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Article

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Ekonomický časopis

Provided in Cooperation with:
Slovak Academy of Sciences, Bratislava

Reference: Káčer, Marek/Ochotnický, Pavol et. al. (2019). The Altman's revised Z'-score model, non-financial information and macroeconomic variables : case of Slovak SMEs. In: Ekonomický časopis 67 (4), S. 335 - 366.

This Version is available at:
http://hdl.handle.net/11159/4214

The Altman's Revised Z'-Score Model, Non-financial Information and Macroeconomic Variables: Case of Slovak SMEs[1]

Marek KÁČER – Pavol OCHOTNICKÝ** – Martin ALEXY***

Abstract

In this paper, we assess the classification performance of the re-estimated Altman's Z'-Score model for a large sample of private SMEs in Slovakia. More specifically, we assess transferability of the revised Z'-Score model (Altman, 1983) and explore the impact of the non-financial company-specific and macro-economic variables. The dataset covers the period from 2009 to 2016 and contains 661 622 company-year observations about 149 618 individual companies with 1 575 failures. The discriminatory power of models is tested in out-of-sample period. We find that even though the model with re-estimated coefficients achieves better discrimination performance, it is not statistically different from the revised Z'-Score model. The non-financial variables improve the discriminatory performance significantly, whereas the macroeconomic variables do not. The latter even worsen the out-of-sample and out-of-time discriminatory performance.

Keywords: *Altman's Z-Score model, failure prediction, default, non-financial information, macro-economic variables, Slovakia*

JEL Classification: G32, G33, C52

Introduction

The corporate failure prediction models are important tools for bankers, investors, creditors, rating agencies, and for companies themselves. Altman (1968) introduced the first multivariate default prediction model 50 years ago. He used

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[1] The paper is the outcome of the research project *Competitiveness, Economic Growth and Firm Survival* (APVV-15-0322) supported by Slovak Research and Development Agency.

linear discriminant analysis to construct Z-Score model. Since that time the model has been widely used and the multivariate approach has been adopted among researchers in finance, banking, and credit risk. One of the reasons why Altman's model is relatively widespread may be that this was the first of its kind. Next it has been actively promoted by its author. However, without the good performance of the model it would not have lasted for so long. The Z-Score model incorporates the main dimensions of financial health of companies and has become a prototype for many of the credit risk and default models.

Failure prediction models use predominantly financial indicators; however, several studies pioneered the utilisation of non-financial variables, too (Grunert, Norden and Weber, 2005; Altman, Sabato and Wilson, 2010). These predictors can be divided into two categories: individual company variables and macroeconomic variables. Several failure prediction models have been constructed for Slovak companies, as well. The models could gradually develop due to experience of market economy in the post-communist period and access to data. However, only a few of them included non-financial variables among determinants of the corporate default (Fidrmuc and Hainz, 2010; Wilson, Ochotnický and Káčer, 2016; Altman et al., 2017). Moreover, none of the studies test the models in the out-of-sample period.

In our study, we build ex ante default prediction models and use a sample of Slovak SMEs covering the period from 2009 to 2016 and comprising of 661 622 company-year observations with 1 575 defaulted companies. Given the inconclusive results of academic literature regarding the transferability of revised Altman's Z'-Score to Slovak corporate environment and lack of studies testing the impact of non-financial company-specific and macroeconomic variables to actual failure prediction we pose three research hypotheses; they are related to the transferability of the revised Z'-Score model (H1), the impact of non-financial company-specific information (H2) and macroeconomic environment (H3) on failure prediction models.

We contribute to the literature in several ways. Using a large recent dataset, we assess the usefulness of the revised Z'-Score model (Altman, 1983) in the Slovak corporate environment. The extant failure prediction studies use predominantly financial ratios and we add non-financial company-specific and macroeconomic variables in the models, too. The incremental discriminatory power of combined models is tested using a large out-of-time holdout sample.

The paper proceeds as follows. In section 1 we describe the development of the literature related to models of corporate default. Based on that we form three hypotheses. In section 2 we describe the variables used in the models and explain the rationale for their inclusion into the models for predicting the future financial performance. In section 3 we elucidate our methodology for assessing the performance of the models using the area under ROC curve and classification accuracy. In section 4 we write up and discuss our results and verify the research hypotheses. The last section concludes the study.

1. Literature Overview

The academic literature aimed at default prediction using financial ratios is rich; notable milestones are: Beaver (1966); Altman (1968); Ohlson (1980); Zmijewski (1984) and Shumway (2001). Beaver (1966) evaluated the evolution of several financial ratios up to five years before default and demonstrated clear differences between defaulting and non-defaulting companies. However, using the univariate analysis (i.e. one financial ratio at a time) is arguably subjective and ambiguous since different ratios may give contradictory information. Altman (1968) combined five carefully chosen financial ratios covering accounting and market data and came up with Altman's Z-Score. Its coefficients were estimated using linear discriminant analysis (LDA). Ohlson (1980) was the first who used logistic regression (logit) to estimate the model's coefficients and Zmijewski (1984) used probit. Shumway (2001) pointed to the fact that using cross-sectional data results in inconsistencies and using panel data (i.e. several observations for a company) is preferable.

Default-prediction models have been extensively studied since Altman (1968). However, in recent years, the introduction of non-financial information as predictor variables, such as size or sector, ownership and financing, have opened new research lines in relation to default prediction (e.g. Grunert, Norden and Weber, 2005; Altman and Sabato, 2007; Altman, Sabato and Wilson, 2010; Altman et al., 2017). Altman, Sabato and Wilson (2010) aimed to produce bankruptcy models specifically for SMEs in the UK in which both financial and non-financial variables are introduced. The incorporation of macroeconomic data adds further interest.

Most recently, Altman et al. (2017) conducted an extensive international performance verification of accounting-based Altman's Z''-Score model (Altman, 1983). This version of Z''-Score has been modified for prediction of financial health of private and public manufacturing and non-manufacturing firms and does not contain the last predictor (sales to total assets) which is apparently sensitive to the industry sector. Their sample covered 28 European countries and three non-European ones for 2007 to 2010 and comprised of nearly six million company-year observations. The dataset was split into the estimation and test parts. Slovakian companies were included in the sample as well. The authors had designated seven research hypotheses. The first two were related to the Z''-Score model itself – one assumed that the coefficients are obsolete and the model with re-estimated coefficients would perform better; the second one hypothesized that logit as an estimation technique with less restrictive assumptions would have better discrimination performance. This model (logit Z-Score) became the baseline model for testing other five hypotheses. It is important to mention that in this study the discrimination performance is measured using the area under the ROC curve (AUC hereafter). The first two hypotheses were not confirmed because in all the data samples the results of the re-estimated models were slightly worse than the original ones. The other five hypotheses were associated with specificity of time, impact of size, age, industry sector and country risk. These hypotheses were confirmed in all data samples because the models with experimental variables related to the respective hypotheses had significantly higher discriminatory performance compared with the baseline model. The authors concluded that the Z''-Score model performs reasonably well in an international context, however for individual countries it is better to estimate specific models, preferably with the additional (i.e. non-financial) variables.

1.1. Default Prediction Models Developed for Slovakia

Slovakia has experienced some profound changes in economic and enterprise structure in a relatively short time frame. Like most developed economies it now has a vibrant SME sector. Post-communist transition, especially the process of privatisation and restructuring in transition economies created corporate sector with a wide range of ownership structures and origins. The literature covering the failure prediction models developed specifically for Slovakia has been gradually growing. We are aware of several published peer-reviewed papers where the development of a default model for Slovakian companies is covered. The objectives of these studies were different, ranging from developing failure prediction models to proving diverse hypotheses using the failure prediction methodology as a tool for analysis. However, all of them have in common that they contributed to issue of the prediction of corporate failure in a Slovakian context.

Fidrmuc and Hainz (2010) investigated the performance of bank loans portfolio to about 700 small and medium sized enterprises (SMEs) in Slovakia in the period from 2000 to 2005. According to their results, the earnings before taxation, cash and bank accounts were significant determinants of loan default from among financial variables. The share of bank loans to total assets was a significant predictor as well, but its impact varied in time and diminished after including random effects. The results of the study further confirmed significant industry fixed effects for retail trade and partially for agriculture and construction sectors. From the viewpoint of legal form and liability, limited liability companies were slightly less risky than joint stock companies, however this relation reversed with increased loan amount. It is the first study we are aware of, which investigated the determinants of bank loan defaults. Even though they analysed bank loans defaults and not corporate defaults, these two categories are closely related.

Harumová and Janisová (2014) estimated a default prediction model for Slovakian companies using logistic regression. Their sample comprised of 53 206 company-year observations for small and medium sized companies based in the Košice and Prešov regions and covered the period from 2008 to 2011. The initial set of 20 financial ratios was eventually reduced to six after the use of the stepwise method of variables selection, and removing those with incorrect size. The reported model equation contained these predictors – accounts payable to sales; earnings before interest, tax, depreciation and amortisation to sales; earnings before tax to total assets; current ratio; sales to total assets; and total liabilities plus accruals and deferrals to depreciation, amortisation and earnings after tax.

Režňáková and Karas (2015) tested the transferability of modified Altman Z-Score model for Visegrad Group countries (Czech Republic, Slovakia, Poland and Hungary). Their sample contained nearly six thousand companies operating in manufacturing industry from 2007 to 2012, with slightly less than 500 firms from Slovakia. They tested three research hypotheses. The first one was related to predictive accuracy of the revised Altman Z-Score model, the second one involved the performance of the re-estimated model with all five Altman's variables for each country and the third one was related to the performance of the reduced models. They confirmed that the transferability of the original model is limited – it does not achieve the original level of predictive accuracy. On the other hand, the re-estimated models perform much better, in all four countries under investigation. The reducing of the original set of Altman's variables does not bring clear results. However, the reduction based on the backward elimination brought rather limited models, as was the case of Slovakia where there was just single predictor – EBIT to total assets.

Another study aimed at portability of Altman's Z-Score model was Boďa and Úradníček (2016). They verified empirically the prediction accuracy of the mixed Z-Score (original Z-Score developed by Altman (1968) with X4 modified as book value of equity to book value of total debt), revised Z'-Score (Altman, 1983) and Z-Score with coefficients re-estimated via the Altman's original procedure. They did so by using the sample comprising of 92 892 company-year observations covering the period from 2009 to 2013. The methodological difference from all the studies considered thus far lied in the definition of a bankrupted company. The authors used the definition of financial distress that combined negative equity, negative earnings after tax and current ratio lower than one.

Using one-year prediction horizon the authors estimated four cross-sectional models. The results are different from the former study (Režňáková and Karas, 2015), in that the revised Altman's Z-Score model achieved the best overall classification accuracy and the best classification accuracy for non-distressed companies. On the other hand, the re-estimated Z'-Score achieved the highest prediction accuracy for distressed companies. Also, the re-estimated models had the best discrimination performance measured by the area under ROC curve.

Wilson, Ochotnický and Káčer (2016) used a failure prediction model to demonstrate the specific features of the transition process in Slovakia – the effect of foreign ownership on corporate performance, the privatization trap and post-transformation recession. To this end they constructed several default models using financial and non-financial information. The sample covering small and medium-sized companies they used was to date the most extensive in terms of the individual companies (44 597), time period (1997 – 2012) and observations (126 649). It is important to mention that the model was predictive in that the dependent variable was the indicator of beginning legal default process in the next accounting period.

The set of relevant financial predictors differed depending on the estimation methodology (logit and Cox's proportional hazard model). In logit model they used cash to total assets, total liabilities to quick assets, trade creditors to total liabilities, retained profit to total assets and net worth to total liabilities whereas in Cox's proportional hazard model they report the quick ratio as significant instead of total liabilities to quick assets. In baseline models the important non-financial predictors covered indicators of modified audit report, company type, industry sector and specific time periods. The reported discrimination performance of logit models via AUC was 0,719 for the model using just financial variables and 0,765 when non-financial variables were included.

Gulka (2016) estimated a well performing failure prediction logit model for Slovak companies as part of his masters' dissertation. He used a balanced estimation sample with 844 observations in total from years 2012 – 2014. The final model contained cash ratio, sales to working capital, financial accounts to total assets, equity to total assets, bank loans to total assets, liabilities to state institutions to total assets and EBITDA to total assets. The model performed very well on a large validation sample containing over 120 thousand company-year observations with an average classification accuracy of about 80% and Type I error rate of about 15%.

Mihalovič (2016) built two models using logit and discriminant analysis with a relatively small sample of 118 defaulted companies matched with equal number of non-defaulted ones based on asset size and industry sector. The model was also a predictive one since the financial data from 2013 was used to predict the defaults that occurred in 2014. Five predictors out of an initial 18 were used in both models: current ratio, current liabilities to total assets, working capital to total assets, current assets to total assets and net income to total assets. Even though the discriminatory performance of the models measured by AUC was fair, both models had a shortcoming. As seems to be the case in other studies, the assumptions underlying the use of discriminant analysis were not fulfilled. And in logistic regression only two variables were statistically significant, albeit one just at 10% level.

Since the default is not an instant event but develops over time, Gavurová et al. (2017) aimed to answer the question whether variables representing (relative) change in time contribute to the performance of failure prediction models. They used two techniques to solve the problem of firms' classification into two categories of bankrupt and non-bankrupt groups – discriminant analysis and decision tree. To our knowledge, this study was the first one that attempted to build a failure prediction model using a more complicated classification framework (decision tree).

The authors compiled a sample comprising data for 1 182 companies, each having at least four time-series observations from 2009 to 2014. Out of 1 182 companies 277 were defaulted. The sample was split into a training and a validation set. The initial set of potential predictors was relatively large with 51 different financial ratios along with their trend counterparts. When using the discriminant analysis, the initial set was firstly screened for statistical significance using Wilk's lambda and secondly for correlation to assure that only variables with sufficient explanatory power and with least correlation enter the estimation. Then two models were estimated – a static and a dynamic one – and their in-sample discriminatory performance was assessed using one- and two-year prediction horizons. Both models for both predictive horizons achieved about 75% of correctly classified firms so the dynamic version was not better than the static one. The decision tree was built using the CHAID algorithm. The static version used just two variables (Loan Capital/Assets and Cash Flow/Loan Capital), whereas the dynamic version used four variables (Loan Capital/Assets, Cash Flow/Loan Capital, Assets/Equity and Financial Assets/Current Liabilities). The prediction accuracy of decision trees was about 85% and the dynamic model in a training sample was slightly better but interestingly, the dynamic version did not contain any trend variable yet used two additional static variables when compared with the static version. The performance of models built using both techniques was noticeably worse on validation sample, yet the more complicated version of the decision tree achieved about 20 percentage points higher accuracy than Altman's Z-Score or Index IN05 models.

T a b l e 1

Summary of the Published Papers for Slovak Economy

Study	Method	Estimation sample	Relevant explanatory variables
Fidrmuc and Hainz (2010)	probit	1 496 obs. 2000 – 2005	bank loans to total assets, earnings before taxation to total assets, cash and bank accounts to total assets, legal form dummy, industry sector dummy, time dummy
Harumová and Janisová (2014)	logit	53 206 obs. 2008 – 2011	*accounts payable to sales; earnings before interest, tax, depreciation and amortisation to sales; earnings before tax to total assets; current ratio; sales to total assets; and total liabilities to earnings after tax
Režňáková and Karas (2015)	LDA	498 obs. 2007 – 2012	EBIT to total assets
Boďa and Úradníček (2016)	LDA	92 892 obs. 2009 – 2013	*working capital to total assets, retained earnings to total assets, EBIT to total assets, net worth to total liabilities, sales to total assets
Wilson, Ochotnický and Káčer (2016)	logit, Cox's proportional hazard model	126 649 obs. 1997 – 2012	cash to total assets, total liabilities to quick assets, trade creditors to total liabilities, retained profit to total assets, net worth to total liabilities, quick ratio
Gulka (2016)	logit	844 obs. 2012 – 2014	cash ratio, sales to working capital, financial accounts to total assets, equity to total assets, bank loans to total assets, liabilities to state institutions to total assets and EBITDA to total assets
Mihalovič (2016)	LDA, logit	236 obs. 2013 – 2014	net income to total assets, current ratio, current liabilities to total assets, working capital to total assets
Altman et al. (2017)	logit	7 976 obs. 2007 – 2010	**working capital to total assets, retained earnings to total assets, EBIT to total assets, net worth to total liabilities
Gavurová et al. (2017)	LDA, decision trees	700 obs. 2009 – 2014	Loan Capital/Assets, Cash Flow/Loan Capital, Bank Loans/Assets, Assets/Equity, Financial Assets/Current Liabilities, Capital/Loan Capital, Inventory/Assets, Current Liabilities/Current Assets (level + trend), EAT/Costs, (Loan Capital – Current Financial Assets)/CF, Long-term Assets/Assets, Liabilities/Assets (level + trend), Assets/Equity, Current Assets/(Current Liabilities + Bank Loans), Turnover/Equity, EAT/Long-term Assets, Turnover/Assets (trend), Financial Costs/Liabilities (trend), Inventory/Turnover1*360 (trend), Long-term Assets/Long-term Liabilities (trend), Turnover/Inventory (trend), Turnover – Costs)/Turnover (trend)
Klieštik et al. (2017)	LDA	265 327 obs. 2012 – 2015	current ratio, cash ratio, return on assets, return on equity, debt to equity ratio, number of days payables and working capital to total assets

Notes: *Authors do not give hints about the statistical significance of variables in the model; **Specific models for Slovakia are not reported in the study

Source: Authors' elaboration.

Klieštik et al. (2017) built the default models in order to understand the changing legal environment in Slovakia. Importantly, their definition of default was similar to that used by Faltus (2015), i.e. the indicator of negative equity has been utilized. The estimation sample was relatively large and comprised of about 265 thousand company-year observations. The relaxed definition of 'default' resulted in about one quarter of those being defaulted. The initial set of predictors comprised of the 11 mostly used financial ratios identified in literature. The method of estimation was multivariate linear discriminant analysis, and four cross-sectional models were estimated for years 2012 to 2015. The set of relevant predictors differed across the models but the variables with the highest incidence included cash ratio, returns on assets, the debt to equity ratio and the working capital to total assets. The models were successful in identifying the non-bankrupt companies but much less so in determining the bankrupted ones (i.e. those with negative equity).

We have already mentioned the extensive study of Altman et al. (2017). Their analysis covered 28 countries and the estimation sample included nearly 8 000 observations of Slovak companies. When focusing on results related to Slovakia the first two hypotheses (H1: the coefficients of Z''-Score are obsolete, H2: logit will have better discrimination performance) were not confirmed; the results of the re-estimated models were slightly worse than the original one. The other five hypotheses were associated with specificity of time, impact of size, age, industry sector and country risk. The results for Slovakia were not so convincing – the noticeable differences were achieved just for models with size and for the complete models. i.e. with all additional variables, but the differences were not statistically significant.[2]

1.2. Development of Hypotheses

H1: Even though Altman's Z-Score models has been widely used in Slovakia (see examples in Boďa and Úradníček, 2016), their transferability was not tested much. We are interested in finding whether newly estimated models using the same variables as the revised Z'-Score model perform better. Even though the variables included in the revised Z'-Score model capture the essential dimensions of firms' financial health and performance, it is assumed that each country would have its own unique characteristics, e.g. liquidity management, investment decisions, capital structure and dividend policy. This could be reflected in different impact of predictors on the default probability, more specifically in the magnitudes and significance of the estimated coefficients. Režňáková and Karas (2015) found that the new estimated model performed better than the Z'-Score model, while Boďa and Úradníček (2016) did not find significant differences in

[2] The statistical significance is the function of sample size. In Altman et al. (2017), the absolute differences between AUC for respective models were similar in all data models and in the model specific for Slovakia. However, due to sample size these differences were statistically significant in all data samples while non-significant in Slovak model.

the models' performances. Altman et al. (2017) suggest that it is appropriate to estimate specific models for individual countries, even though they did not find significant differences in the models' performances in Slovakia and in most of 28 analysed countries. We test whether the logit model using variables of the revised Z'-Score model performs better than the original model. We use logistic regression in which assumptions are less restrictive to the data sample we use. At the same time our training sample is 2009 – 2014 and the test sample is 2015 – 2016. Altman and Hotchkiss (2006, p. 240) note that average financial ratios vary over time, therefore it is ideal that the training period directly precedes test period, which was not the case in any of above-mentioned studies. *Therefore, our first hypothesis (H1) is that the re-estimated version of Z'-Score will perform better than the original revised Z'-Score model.*

H2: Earlier studies concluded that non-financial company-specific variables contribute significant information to failure prediction. Grunert, Norden and Weber (2005) suggest that models combining both financial and non-financial factors result in more accurate predictions of default. Their sample consisted of 409 company-year observations and they used the bootstrap method to test the robustness of results. Altman, Sabato and Wilson (2010) found that adding non-financial firm-specific characteristics to a risk model makes a significant contribution to increasing the default prediction power. Their study focused on UK SMEs and their sample consisted of more than 5.8 million company-year observations. Fidrmuc and Hainz (2010); Wilson, Ochotnický and Káčer (2016) and Altman et al. (2017) all conclude that additional non-financial variables improve classification accuracy of models. None of these studies tested the usefulness of non-financial company-specific variables in validation samples (ex ante predictions) of Slovak companies. *That is why we design our second hypothesis (H2), which assumes that non-financial company-specific variables improve discriminatory performance and classification accuracy.*

H3: Variables related to the macroeconomic conditions are usually utilized in aggregate models explaining the average failure rates (Altman, 1983; Virolainen, 2004; Jakubík and Seidler, 2009). Since in any period these variables are effecting all individual companies equally and there is no cross-sectional variation, they have not been utilized very much in previous research. Wilson, Ochotnický and Káčer (2016) used interest rate in micro-econometric study related to Slovak companies and found it to be highly statistically significant. Similarly, Altman et al. (2017) used country rating rank as a measure of country risk and their results confirmed it was significant, too. However, we are not aware of any study in Slovakia, that used any proxies of the macroeconomic conditions in truly failure prediction models. More specifically none of the studies tested their significance in the out of time-frame of the training/estimation sample. *That is why, we design our third hypothesis (H3), which assumes that macroeconomic variables improve discriminatory performance and classification accuracy.*

2. Determinants of Corporate Default

The use of financial variables as predictors for default in developed economies has a long history. Usually the ratios used for default prediction are the measures of liquidity, profitability, leverage and activity. The non-financial variables often offer additional information to the financial ratios and that is why they are used for default modelling, as well (Altman, Sabato and Wilson, 2010). The usual financial ratios employed for corporate default prediction are those related to liquidity, profitability, leverage and activity. The use of financial ratios is not without its issues. Financial ratios are calculated using information from previously filed financial statements. As such they may not provide the real picture of the present or future situation of a company. Also, the balance sheet data provides a “snapshot” of a company’s financial situation at the end of reporting period but the underlying financial variables themselves are subject to fluctuations during a year and the values reported at the end of the year may not represent the typical ones. Moreover, the assets are not reported at their current value. The book value may be very different from what they may be sold for in the market, especially under distress.

X1 – working capital to total assets

Working capital is an indicator of a company’s ability to meet its current obligations. Algebraically, it is a difference between current assets and current liabilities. Obviously, the larger value of the variable is better than a smaller one.[3] In order to allow for comparison, the working capital is divided by total assets. Altman (1968) notes that “a firm experiencing consistent operating losses will have shrinking current assets in relation to total assets” (p. 594).

X2 – retained earnings to total assets

This variable is a measure of a company’s ability to accumulate profit. Altman (1968) considers it a measure of age, as well. Thus, young companies will have a smaller value of the variable. It is expected that a low value of the variable may

[3] However, one may imagine also a different scenario, one in which the working capital actually increases once a firm is experiencing difficulties – firstly, a company may face difficulties to obtain trade credit financing from its suppliers. Or secondly, a distressed firm may hoard inventories, i.e. the produced goods and it is unable to sell it. Or thirdly, distressed firm may offer trade credit to its customers. In all these situations the working capital may increase and hence these scenarios may explain why working capital may not work well.

signal problems. Firstly, a young firm has not accumulated enough assets and hence may have a higher propensity to fail. Secondly, if an older company has a low volume of retained earnings, it may signal its inability to succeed in the marketplace. Such a company may be less resistant to shocks and a competitive environment and more prone to failure.

X3 – earnings before interest and tax to total assets

This variable represents return on assets. This ratio is arguably more stable than earnings after tax. The logic of inclusion of this variable in the failure prediction model lies in the fact that if a company is not able to generate earnings, its prospects are bleak and the end is near. Again, the lower the value, the higher expected propensity to fail.

X4 – book value of equity (net worth) to total liabilities

Indebtedness is a traditional predictor of defaulting. There are two major reasons why a company with a higher proportion of debt is supposed to have a higher propensity to fail. Firstly, a highly indebted firm will have to pay higher paybacks because of a higher volume of debt and because of the higher perceived rate of riskiness when compared with a company that is less indebted. Hence, in the case of cash-flow difficulties, a highly indebted firm will have a higher probability of missing the payment and eventually defaulting on the loan. Secondly, there are reasons on the side of management incentives as well. The firm's management knows that even in the case of success, a large part of potential profit will be used to pay back the loan. The difference between the success and failure thus decreases. In this situation, the management may not exert the sufficient effort to succeed (Fidrmuc and Hainz, 2010).

X5 – sales to total assets

The fifth variable in Altman's Z-score represents the ability of assets to generate sales. It shows the ability of the firm to compete in the marketplace. This variable is included in the original Z-Score (Altman, 1968) and the revised Z''-Score model for private firms (Altman, 1983, p. 122). However, it is deemed industry-sensitive and hence not included in the further revised Z'-Score model for private non-manufacturing companies (Altman, 1983, p. 124).[4] Altman (1968) states that even though the variable was least significant in univariate tests, it worked well in combination with other variables and proved to be the second best predictor in the multivariate model.

[4] The revised models differ not just in the set of utilized variables but also in the estimated coefficients and bounds for grey zone. The equation of the revised Z'-Score is: Z' = 0.717 * X1 + 0.847 * X2 + 3.107 * X3 + 0.420 * X4 + 0.998 * X5 and the lower and upper bounds for the grey zone (i.e. the area of uncertainty) are 1.23 and 2.9, respectively. On the other hand, the formula for the revised Z''-Score is Z'' = 6.56 * X1 + 3.26 * X2 + 6.72 * X3 + 1.05 * X4 and the lower and upper bound for the grey zone are 1.1 and 2.6, respectively.

2.1. Other Potential Predictors of Default

The financial variables, such as those mentioned above, capture important dimensions of firms' performances related to liquidity, profitability, leverage and activity. Given that a default is primarily a financial event; in which a company fails to pay its obligations in full and on time, it is not surprising that the financial variables are primary predictors of default and as such are incorporated in failure prediction models. Yet, there are valid reasons to complement these variables with non-financial ones. Firstly, non-listed companies lack market-related information and only the accounting data is available. Secondly, many small and medium--sized enterprises report only a limited set of accounts (lower reporting requirements). In this situation, the non-financial variables may provide a vital additional information for default studies (Grunert, Norden and Weber, 2005; Altman, Sabato and Wilson, 2010; Wilson, Ochotnický and Káčer, 2016; Altman et al., 2017). These variables can be roughly divided into two groups – those related to an individual firm and those related to the macroeconomic environment.

2.2. Non-financial Company-specific Variables

Legal form

There are several reasons why legal form may be informative for failure prediction. The first one is related to degree of liability. If the degree of liability differs across legal forms then it makes sense to hypothesize that the corporate form with higher liability is less likely to fail. This argument is elaborated in Fidrmuc and Hainz (2010). However, their sample included natural persons (sole traders) along with legal entities (limited liability companies, joint stock companies and cooperatives) and their hypothesis was related to a lower failure rate of natural persons. Our sample contains two types of firms – limited liability companies and joint stock companies. In terms of incentives, it is not entirely clear why one legal form should be more likely to fail than the other one. On the one hand, joint stock companies are more complex, the shareholders and management are more distant from each other, they may have conflicting incentives and the management may take excessive risks (agency problems). On the other hand, these companies tend to have control systems in place to mitigate such concerns. Conversely, in limited liability companies, the management and the owners are closer to each other and agency problems are not that pronounced. Also, many of these companies are family firms and these companies tend to be more risk averse and conservative. However, the minimum equity endowment for limited liability company is rather low and thus as the name suggests liability is limited. Although it is not entirely clear which legal form is riskier, including the indicator of joint stock companies will capture their residual failure rate.

Age

There are two conflicting tendencies when it comes to impact of age on the business failure. The first one is related to fact that a company accumulates experience and profit in time and thus the older the firm is, the more experiences and profit it accumulates and consequently its propensity to fail decreases. From another perspective, it takes time to fail and a young company usually has a start-up capital to live from even though it is not generating sufficient revenues. If we combine these two tendencies, the failure rate should be relatively small in the first years, corresponding to time when the company is backed up with start-up capital. After the capital is spent, the company is still in the process of gaining experiences and hence this period should be associated with the highest failure rates. The surviving companies become established and the failure rates fall. In order to operationalize this narrative we include the quadratic function of age in the models, too.

Size

Altman, Sabato and Wilson (2010) modelling defaults of UK private small and medium-sized companies note that "… businesses with low asset values are less likely to be pursued through the legal process of insolvency since creditors would have little to gain" (p. 117). The creditors may prefer to utilize other means than litigation, such as out-of-court settlement. However, the association may be a non-linear one and that is why we use two dummy variables (small and medium companies, micro companies are the reference category).

Industry sector

The idea of the inclusion of industry sector indicators is to represent industry-level failure rates in to the default prediction. The industry indicators were used in Fidrmuc and Hainz (2010); Wilson, Ochotnický and Káčer (2016) or Altman et al. (2017). Altman et al. (2017) note that the asset turnover ratio (variable X5) is particularly sensitive to differences among industry sectors. We include a dummy variable for industries defined by two-digit NACE with over 20 000 company-year observations in our sample.

2.3. Macroeconomic Variables

In several ex-post studies the macroeconomic conditions are represented by fixed time effects (Grunert, Norden and Weber, 2005; Fidrmuc and Hainz, 2010; Altman et al., 2017). However, this approach cannot be used in predictive studies since the coefficients are estimated retrospectively and are not known in advance. That is why we attempt to decompose these dummy variables using the proxies for credit situation and economic expectations.

Credit situation

The situation in the credit market directly influences the failure rate. The availability and, more importantly, cost of credit are relevant for an indebted firm. If such a company experiences operating difficulties, the worsened situation in the credit market associated with increased interest rates may trigger the bankruptcy. The variable was used in Wilson, Ochotnický and Káčer (2016). However, in that study it was used to test the hypotheses related to transformation ex post whereas in our case it will be used for prediction ex ante. To take into account this dimension of macroeconomic environment, we use annual average interest rate for loans to non-financial corporations.

Macroeconomic conditions and expectations

The economic expectations are a very powerful determinant of economic activity. From a corporate perspective, negative expectations translate into smaller sales, less orders, lower economic growth and generally worsening economic environment. Such conditions may increase competitive pressures, and this may aggravate the difficulties of a distressed company. That is why, in combination with other variables, macroeconomic conditions may be a relevant predictor of default for individual companies too. However, GDP growth or change are not appropriate for prediction models, since there are significant lags in reporting of GDP. A forward-looking measure, such as expectations, seems to be a better choice. In our models we include employment expectation for the next three months.

3. Dataset and Methodology

3.1. Dataset

The dataset used for this study was compiled using several sources. The financial and non-financial information about individual companies were obtained from databases Albertina Platinum (CDs 3/2013 and 9/2015) and Finstat Premium (downloaded 24 October 2017). The initial dataset contained over 1.4 million company-year observations for over 254 thousand unique companies covering period from 1997 to 2016.[5] To ensure the homogeneity of the sample we employed a few restrictions, as outlined in Table 2. Firstly, because the frequencies of observations before 2009 were significantly smaller than after 2009, we use observations from 2009 onwards. Secondly, we removed observations that could not be qualified as SMEs.[6] Thirdly, since we wanted to focus on active companies, we removed observations for companies with turnover or total assets lower

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[5] If more than one database contained information about a given company and year, we used information from the newer database.

than five thousand Eur. Fourthly, we kept just limited liability companies and joint stock companies, since other legal forms such as public or private partnerships, or cooperatives, are rather specific and we have very few default events for them. Finally, the observations with missing values for any of the relevant variables were eliminated. The resulting sample comprises 661 622 company-year observations for 149 618 unique companies.

T a b l e 2

Construction of Sample

Description	Company-year observations	Number of unique companies
Initial sample	1 413 730	254 329
- Observations before 2009	102 053	
- Non-SME companies	327 122	
- Total assets or turnover lower than 5000 Eur	224 892	
- Legal form other than limited or joint stock company	12 465	
- Missing values for relevant variables	86 427	
Final sample	661 622	149 618
Training sample (2009 – 2014)	491 349	130 959
Validation sample (2015 – 2016)	170 273	94 340

Notes: The table shows construction of the sample used in this paper and the restrictions imposed.
Source: Authors' elaboration.

T a b l e 3

Breakdown of Sample by Year and Default

	Year	Non-defaulted	Defaulted	Total
Training sample	2009	67 932	218	68 150
	2010	73 329	214	73 543
	2011	79 356	231	79 587
	2012	82 229	275	82 504
	2013	89 980	258	90 238
	2014	97 219	108	97 327
Validation sample	2015	83 517	159	83 676
	2016	86 485	112	86 597
	Total	660 047	1 575	661 622

Notes: The table shows frequencies of company-year observations in the sample according to years and default status.
Source: Authors' elaboration.

The sample was split into a training sample (from 2009 to 2014) and a validation sample (2015 and 2016). The validation sample contains both the company-year observations for companies from the training sample and for new companies. The breakdown by year and default status is shown in Table 3. The size of our sample is larger than in any of the studies about Slovak companies we are aware of.

[6] To qualify as SME, the definition of European Commission (EC) is used on the basis of number of employees, turnover and total assets. More specifically, a small or medium-sized company has number of employees smaller than 250 and either turnover lower than 50 million Eur or total assets lower than 43 million Eur.

3.2. Construction of Dependent Variable

Default-prediction models (or bankruptcy models) assign firms to one of two groups: A ‘good firm’ group that is not likely to experience financial distress, and thus survive in the long term discharging its obligations to creditors, or a ‘bad firm’ group that has a high likelihood of bankruptcy and/or default caused by financial distress. The dependent variable is binary and it represents the event of default (Equation 3). The definition of default differs significantly across failure prediction studies for Slovak companies. Fidrmuc and Hainz (2010) use an indicator of a bank loan default, Boďa and Úradníček construct their own indicator of financial distress, Wilson, Ochotnický and Káčer (2016) and Gulka (2016) utilize the legal definition of default and finally Klieštik et al. (2017) use an indicator of negative equity. While each definition has its advantages and shortcomings, we use the legal definition of default in this study because it is a clear indication of insolvency.[7] To construct the variable, we checked the legal documents related to the companies marked as defaulted in our database and define the year of default as the year when the bankruptcy proceeding started, i.e. when the case was filed to the court either by the company itself (voluntary) or by any of its creditors (involuntary).[8] Finally, we mark as bankrupt the financial accounts immediately preceding the year of default. This setting alleviates potential endogeneity concerns and at the same time assures that the model is a predictive one.[9]

3.3. Estimation Methods

We follow literature (Altman and Sabato, 2007; Altman, Sabato and Wilson, 2010; Altman et al., 2017) by estimating logit using a combination of variables reflecting both financial and non-financial characteristics, and at the same time considering macroeconomic conditions. The two most frequently used estimation methods in previous studies of Slovak companies were linear discriminant analysis (LDA) and logistic regression (logit). LDA is based on the assumption that each class can be modelled by a normal distribution, although financial ratios do not to tend to be normally distributed. Another assumption is that all the classes share the same covariance matrix, which does not seem to be the case in

[7] According to current Slovak legislation, a company is considered bankrupt if it is either insolvent or over-indebted. Insolvency is defined as more than 30 days arrears on at least two financial obligations to more than one creditor. The company is over-indebted if it has more than one creditor and its liabilities exceed the assets.

[8] Even though the year when a legal bankruptcy process starts is rarely the year when the company is declared bankrupt, in failure prediction studies it is important to signal the earliest possible sign of problems and when a legal process starts, a company is insolvent or over-indebted already.

[9] The predictive ability of the model is important also because of the lags between the end of financial year and the submission of the financial accounts.

empirical studies either.[10] Logit does not have these restrictive assumptions and it is the most commonly used estimation method for modelling binary outcomes. That is why we prefer to use logit.

Table 4
Description of Explanatory Variables

Variable	Description
X1	Working capital to total assets, winsorized at the 5th and the 95th percentile
X2	Retained earnings to total assets, winsorized at the 5th and the 95th percentile
X3	Earnings before interest and tax to total assets, winsorized at the 5th and the 95th percentile
X4	Net worth to total liabilities, winsorized at the 5th and the 95th percentile
X5	Sales to total assets, winsorized at the 5th and the 95th percentile
Joint stock company	Indicator of joint stock company, equals to one for joint stock company, zero otherwise
Age (log)	Age in years (difference between year of founding and year of financial accounts), natural logarithm
Small company	Indicator of small company (i.e. number of employees below 50 and either total assets or turnover lower than 10 million Eur), equals to one for small company, zero otherwise
Medium-sized company	Indicator of small company (i.e. not small company, number of employees below 250 and either total assets lower than 43 million Eur or turnover lower than 50 million Eur), equals to one for medium-sized company, zero otherwise
Manufacturing	Indicator of industry sector C – Manufacturing, equals to one for this sector, zero otherwise
Construction	Indicator of industry sector F – Construction, equals to one for this sector, zero otherwise
Retail & wholesale	Indicator of industry sector G – Wholesale and retail trade; repair of motor vehicles and motorcycles, equals to one for this sector, zero otherwise
Transport	Indicator of industry sector H – Transportation and storage, equals to one for this sector, zero otherwise
Information	Indicator of industry sector J – Information and communication, equals to one for this sector, zero otherwise
Real estate	Indicator of industry sector L – Real estate activities, equals to one for this sector, zero otherwise
Professional	Indicator of industry sector M – Professional, scientific and technical activities, equals to one for this sector, zero otherwise
Administrative	Indicator of industry sector N – Administrative and support service activities, equals to one for this sector, zero otherwise
Health	Indicator of industry sector Q – Human health and social work activities, equals to one for this sector, zero otherwise
Interest rate	Annual average interest rate for loans to non-financial corporations, stock and new loans
Employment expectation	Monthly indicator of economic sentiment – expectation for employment for next three months

Notes: The table shows the description of explanatory variables. The source of data used for calculation of the financial and non-financial variables related to individual companies are databases Albertina Platinum (CDs 3/2013 and 9/2015) and Finstat Premium from Finstat (downloaded 24 October 2017). The source of interest rate and employment expectation is statistics data gathered by National Bank of Slovakia.

Source: Authors' elaboration.

The model specification used for the default prediction is as follows:

$$P(d_{i,t+1} = 1 \mid \Omega_t) = 1/\{1 + \exp[-(\beta_0 + \beta_1 F_{i,t} + \beta_2 N_{i,t} + \beta_3 M_{i,t})]\} \qquad (1)$$

[10] See results in Boďa and Úradníček (2016); Mihalovič (2016) or Klieštik et al. (2017).

where

d – default in the following period,
F – financial variables,
N – non-financial variables,
M – macroeconomic variables.

The explanatory variables used for the model are described above in the text and in Table 4. Since accounting ratios are often subject to outlying and extreme values that can potentially bias our multivariate estimates, particularly for private companies, we apply a consistent strategy for dealing with outliers (winsorization at the 5th and 95th percentile).

3.4. Criteria for Models' Evaluation

The true performance test of a failure prediction model lies in using observations that have not been used for estimation of its parameters. There are several ways how to evaluate the out-of-sample discriminatory performance of binary classification models. The earlier studies mostly used the percentages of correctly and incorrectly predicted outcomes, or overall correctly predicted cases. However, to evaluate classification accuracy one needs to determine a specific threshold and the classification accuracy will be basically a function of the threshold. We'll put forward suggestions for specific cut-off points later in this section. But before that we assess the out-of-sample discriminatory performance of the models using area under ROC (receiver operating characteristic) curve which does not need a specific threshold.

ROC curve is a graphical representation of combinations of the true positive rate and the false positive rate across the whole spectrum of possible cut-off points. A perfect model (one which perfectly discriminates between defaulted and non-defaulted companies) has the area under ROC curve (AUC) equal to one, a completely random model (one which does not discriminate at all) has the AUC equal to 0.5. Therefore, the closer the AUC to one, the better. For completeness and to facilitate comparison with the earlier studies, we calculate and present summaries of correctly/incorrectly classified observations from validation sample. However, firstly we have to determine the cut-off point for estimated propensity scores. Since we build the default prediction model, to simulate the deployment of the model in practice we use just the information from training sample to decide the cut-off point. We use three alternative cut-off points:[11]

[11] Determination of the cut-off points has not been entirely transparent in all previous studies; it was explicitly stated only in some of them. The most thorough treatment of the choice of optimal cut-off point was given in Mihalovič (2016). In some cases, the cut-off point choice was very straightforward (Gulka, 2016). Boďa and Úradníček (2016) chose cut-off value so that the sum of Type I and Type II error rates was minimal.

1. Proportion of defaulted companies in training sample – this choice corresponds in a way to using threshold of 0.5 for balanced proportions of defaulted and non-defaulted companies.

2. Value of cut-off point maximizing Youden index J (Youden, 1950). This cut-off point maximizes average true positive rate and true negative rate, i.e. proportions of correctly classified cases in both groups which makes sense in samples such as ours with strongly imbalanced proportions of defaulted and non-defaulted companies.

3. Cut-off point at the lower quartile of estimated scores for defaulted companies – this cut-off point makes the acceptable rate of false negatives explicit; in this case the false negatives rate in the training sample was set to 25% (arbitrary number). The choice of this cut-off point assumes approximately stable false negative rates in the training and test samples.

We report two aggregate measures for classification accuracy – the average accuracy and the overall accuracy. Earlier studies that included analysis of classification accuracy utilized mostly the overall accuracy. Comparison of overall correctly predicted cases is appropriate for situations when the proportions of failed and non-failed companies are approximately balanced[12] and at the same time costs of making Type I or Type II error[13] are similar. Since our sample contains much more non-defaulted companies, overall accuracy alone could be misleading. That is why we report average accuracy in addition to overall accuracy. The average accuracy is the simple average of the true positive and true negative rate whereas the overall accuracy is the weighted average of the true positive and true negative rate. The weights are the frequencies of defaulted and non-defaulted companies, respectively.

4. Results and Discussion

4.1. Descriptive Statistics of the Explanatory Variables for Estimation (training) Sample

Table 5 shows descriptive statistics of the explanatory variables for the training sample (i.e. period from 2009 to 2014). The descriptive statistics (mean, standard deviation and quartiles) are calculated both for failed and non-failed companies.

[12] In a dataset with 99% of non-defaulted companies and 1% of defaulted ones, a naïve model predicting all instances as non-defaulted will achieve 99% overall accuracy. In case of severely imbalanced groups it makes more sense to report the average of correctly predicted cases in both groups.

[13] The Type I error in binary classification is committed when failed company is marked as non--failed, i.e. it is false negative. The Type II error is false positive, i.e. non-failed company is marked as failed. In credit risk modelling the Type I error is more costly, since there are higher costs associated with lending a company that will bankrupt eventually than it is to let go of potentially profitable client just because the model marked him as bankrupted.

The descriptive statistics for the failed companies relate to the last available company-year observation before the company filed for bankruptcy. To convey information about the statistical significance of difference in means between the two groups p-values for t-test are displayed in the penultimate column. To measure the real effect of the difference Cohen's d statistic[14] is shown in the last column.

The first four financial variables seem to discriminate well between failed and non-failed companies in that means of these variables are lower for the group of defaulted companies than for non-defaulted ones. The difference between the means is statistically significant as indicated by t-test (Table 5, penultimate column). However, they differ in the real effect size measured by Cohen's d statistic. The strongest effect has X3 (EBIT to total assets), followed by X1 (working capital to total assets). On the other hand, X5 (sales to total assets) does not discriminate well,[15] at all. Even though the mean and the first two quartiles are higher for the non-defaulted group, the upper quartile is slightly higher for the defaulted group. The difference in means is not even statistically significant.

From among the non-financial variables the indicator of legal form (joint stock dummy) along with size indicator (small company dummy) seem to have the greatest influence. The next variable is the indicator of manufacturing sector followed by age. The effect of other variables is relatively small with the real effect size below 0.3 standard deviation. However, even though many of them do not perform well individually, they may do so in conjunction with other variables.

4.2. Estimation Results

Table 6 presents the estimation results. We estimated five models. The first model contains the financial variables from the revised Altman's Z'-Score model (Altman, 1983). The model achieves relatively low McFadden pseudo-R^2, even though its in-sample discriminatory ability measured by the area under ROC curve (AUC) is rather high. However, the first two variables – X1 (working capital to total assets) and X2 (retained earnings to total assets) – attract opposite signs, albeit the first one is not statistically significant and the second one only marginally so. Since the univariate discriminatory performance of these two variables was fair, the opposite signs in multivariate regression may be due to multicollinearity. Indeed, the pairwise correlation coefficients of the first three variables are above 0.6. Since the first two variables do not contain much additional information to variable X3 (EBIT to total assets), we excluded them and re-estimated the model without them.

[14] Given the size of the sample even relatively small difference in means will be statistically significant. Unlike t-test, Cohen's d statistic indicates standardized difference and conveys information about the real effect size.

[15] In Altman (1968) this variable did not perform well in univariate analysis, either.

Table 5

Descriptive Statistics of Explanatory Variables

	Non-defaulted companies N = 490 045					Defaulted companies N = 1 304					t-test p-value	Cohen's d
Variable	Mean	SD	p25	p50	p75	Mean	SD	p25	p50	p75		
X1	0.05	0.69	–0.19	0.15	0.51	–0.50	0.87	–0.90	–0.21	0.08	0.00	0.80
X2	–0.15	0.78	–0.13	0.00	0.16	–0.57	1.18	–0.58	–0.08	0.02	0.00	0.54
X3	0.02	0.28	–0.04	0.03	0.14	–0.25	0.36	–0.46	–0.12	0.00	0.00	0.97
X4	1.99	4.71	0.01	0.34	1.51	–0.01	1.63	–0.47	–0.12	0.10	0.00	0.42
X5	1.79	1.46	0.70	1.41	2.46	1.75	1.68	0.44	1.22	2.49	0.37	0.03
Joint stock company	0.04	0.19	0	0	0	0.13	0.34	0	0	0	0.00	–0.53
Age (log)	1.88	0.74	1.39	1.95	2.48	2.14	0.67	1.61	2.20	2.71	0.00	–0.36
Small company	0.11	0.32	0	0	0	0.28	0.45	0	0	1	0.00	–0.53
Medium-sized company	0.03	0.17	0	0	0	0.07	0.26	0	0	0	0.00	–0.26
Manufacturing	0.10	0.30	0	0	0	0.24	0.43	0	0	0	0.00	–0.46
Construction	0.09	0.28	0	0	0	0.16	0.36	0	0	0	0.00	–0.25
Retail & wholesale	0.29	0.45	0	0	1	0.27	0.45	0	0	1	0.21	0.04
Transport	0.04	0.20	0	0	0	0.05	0.21	0	0	0	0.41	–0.02
Information	0.05	0.21	0	0	0	0.02	0.15	0	0	0	0.00	0.12
Real estate	0.06	0.23	0	0	0	0.05	0.22	0	0	0	0.25	0.03
Professional	0.16	0.37	0	0	0	0.07	0.25	0	0	0	0.00	0.26
Administrative	0.07	0.25	0	0	0	0.05	0.22	0	0	0	0.02	0.07
Health	0.05	0.21	0	0	0	0.01	0.07	0	0	0	0.00	0.20
Interest rate	3.48	0.20	3.33	3.38	3.54	3.50	0.20	3.33	3.45	3.54	0.00	–0.11
Employment expectation	–7.38	8.05	–9.31	–3.75	–2.93	–8.12	8.60	–9.31	–6.59	–2.93	0.00	0.09

Notes: The table shows descriptive statistics of the explanatory variables in the training (estimation) sample. SD stands for standard deviation, p25, p50 and p75 stand for lower quartile, median and upper quartile, respectively. The means for dummy variables indicate the proportion of companies with given characteristic in the group. The figures in penultimate column indicate p-values of the t-test of the difference of means for explanatory variables in the defaulted and non-defaulted groups. The figures in the last column show Cohen's d for the difference. The variables are described in Table 4.

Source: Authors' elaboration.

T a b l e 6

Estimation Results

	Model 1	Model 2	Model 3	Model 4	Model 5
X1	0.0373 (0.51)				
X2	0.0658* (1.68)				
X3	–1.812*** (–12.75)	–1.497*** (–8.08)	–2.077*** (–12.49)	–1.477*** (–7.84)	–2.056*** (–12.25)
X4	–0.750*** (–2.99)	–0.738*** (–3.84)	–0.706*** (–4.15)	–0.748*** (–3.82)	–0.714*** (–4.15)
X5	–0.141*** (–6.24)	–0.103*** (–4.81)	–0.123*** (–5.63)	–0.102*** (–4.77)	–0.122*** (–5.57)
Joint stock company			1.181*** (12.03)		1.169*** (11.97)
Small company			1.074*** (14.61)		1.062*** (14.44)
Medium-sized company			0.863*** (6.43)		0.848*** (6.35)
Age (log)			0.862*** (3.92)		0.877*** (4.04)
Age (log) squared			–0.107* (–1.94)		–0.105* (–1.92)
Manufacturing			1.123*** (10.12)		1.108*** (10.01)
Construction			1.076*** (9.27)		1.075*** (9.28)
Retail & wholesale			0.432*** (3.98)		0.424*** (3.91)
Transport			0.590*** (3.72)		0.586*** (3.71)
Information			–0.0643 (–0.30)		–0.0645 (–0.31)
Real estate			0.278* (1.74)		0.277* (1.73)
Professional			–0.173 (–1.18)		–0.170 (–1.16)
Administrative			0.193 (1.24)		0.200 (1.29)
Health			–0.666* (–1.91)		–0.665* (–1.91)
Interest rate				0.565*** (4.00)	0.653*** (5.03)
Employment expectation				–0.0123*** (–3.73)	–0.0130*** (–4.32)
Constant	–5.648*** (–77.94)	–5.718*** (–97.88)	–7.767*** (–31.82)	–7.783*** (–15.66)	–10.17*** (–19.29)
Observations	491 349	491 349	491 349	491 349	491 349
Log-likelihood	–8 386.2	–8 386.4	–7 881.3	–8 372.9	–7 866.8
McFadden's R^2	0.072	0.072	0.128	0.074	0.130
AUC (training)	0.791	0.791	0.834	0.792	0.835
AUC (validation)	0.797	0.797	0.840	0.801	0.844

Notes: The table shows estimation results for default model. The parameters are estimated using logistic regression. The training sample covers time-period from 2009 to 2014. The z-statistics are denoted in parentheses and the statistical significance is indicated with asterisks (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$). The standard errors used to compute the z-statistics are calculated using robust standard errors clustered within companies to control for correlations of errors within companies. The dependent variable is the indicator of default and the explanatory variables are described in Table 4.

Source: Authors' elaboration.

The second model contains just three of the original five variables, yet the log-likelihood, McFadden R^2 and AUC are nearly unchanged. This confirms the notion that the variables X1 and X2 do not contribute new information to the model.[16] This can be due to specific features of business environment in Slovakia such as management of liquidity or dividend policy.

Model 3 contains non-financial variables besides financial ones. The discriminatory performance improved significantly when compared to model 2, so clearly these variables provide additional information for the models. The results suggest that joint stock companies are more risky than limited liability companies. The propensity to fail firstly increases with higher age and then decreases since the estimated coefficient for squared age is negative. However, it is only marginally statistically significant. The breakpoint when the relationship reverses is at about 55 years, so for the majority of the sample the relationship is monotonic (with decreasing marginal change), which is a rather unexpected result. At the same time, small and medium-sized companies are more risky than micro companies on average. The coefficients for industrial sector indicators suggest that manufacturing, construction, retail and wholesale, and transport industries are riskier than others, whereas companies from the health sector have lower propensity to fail.

Model 4 contains macroeconomic variables in addition to financial ones. These variables are statistically significant with expected signs, i.e. in periods with higher interest rate the propensity to fail is higher on average, whereas if the expectations about employment are positive, the companies fail less. However, the macroeconomic variables improve the discriminatory performance only modestly when compared to model 2 containing just financial variables.

Model 5 is the full model with all variables included. The coefficients have stable signs and most variables have stable coefficients too when compared to previous models. In terms of discriminatory power this model does not seem to be different from model 3.

In Table 6, for each estimated model, AUC for training sample (in-sample performance) is in the penultimate row and AUC for validation sample (out-of--sample performance) is presented in the last row. Comparison of these two figures enables us to assess whether our models captured true associations among variables or just random noise specific to the estimation sample.

The performance of models in the validation sample is similar to that in the training sample, i.e. the models are not over-fitted and the relations between

[16] The issues of opposite signs and/or using just subset of Altman's variables are not new and similar results were reported in earlier studies (e.g. Režňáková and Karas, 2015; Boďa and Úradníček, 2016; Kliештik et al., 2017). Even though in the literature there is no consensus whether it is preferable for predictive model to keep variables with opposite sign or it is better to remove them, we prefer the latter option.

variables in the training sample captured by the models continue to hold in the validation sample as well. From the perspective of the achieved values of AUC, there are no generally accepted thresholds or intervals since these values are context-specific. Usually stand-alone values over 0.8 are considered very good and over 0.9 excellent. However, from the viewpoint of our research hypotheses, AUC enables simple comparison of the models along with tests of statistical significance. Thorough treatment of models' out-of-sample performance and verification of our research hypotheses is the topic of the next section.

4.3. Out-of-sample Discriminatory Performance

Table 7 shows the AUCs for validation sample along with their standard errors and 95% confidence intervals. Model 1 and model 2 containing only financial variables have AUC in validation sample 0.797. When we add non-financial firm-specific variables, the AUC increases to 0.84 (model 3). If we add macroeconomic variables instead (model 4), the increase in AUC is very marginal (from 0.797 to 0.801). The best AUC gives a combination of non-financial company-specific information and macroeconomic variables with the value of 0.844 (model 5). However, the difference in comparison with model 3 is small. It seems that the macroeconomic variables do not contribute much to the value of AUC. A look at confidence intervals for AUCs confirms this conjecture. While the models 2 and 4, and models 3 and 5, are not statistically significantly different in terms of AUC, models 2 and 3, and models 4 and 5 are. The outcome is that the non-financial variables significantly increase the discriminatory performance of the models, while macroeconomic variables do not.

Table 7

Area under ROC Curve and Its Confidence Interval for Validation Sample

			95% Confidence interval	
	AUC	Std. error	Lower bound	Upper bound
Model 1	0.797	0.012	0.773	0.822
Model 2	0.797	0.012	0.773	0.821
Model 3	0.841	0.011	0.818	0.863
Model 4	0.801	0.012	0.777	0.825
Model 5	0.844	0.011	0.822	0.866
Revised Z'-Score	0.780	0.013	0.754	0.805
Revised Z''-Score	0.760	0.012	0.736	0.784

Notes: The models estimated in this paper (see Table 6 for details) were validated using AUC criterion, which stands for area under ROC (receiver operating characteristic) curve. Standard errors were calculated using Hanley and McNeil (1982) procedure as implemented in Stata's command roctab (StataCorp, 2017). The revised Z'-Score is a re-estimated Z-Score for private companies: Z' = 0.717 * X1 + 0.847 * X2 + 3.107 * X3 + 0.420 * X4 + 0.998 * X5. The revised Z'-Score is adapted for non-manufacturing private firms: Z' = 6.56 * X1 + 3.26 * X2 + 6.72 * X3 + 1.05 * X4. The variables are defined in Table 4. Both models were published in Altman (1983).

Source: Authors' elaboration.

Table 8

Classification Accuracy of the Models Using Validation Sample

				Failed companies, N = 271				Non-failed companies, N = 170 002			
		Average accuracy	Overall accuracy	Failure accuracy (True positives)		Type I error (False negatives)		Non-failure accuracy (True negatives)		Type II error (False positives)	
		%	%	Obs.	%	Obs.	%	Obs.	%	Obs.	%
Model 1	cut-off 1	72.49	75.96	187	69.00	84	31.00	129 150	75.97	40 852	24.03
	cut-off 2	72.33	74.54	190	70.11	81	29.89	126 724	74.54	43 278	25.46
	cut-off 3	72.47	76.66	185	68.27	86	31.73	130 348	76.67	39 654	23.33
Model 2	cut-off 1	72.50	75.98	187	69.00	84	31.00	129 194	76.00	40 808	24.00
	cut-off 2	73.16	72.89	199	73.43	72	26.57	123 907	72.89	46 095	27.11
	cut-off 3	72.48	75.95	187	69.00	84	31.00	129 139	75.96	40 863	24.04
Model 3	cut-off 1	77.06	75.15	214	78.97	57	21.03	127 751	75.15	42 251	24.85
	cut-off 2	76.59	77.89	204	75.28	67	24.72	132 421	77.89	37 581	22.11
	cut-off 3	76.46	76.53	207	76.38	64	23.62	130 109	76.53	39 893	23.47
Model 4	cut-off 1	64.33	90.57	103	38.01	168	61.99	154 118	90.66	15 884	9.34
	cut-off 2	65.57	89.00	114	42.07	157	57.93	151 423	89.07	18 579	10.93
	cut-off 3	64.75	90.30	106	39.11	165	60.89	153 653	90.38	16 349	9.62
Model 5	cut-off 1	74.36	85.58	171	63.10	100	36.90	145 553	85.62	24 449	14.38
	cut-off 2	72.96	87.20	159	58.67	112	41.33	148 320	87.25	21 682	12.75
	cut-off 3	72.94	87.16	159	58.67	112	41.33	148 253	87.21	21 749	12.79
Revised Z'-Score		71.72	59.35	228	84.13	43	15.87	100 827	59.31	69 175	40.69
Revised Z''-Score		71.75	62.71	219	80.81	52	19.19	106 562	62.68	63 440	37.32

Notes: The models estimated in this paper (see Table 6 for details) were assessed using three different cut-off points: cut-off point 1 – proportion of defaulted companies in training sample; cut-off point 2 – value of optimal cut-off point maximizing Youden index (Youden, 1950) in training sample, as implemented in user-written Stata command cutpt (Clayton, 2013); cut-off point 3 – lower quartile of estimated probability scores for defaulted companies in training sample; The revised Z'-Score is a re-estimated Z-Score for private companies: Z' = 0.717 * X1 + 0.847 * X2 + 3.107 * X3 + 0.420 * X4 + 0.998 * X5. The revised Z''-Score is adapted for non-manufacturing private firms: Z'' = 6.56 * X1 + 3.26 * X2 + 6.72 * X3 + 1.05 * X4. The variables are defined in Table 4. Both models were published in Altman (1983). The cut-off points for these models were averages of the lower and upper bounds for the grey zone, i.e. 2.065 and 1.85, respectively. The average accuracy is a simple average of true positives and true negatives percentages to account for unbalanced sample. The overall accuracy is a weighted average of true positives and true negatives percentages; the weights are number of observations in each group.

Source: Authors' elaboration.

Besides comparison of the models estimated in this paper, Table 7 shows AUCs and their standard errors for both Altman's revised Z-Score models for private companies (Altman, 1983). Both models have an AUC lower than any of our models. While the AUC for model comprising all five variables (Z'-Score) lies in the confidence interval of AUC of models 2 and 4, the AUC of model comprising just four variables (revised Z''-Score) does not. It is interesting that Z''-Score does not discriminate all that well and our model with just three variables (model 2) dominates it.

Table 8 shows the classification accuracy figures for each model and each cut-off point choice using validation sample.[17] Also, in the last two rows, we show the classification accuracy for revised Z-Score models. The pattern outlined in Table 8 is revealing, in a way. Model 1 and model 2 containing just financial variables achieve a similar average accuracy, about 72.5%, depending on the cut-off point choice, with Type I error 30%. Interestingly, the reduced model 2 achieves a slightly better score confirming our conjecture that it is better to remove non-significant variables or those with negative signs. Model 3 is the most accurate – the average accuracy is about 76.7% with Type I error slightly over 23%. Model 4 has an unacceptable high Type I error rate – over 60%, even for the cut-off point which is equal to lower quartile of predicted probabilities for defaulted companies (in the training sample). The average accuracy is slightly smaller than 65%, even though the overall accuracy is about 90%. The reason for this is considerably lower Type II error rate of this model. This is clearly an example where the overall accuracy may be misleading – it is heavily leaned towards true negative rate, since there are much more non-defaulted companies. Model 5, which contains all explanatory variables, achieved better results when compared to model 4 with Type I error rate slightly less than 40%, even though Type II error rate was relatively low too. The average accuracy is about 73.5%. These results suggest that models with macroeconomic variables have less desirable properties in terms of stability of classification accuracy.

4.4. Verification of Research Hypotheses

Our first hypothesis (H1) assumes that the re-estimated model of Z'-Score will perform better than the original revised Z'-Score model. We can test H1 by comparing discriminatory performance and classification accuracy of the revised Z'-Score model and the newly estimated models (model 1 and model 2). The newly estimated models containing the same variables achieve better results; both AUC (see Table 7) and average classification accuracy (see Table 8) are higher. However, considering confidence intervals, the AUC for revised Z'-Score model is not statistically different from that of newly estimated model (see confidence intervals

[17] Table 9 displays identical quantities for training sample.

in Table 7). Even though the revised Z-Score models were developed 35 years ago in a completely different economic environment (Altman, 1983), they are still very good and reliable benchmark models of companies' financial health. Similar results were obtained by Boďa and Úradníček (2016) and Altman et al. (2017). Thus, we conclude that the validity of our H1 is supported only partially; better results are obtained using re-estimated models, but using the traditional significance level of 5% we cannot reject statistical hypothesis of equal results.[18]

Our second hypothesis (H2) states that non-financial company-specific variables improve discriminatory performance and classification accuracy. This hypothesis is tested by comparing results of model 3 vs model 2 and model 5 vs model 4. Looking at Table 7 we see that in both cases AUC increases by about 0.04. The confidence intervals show that this difference is statistically significant. Additionally, the average classification accuracy of models with non-financial company-specific variables dominate those without them (see Table 8). These results are consistent with those for the training sample (see Table 9). Our results provide strong evidence in favour of H2. Earlier studies confirmed that non-financial variables are significant predictors of the default (Fidrmuc and Hainz, 2010; Wilson, Ochotnický and Káčer, 2016; Altman et al., 2017). We extend this finding to recent time-period (2015 – 2016). In addition, we found that these indicators improve significantly out-of-sample predictions and hence are of practical importance.

Our third hypothesis (H3) supposes that macroeconomic variables improve discriminatory performance and classification accuracy. This hypothesis is tested by comparing results of model 4 vs model 2 and model 5 vs model 3. We see that increments in AUC in both training and validations samples due to macroeconomic variables are very small indeed (see last two rows of Table 6). Thus, it is not surprising that these differences are not statistically significant (see Table 7). Comparing classification accuracy of models with macroeconomic variables for training and validation samples clearly shows instability of average accuracy for any of the cut-off points (see Table 8 and Table 9). A particularly disturbing feature is the instability of false negative rate. This is not a desirable property in terms of failure prediction. Similar to Wilson, Ochotnický and Káčer (2016) and Altman et al. (2017), the macroeconomic variables seem to work well when used ex-post. In this setting they are statistically significant and attract expected sign. However, we found that they are not very useful for ex-ante prediction. Based on our results they do not seem to contribute to the prediction accuracy, on the contrary, they worsen the performance of the models. In the light of this evidence we reject hypothesis H3.

[18] We have previously mentioned that standard errors and confidence intervals are function of sample size. The size of our validation sample is rather large (more than 170,000 company-year observations) when compared to other studies (Boďa and Úradníček, 2016; Altman et al., 2017). Thus, we consider our results robust in relation to testing the hypothesis H1.

T a b l e 9

Classification Accuracy of the Models Using Training Sample

				Failed companies, N = 1 304				Non-failed companies, N = 490 045			
		Average accuracy	Overall accuracy	Failure accuracy (True positives)		Type I error (False negatives)		Non-failure accuracy (True negatives)		Type II error (False positives)	
		%	%	Obs.	%	Obs.	%	Obs.	%	Obs.	%
Model 1	cut-off 1	72.38	68.56	994	76.23	310	23.77	335 885	68.54	154 160	31.46
	cut-off 2	72.66	66.97	1 022	78.37	282	21.63	328 051	66.94	161 994	33.06
	cut-off 3	72.20	69.42	978	75.00	326	25.00	340 094	69.40	149 951	30.60
Model 2	cut-off 1	71.97	68.96	978	75.00	326	25.00	337 863	68.95	152 182	31.05
	cut-off 2	72.70	65.60	1 041	79.83	263	20.17	321 297	65.56	168 748	34.44
	cut-off 3	71.96	68.93	978	75.00	326	25.00	337 711	68.91	152 334	31.09
Model 3	cut-off 1	75.23	74.01	997	76.46	307	23.54	362 640	74.00	127 405	26.00
	cut-off 2	75.58	76.93	968	74.23	336	25.77	377 008	76.93	113 037	23.07
	cut-off 3	75.25	75.49	978	75.00	326	25.00	369 936	75.49	120 109	24.51
Model 4	cut-off 1	71.94	69.42	971	74.46	333	25.54	340 142	69.41	149 903	30.59
	cut-off 2	72.49	64.57	1 049	80.44	255	19.56	316 205	64.53	173 840	35.47
	cut-off 3	71.73	68.49	978	75.00	326	25.00	335 524	68.47	154 521	31.53
Model 5	cut-off 1	75.41	74.05	1 001	76.76	303	23.24	362 863	74.05	127 182	25.95
	cut-off 2	75.68	76.35	978	75.00	326	25.00	374 174	76.36	115 871	23.64
	cut-off 3	75.64	76.29	978	75.00	326	25.00	373 855	76.29	116 190	23.71
Revised Z'-Score		68.92	52.42	1 115	85.51	189	14.49	256 436	52.33	233 609	47.67
Revised Z''-Score		69.12	55.65	1 078	82.67	226	17.33	272 367	55.58	217 678	44.42

Notes: The models estimated in this paper (see Table 6 for details) were assessed using three different cut-off points: cut-off point 1 – proportion of defaulted companies in training sample; cut-off point 2 – value of optimal cut-off point maximizing Youden index (Youden, 1950) in training sample, as implemented in user-written Stata command cutpt (Clayton, 2013); cut-off point 3 – lower quartile of estimated probability scores for defaulted companies in training sample; The revised Z'-Score is a re-estimated Z-Score for private companies: $Z' = 0.717 * X1 + 0.847 * X2 + 3.107 * X3 + 0.420 * X4 + 0.998 * X5$. The revised Z''-Score is adapted for non-manufacturing private firms: $Z'' = 6.56 * X1 + 3.26 * X2 + 6.72 * X3 + 1.05 * X4$. The variables are defined in Table 4. Both models were published in Altman (1983). The cut-off points for these models were averages of the lower and upper bounds for the grey zone, i.e. 2.065 and 1.85, respectively. The average accuracy is a simple average of true positives and true negatives percentages to account for unbalanced sample. The overall accuracy is a weighted average of true positives and true negatives percentages; the weights are number of observations in each group.

Source: Authors' elaboration.

4.5. Limitations of the Study and Ideas for Future Research

In this study, we attempted to assess the contribution of company-specific non-financial information and macroeconomic variables to the performance of the failure prediction model. While we believe that the research objective has been achieved, there are several potential issues that need to be taken into account when interpreting the results. Firstly, even though the components of the revised Z'-Score model represent the relevant dimensions of firms' financial situation (liquidity, profitability, leverage and activity) a financial variable omitted from the model could partially or altogether invalidate our findings. Namely, the significant correlation between the omitted variable and the non-financial company-specific variables could result in decreased significance of the latter. Indeed, as Almamy, Aston and Ngwa (2016) found in the context of the UK listed companies, the cash flow from operations to total liabilities ratio significantly improves the predictive performance of the original Z-Score model. Secondly, the sample includes companies from various industry sectors but such an aggregate model may disguise interesting variations in the profiles of failing companies (e.g. the financial ratios depend heavily on the industry sector). Yet to the best of our knowledge, the distress or failure prediction models of Altman type are estimated for the samples covering multiple sectors. While it is true that in some sectors (such as Information and Health) the number of defaults is relatively small and there was a danger that the industry indicators of these sector would capture the idiosyncratic characteristics of failing companies, this was not the case since indicators of these sectors were not statistically significant. Moreover, the industry sector differences may manifest in the non-linear relationships, yet this is beyond the scope of our paper. Thirdly, even though the estimation sample covers the aftermath of the recent financial crisis associated with an economic slowdown and the ensuing recovery, it does not cover the whole business cycle. Moreover, the impact of company-specific and macroeconomic variables may be beyond simple additive relationships. Indeed, significant non-linear effects have been demonstrated by Hwang (2012). It remains an open question whether the above-mentioned points are relevant for Slovak SMEs. In any case, each of the points may prove to be a fruitful idea for future research.

Conclusion

So far, little has been known about the impact of non-financial company-specific information and macroeconomic variables on out-of-sample predictions in the context of Slovak SMEs. Our study aims to fill this gap. In addition, we provide updated evidence on the usefulness of the revised Z'-Score model in

Slovakia. We posed three research hypotheses: H1 anticipates that the re-estimated model of Z'-Score will perform better than the original model. H2 and H3 assume that non-financial company-specific variables (H2) and macroeconomic variables (H3) improve discriminatory performance and classification accuracy.

We use data from period 2009 – 2016 including 661 622 company-year observations about 149 618 individual SMEs with 1 575 failures. In relation to the first hypothesis we found that even though the model with re-estimated coefficients performs better than the original one, the difference in discriminatory performance measured by AUC is not statistically significant. In this regard, we confirm the results of Boďa and Úradníček (2016) and Altman et al. (2017). Non-financial company-specific variables (size, age, legal form and industry sector) significantly improved prediction performance, confirming our second hypothesis. Thus, we extended the findings of Fidrmuc and Hainz (2010), Wilson, Ochotnický and Káčer (2016) and Altman et al. (2017) in out-of-sample period. Our results suggest that macroeconomic variables (interest rates and economic expectations) seem to work well when used ex-post which is consistent with Wilson, Ochotnický and Káčer (2016) and Altman et al. (2017). However, we found that these variables do not significantly contribute to the discriminatory performance and even worsen classification accuracy in out-of-sample period. As a conclusion, we reject our third hypothesis.

Our results confirm that including non-financial firm-specific variables in default models of SMEs significantly improves the model's prediction performance. Such variables can be accessed relatively easily and updated frequently, thus allowing interested parties (banks, creditors etc.) to adjust their credit models. Future studies could perhaps enhance the set of these variables e.g. incorporating governance variables (information about owners and/or managers).

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