. Nevertheless, we believe that our typology is more complex and systematic, because it covers all possible relationships of TIF/TFP development on the one hand and GDP development on the other. As we show in Section 2, various (especially combined) relationships between TIF/TFP development and GDP development do really occur, and therefore ought to be named appropriately.[18]

Conclusion

The article presents a complex typology of relationships between the development of total factor productivity (*TFP*) and total input factor (*TIF*) on the one hand and GDP development on the other. This typology is based on a multiplicative link, where the GDP is defined as the product of *TFP* times *TIF*. The typology includes, inter alia, pure developments, where the GDP changes either only as a result of a *TFP* change or a *TIF* change, or the change of *TFP* is equal to *TIF*, i.e. with both factors equally counteracting each other, resulting in GDP stagnation. In addition, the article describes all of the mixed developments which make the GDP change (grow or decline) as a result of a current (growing or declining) change in *TFP* and *TIF*. This typology is used as the basis of the proposed dynamic parameters of intensity and extensity. These quantify the percentage share of the impact of intensive factors ($I(TFP)$) and extensive factors ($I(TIF)$) in GDP change. The values of these parameters range from –100% to 100% and are easy to interpret. The versatility of this concept is compared to growth accounting, which does not separate the measurements of development intensity from the problem of substitution of technology for labour; explicitly, it does not address the problem of expressing intensive and extensive impacts, and is not sufficiently accurate for higher growth rates (over approximately 5%). Moreover, matters are complicated by the necessity of working with the weights

[17] We have found no other complex typology of developments – in Czech or foreign bibliographies – on total factor productivity and on measuring the share of impact of intensive factors. In contrast, the terms intensive and extensive are used very often.

[18] The nomenclature presented in the text can also be used at the corporate (or sectoral) level. This is where the relationship between the company's inputs and their productivity (efficiency) on the one hand and the company's output on the other is also relevant for us. Inputs can be expressed as cost and outputs as income. Even at the corporate level, inputs and efficiency may unequally contribute to growth or decline, or mutually and unequally compensate each other. Even pure developments are theoretically possible at this level. For details see Kotěšovcová, Mihola and Wawrosz (2015).

of the impacts of growth sub-factors. In growth accounting, the expression of the shares of impacts of individual factors in GDP change as a share of rates of growth *G*(*TPF*) or *G*(*TIF*) and *G*(*Y*) is also problematic, as these shares are not standardised into an easily interpretable interval, with their values ranging from $-\infty$% to ∞%. Our methodology including dynamic parameters offers a logical indisputable system of all relationships between *I*(*TFP*), *I*(*TIF*) and *I*(*Y*), and assigns each situation a clear and meaningful title and values for both parameters. It can thus be seen as a possible alternative to growth accounting.

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Appendix

Table A1

Different Values of Expression (9) and the Dynamic Parameter of Intensity for Purely Intensive-extensive Growth or Decline (in %)

Purely intensive-extensive growth						**Purely intensive-extensive decline**				
G(SPF)	**G(SIF)**	**G(Y)**	**i_f**	**i**		**G(SPF)**	**G(SIF)**	**G(Y)**	**i_f**	**i**
1	1	2	50	50		–1	–1	–2	50	–50
5	5	10	49	50		–5	–5	–10	51	–50
10	10	21	48	50		–10	–10	–19	53	–50
15	15	32	47	50		–15	–15	–28	54	–50
20	20	44	45	50		–20	–20	–36	56	–50

Source: Created by the author.

Table A2

Different Values of Expression (9) and the Dynamic Parameter of Intensity for Purely Extensive-intensive Compensation or Purely Intensive-extensive Compensation (in %)

Purely extensive-intensive compensation						**Purely intensive-extensive compensation**				
G(SPF)	**G(SIF)**	**G(Y)**	**I_f**	**i**		**G(SPF)**	**G(SIF)**	**G(Y)**	**I_f**	**i**
1	–1	–0.01	–10,000	50		–1	1	–0.01	10,000	–50
5	–5	–0.25	–2,000	50		–5	5	–0.25	2,000	–50
10	–10	–1.00	–1,000	50		–10	10	–1.00	1,000	–50
15	–15	–2.25	–667	50		–15	15	–2.25	667	–50
20	–20	–4.00	–500	50		–20	20	–4.00	500	–50

Source: Created by the authors.