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Book

On the measurement of productivity growth in the Norwegian National Accounts

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On the measurement of productivity growth in the Norwegian National Accounts

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Preface

In response to a EUROSTAT call for research proposals in Economic Governance and Economic and Social Performance (ESTAT-2019-PA4-C-IGA), Statistics Norway launched a project in July 2019, for making available productivity and growth indicators building on the experiences with the KLEMS model. As the first stage of the project, this document undertakes a critical review of the current methodologies for compiling measures of productivity growth at Statistics Norway, with the view of identifying better methodologies and data sources that can be applied in the future.

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Statistisk sentralbyrå, 27 April 2020

Lasse Sandberg

Abstract

This document aims to take a review with critique of the current methodologies for measuring productivity growth at Statistics Norway, with the view of identifying the availability of better methodologies and data sources. In general, many of the currently applied methods are acceptable. However, various measures of labor and total factor productivities are calculated not always in an internally consistent way. In addition, some detailed methods need to be corrected and updated, in line with the framework of modern growth accounting. Based on the review results, a number of comments and suggestions are given for possible improvements in compiling growth and productivity accounts at Statistics Norway in the future. For instance, the current method of calculating capital services for KNR industry should be updated, and the inconsistency across the level of aggregation should be removed.

Contents

1. Introduction

Measures of productivity growth constitute essential indicators for the analysis of economic growth and wellbeing in an economy. For instance, value added per hour worked, one measure of labor productivity, is closely related to per capita income, probably the most common measure of living standards in an economy. Moreover, long-term trend in total factor productivity is frequently used to assess an economy's underlying productive capacity, which is an important indicator of the growth possibilities of the economy. Therefore, measures of productivity growth are of great interest to academics, policy makers, and even the public as well.

Productivity is commonly defined as the ratio of a volume measure of output to a volume measure of input for a production unit. While there is no disagreement on this general notion, there are many different approaches to measuring productivity in practice. For example, productivity could be a single factor measure (such as labor productivity), or multifactor ones (such as total factor productivity[1]). It could be measured by gross output or by value added. Further, it could be calculated at the establishment, firm, industry, sector, or the economy-wide level. Hence, the calculation, aggregation, and interpretation of various productivity measures require careful considerations.

With detailed and good-quality disaggregated industry level data available in the Norwegian national accounts compilation system, a bottom-up approach has been followed for compiling the measures of productivity growth at Statistics Norway. Thus, estimates of productivity growth are made at the lower-industry level first, and are then aggregated and finally published at the aggregated higher-industry or sector or the economy-wide level.

The measures of productivity growth at Statistics Norway are also used as inputs for economic analysis and are published, both internally in annual economic report (see e.g. Statistics Norway, 2019), and externally in reports presented each year by the Norwegian Technical Calculation Committee for Wage Settlements.[2]

In March 2019, an EU call was issued for projects to be funded under the European Statistics Programme in the area of Economic Governance and Economic and Social Performance. Among the activities that can be financially supported according to this call, methodological and practical work are encouraged for making available productivity and growth indicators building on the experiences with the KLEMS model.[3] As a response to the call, Statistics Norway launched a project in July 2019, and applied for financial support from Eurostat.[4]

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[1] To be more appropriate, total factor productivity (TFP) should be called multifactor productivity (MFP), simply because not all factors contributing to output growth are caught up in this productivity measure.

[2] The Technical Calculation Committee for Wage Settlements (*Det tekniske beregningsutvalget for inntektsoppgjørene (TBU)* in Norwegian) facilitates that the social partners and the authorities have the best possible common understanding of the situation in the Norwegian economy, before reaching any agreements on wage settlements between parties. The committee's reports are submitted to the Norwegian Ministry of Labor and Social Affairs.

[3] Using the modern growth accounting framework, the KLEMS model for an economy provides detailed production input measures including capital (K), labor (L), energy (E), materials (M) and services (S), as well as the output measure, at the disaggregated industry level. A key objective of the KLEMS model is to move beneath the aggregate economy level and examine the productivity performance of individual industries and their contributions to economic growth (see e.g. O'Mahony and Timmer, 2009; Timmer *et al*., 2010; Liu, 2017, 2018, 2019, 2020).

[4] The project has the title of 'Improving productivity accounts, data quality related to transactions of MNEs, and existing data sources for goods for processing and merchanting' and the acronym of EU project 874652 - 2019/NO/IGA. The work for the construction of improved growth and productivity accounts is one part of the project.

This document aims to review the current methodologies for measuring productivity growth as applied at Statistics Norway, with the view of identifying the availability of better methodologies and data sources. Based on the review results and in line with the framework of modern economic growth accounting (see e.g. Jorgenson and Griliches, 1967; Diewert, 1976; Caves *et al*., 1982; Jorgenson *et al*., 1987, 2005; Schreyer, 2001, 2009), comments and suggestions will be given for possible improvements in compiling growth and productivity accounts at Statistics Norway in the future.

Note that the review is, for the time being, carried out by focusing on productivity growth (for intertemporal comparison at one place) only, rather than on productivity level (for inter-space comparison at one point of time), although the latter is of essential importance when making comparative analysis across countries.

The rest of the document is organized as follows. In Section 2, a short description is given of the classification of industries in the Norwegian national accounts compilation system, based on which productivity growth measurement has been implemented. Section 3 discusses the measurement of industry labor productivity growth. Section 4 is about the measurement of industry total factor productivity growth.

Aggregation of the productivity growth measures from lower-industry level to higher-industry level, or further to sector and even to the economy-wide level is discussed in Section 5, then averaging over time of productivity growth measures is briefly discussed in Section 6. The overall structure in each of Sections 3-6 is the same, with the first subsection introducing the current practices at Statistics Norway and the second subsection giving comments. Finally, some concluding remarks and suggestions are given in Section 7.

2. Industry classification

The classification of industries in the Norwegian national accounts compilation system is an aggregated version of NACE rev.2[5], which specifies around 150 industries being applied in the *annual* national accounts (see Simpson and Todsen, 2012), and 79 aggregated industries being used in the *quarterly* national accounts (see Korsnes, 2014). For convenience, the former industries are thus called annual-national-accounts industries, and the latter ones quarterly-national-accounts industries.

The code and description of the 79 quarterly-national-accounts industries are listed in the first and second columns in Table 1, respectively. Among the total of 79 industries, 62 industries (with code of 23xx) deal with market activities, including production for own consumption. For nonmarket activities which cover 17 industries, central government deals with 6 industries (with code of 24xx), so does local government (with code of 25xx); the NPISHs (Non-profit Institutions Serving Households) have 5 industries (with code of 26xx).

Annual growth of labor productivity, both gross output-based and value added-based, is estimated first at the annual-national-accounts industries level. The estimated results are, after simple aggregation, published at A38, A64, and A88

[5] The term NACE is derived from the French *Nomenclature statistique des activités économiques dans la Communauté européenne*, which is the Statistical classification of economic activities in the European Community. NACE rev.2 is a (second) revised classification and was adopted at the end of 2006.

Table 1. Industries and sectors in the Norwegian national accounts

Industry		Sector (aggregate of industries)												
Code	Description	NR23FN	NRLKNR_NR23IND	NR23FN_AV	NR23JORD	NR23FISK	NR23BERG	NR23ELGV	NR23BOA	NR23FN_PT	NR23VAH	NR23ITR	NR23IKT	NR23FIN
Market activities (including production for own consumption)														
2301	Agriculture, Hunting	x		x	x									
2302	Forestry	x		x	x									
2303	Fishing	x		x		x								
2304	Aquaculture	x		x		x								
2305	Mining and quarrying	x		x			x							
2306	Extraction of oil and gas													
2307	Service activities incidental to oil and gas	x								x				
2310	Food products, beverages and tobacco	x	x											
2312	Fish farming	x	x											
2313	Textiles, wearing apparel, leather	x	x											
2315	Manufacture of wood and wood products	x	x											
2316	Wood processing	x	x											
2317	Graphic production	x	x											
2318	Production of coal and refined petroleum	x	x											
2319	Chemical raw goods	x	x											
2320	Chemical products	x	x											
2321	Production of pharmaceutical products	x	x											
2322	Rubber and plastic products	x	x											
2323	Other chemical and mineral products	x	x											
2324	Metal raw goods	x	x											
2325	Metal products	x	x											
2326	Computer and electronics	x	x											
2327	Electrical equipment	x	x											
2328	Machinery and equipment	x	x											
2329	Production of transport equipment	x	x											
2330	Building of ships	x	x											
2331	Building of oil platforms and modules	x	x											
2332	Other industry production	x	x											
2333	Repair/installation of machinery/equipment	x	x											
2335	Production of electricity	x		x				x						
2336	Transport and sale of electricity	x		x				x						
2337	Other energy, district heating and gas	x		x				x						
2338	Water supply, sewerage, waste	x								x				
2341	Building development	x		x					x					
2342	Construction	x		x					x					
2344	Wholesale/retail trade, repair of motor v.	x								x	X			
2346	Passenger transport	x								x		x		
2347	Goods transport	x								x		x		
2348	Pipe transportation of oil and gas													
2349	Foreign maritime													
2350	Domestic maritime transport	x								x		x		
2351	Air transport	x								x		x		
2352	Services connected to transport	x								x		x		
2353	Post and distribution	x								x		x		
2356	Hotel and restaurant	x								x				
2358	Publishing business	x								x			x	
2361	Telecommunication	x								x			x	
2362	Information services	x								x			x	
2364	Financial services	x								x				x
2367	Managing real estate	x								x				

2368	Owner-occupied housing services (imputed)													
2369	Housing services (rented)													
2370	Architecture/legal/accounting/consulting	x								x				
2372	Research and Development	x								x				
2373	Marketing/veterinary and other services	x								x				
2377	Leasing, travel and other business services	x								x				
2385	Education/training	x								x				
2386	Health services	x								x				
2387	Social welfare services	x								x				
2390	Cultural/sports/leisure activities	x								x				
2394	Membership and other private activities	x								x				
2397	Paid household works	x								x				
Nonmarket activities (central government)														
2482	Defense													
2484	Public administration													
2485	Education/training													
2486	Health services													
2487	Social welfare services													
2490	Cultural and other activities													
Nonmarket activities (local government)														
2538	Water supply, sewerage, waste													
2584	Public administration													
2585	Education/training													
2586	Health services													
2587	Social welfare services													
2590	Cultural and other activities													
Nonmarket activities (NPISHs))														
2685	Education/training													
2686	Health services													
2687	Social welfare services													
2690	Cultural and other activities													
2694	Membership related activities													

Notes: ‘x’ stands for ‘included’.

NR23FN = Market activities in mainland Norway (excluding housing services)
NRLKNR_NR23IND = Industrial activities
NR23FN_AV = Other goods production industries in mainland Norway
NR23JORD = Agriculture and forestry
NR23FISK = Fishing and aquaculture
NR23BERG = Mining and quarrying
NR23ELGV = Electricity and district heating and gas
NR23BOA = Building development and construction
NR23FN_PT = Private services in mainland Norway (excluding housing services)
NR23VAH = Wholesale/retail trade, repair of motor vehicles
NR23ITR = Domestic transport
NR23IKT = Information and communication technology
NR23FIN = Financial services

Source: Statistics Norway

industry levels[6] in Table 09174 (Wages and salaries, employment and productivity, by industry) in the online databank at Statistics Norway.[7]

On the other hand, annual growth of total factor productivity is estimated only for the 79 quarterly-national-accounts industries. One reason is that data for the latest years are only available at the level of quarterly-national-accounts industries.

[6] A38, A64 and A88 industry levels refer to the industry classification levels at Eurostat. A38 and A64, but not A88, industry levels, are used for data reporting to Eurostat by countries through the ESA Transmission Programme (Eurostat, 2013).

[7] See https://www.ssb.no/en/statbank/table/09174/.

As shown in Table 1, special interests have been given to some sectors that are defined as various aggregates of those industries dealing with market activities (i.e. industries with code of 23xx in Table 1). In addition, because these sectors are regarded as being of critical importance for economic analysis, both annual growth of value added-based labor and total factor productivities are estimated within the same framework for the 79 quarterly-national-accounts industries first, and the aggregated estimates for these sectors in concern are published regularly at Statistics Norway.[8]

3. Industry labor productivity growth

3.1. Current method

In the Norwegian national accounts compilation system, the volume index of an industry *j*'s gross output ($\frac{Y_{j,t}}{Y_{j,t-1}}$) and that of intermediate input ($\frac{X_{j,t}}{X_{j,t-1}}$) are constructed by means of the (annually chained) Laspeyres volume index formula:

(1) $$\frac{Y_{j,t}}{Y_{j,t-1}} = \frac{\sum_i P_{i,j}^{Y,t-1} Y_{i,j}^{t}}{\sum_i P_{i,j}^{Y,t-1} Y_{i,j}^{t-1}},$$

$$\frac{X_{j,t}}{X_{j,t-1}} = \frac{\sum_x P_{x,j}^{X,t-1} X_{x,j}^{t}}{\sum_x P_{x,j}^{X,t-1} X_{x,j}^{t-1}},$$

where $P_{i,j}^{Y,t-1}$ is the price received by industry *j* for selling product *i* at time *t-1*, and $Y_{i,j}^{t}$ is the volume of product *i* produced by industry *j* at time *t*; $P_{x,j}^{X,t-1}$ is the price paid by industry *j* for using intermediate input *x* at time *t-1*, and $X_{x,j}^{t}$ is the volume of intermediate input *x* used by industry *j* at time *t*.

In practice, detailed price index and current transaction value are available for each product *i* and *x*, so it is not difficult to compile the volume index of industry *j*'s gross output and that of intermediate input by using (1).

With the volume index being defined as in (1), the implicit price index, which is Paasche index, of industry *j*'s gross output ($\frac{P_{j,t}^{Y}}{P_{j,t-1}^{Y}}$) and that of intermediate input ($\frac{P_{j,t}^{X}}{P_{j,t-1}^{X}}$) can be written as:

(2) $$\frac{P_{j,t}^{Y}}{P_{j,t-1}^{Y}} = \frac{\sum_i P_{i,j}^{Y,t} Y_{i,j}^{t}}{\sum_i P_{i,j}^{Y,t-1} Y_{i,j}^{t}},$$

$$\frac{P_{j,t}^{X}}{P_{j,t-1}^{X}} = \frac{\sum_x P_{x,j}^{X,t} X_{x,j}^{t}}{\sum_x P_{x,j}^{X,t-1} X_{x,j}^{t}},$$

such that the following identities can be held:

(3) $$\frac{\sum_i P_{i,j}^{Y,t} Y_{i,j}^{t}}{\sum_i P_{i,j}^{Y,t-1} Y_{i,j}^{t-1}} = \frac{P_{j,t}^{Y}}{P_{j,t-1}^{Y}} \cdot \frac{Y_{j,t}}{Y_{j,t-1}},$$

$$\frac{\sum_x P_{x,j}^{X,t} X_{x,j}^{t}}{\sum_x P_{x,j}^{X,t-1} X_{x,j}^{t-1}} = \frac{P_{j,t}^{X}}{P_{j,t-1}^{X}} \cdot \frac{X_{j,t}}{X_{j,t-1}}.$$

[8] See https://www.ssb.no/nasjonalregnskap-og-konjunkturer/statistikker/nr/tilleggsinformasjon/produktivitetsendringer-for-naringer/.

Equation (3) indicates that the value change between two periods will exactly decompose into a Paasche price index times a Laspeyres volume index, which is a desirable characteristic for index formulas.[9]

Based on (1), annual (percentage) changes of the volume of industry *j*'s gross output and that of intermediate input can be chained together. In addition, by choosing a reference year (e.g. 2015) in which the price index is set equal to 1, the volume level of industry *j*'s gross output ($Y_{j,t}$) and that of intermediate input ($X_{j,t}$) at time *t* can be derived in so-called constant (e.g. 2015) prices.

In the following, the super- and/or subscript of time *t* will be suppressed in equations simply for the clarity of exposition, unless the specific context requires time *t* to be referred to explicitly to avoid misunderstanding.

In the Norwegian national accounts compilation system, the volume of value added in industry *j* (Z_j) is defined as the difference between the volume of industry *j*'s gross output (Y_j) and that of intermediate input (X_j) in constant prices:

(4) $$Z_j = Y_j - X_j.$$

The relationship between the nominal values of value added, gross output, and intermediate input in industry *j* is as follows:

(5) $$P_j^Z Z_j = P_j^Y Y_j - P_j^X X_j,$$

where P_j^Z is the (implicit) price index of value added for industry *j*, $P_j^Y Y_j (= \sum_i P_{i,j}^Y Y_{i,j})$ and $P_j^X X_j (= \sum_x P_{x,j}^X X_{x,j})$ are current values of gross output and intermediate inputs for industry *j*, respectively.

At the industry level, two types of labor productivity statistics are compiled: one is gross output-based, and the other value added-based. The gross output-based labor productivity in an industry *j*, LP_j^Y, is defined as:

(6) $$LP_j^Y = \frac{Y_j}{H_j},$$

where H_j is the labor inputs, which is measured as total hours worked in industry *j*.

The growth rate of the gross output-based labor productivity is then measured as:

(7) $$\Delta \ln LP_j^Y = \Delta \ln Y_j - \Delta \ln H_j,$$

where $\Delta q = q_t - q_{t-1}$ denotes the period (e.g. annual) change of variable q between *t-1* and *t*, such that $\Delta \ln q$ indicates the logarithmic growth rate of variable q.

The value added-based labor productivity in industry *j*, LP_j^Z, is defined as:

(8) $$LP_j^Z = \frac{Z_j}{H_j}.$$

[9] For general discussions on the pros and cons of Laspeyres and Paasche indexes, as well as other index number formulas, see e.g. United Nations (2009), International Labor Organization *et al*. (2004a, 2004b, 2009).

The corresponding growth rate of the value added-based labor productivity is measured as:

$$\Delta \ln LP_j^Z = \Delta \ln Z_j - \Delta \ln H_j. \quad (9)$$

3.2. Comments

Sectoral output and input

Once the concept of gross output is used for productivity measurement at the industry level, the question arises about how to treat those transactions that occur within industries, i.e. intra-industry deliveries of intermediate inputs. Clearly, gross output can be made larger and larger by basing the industry aggregate on increasingly smaller statistical units because of the well-known existence of double-counting. Consequently, an industry gross output measure based on establishments would be larger than one based on firms and one based on firms larger than one based on groups of firms, etc.

To overcome this problem, the concept of sectoral output is introduced for measuring industry-level output by excluding intra-industry deliveries (see Gollop, 1979). In other words, the sectoral output of an industry at a given level of aggregation only reflects deliveries outside of the industry, while sectoral input is net of intra-industry deliveries.

Conceptually, the adoption of sectoral output/input amounts to a process of integration of different units or industries - as one moves up the hierarchy of the activity classification, larger and larger units are formed and treated as if they were single firms. At every level of aggregation, only flows out of (as sectoral output) or into the sector (as sectoral input) are considered. Therefore, at the level of the entire economy, measures of sectoral output and of value added converge, although not entirely in the presence of imported intermediate inputs (see Schreyer, 2001).

As introduced in Section 2, published measures of productivity growth at Statistics Norway are for those industries or sectors that are aggregates of around 150 annual-national-accounts industries. When implementing the aggregation based on gross output concept, no concern has ever been paid, and measures taken, for excluding intra-industry deliveries of intermediate inputs. As a result, the gross output-based labor productivity level as currently estimated at Statistics Norway is most likely upward-biased.

However, by means of a brief investigation into the Norwegian national accounts database, preliminary results have shown that both the scale and magnitude of the potential biasedness, though vary across industries, are not significantly large. Moreover, the impact due to the biased productivity level on the estimate of the corresponding growth rate may even be less significant. Nonetheless, further investigation on this issue should be taken in the future.

Finally, it is worth mentioning that the sectoral output concept not only helps for constructing gross output-based labor productivity measures, but also offers one possibility for consistent aggregation of gross-output based total factor productivity growth across industries. However, the construction of the gross-output based total factor productivity growth at industry level has not yet been undertaken at Statistics Norway, despite the usefulness of these indicators for economic analysis. As such, the issue needs also to be addressed in the future.

Index number formula

Both economic and index number theories can be used to facilitate better choice of index number formula. It has been pointed out that using a Laspeyres or Paasche index to calculate output (or input) indices implies an underlying fixed-coefficient technology for the production structure - clearly a strongly simplifying assumption because it excludes the possibility of substitution between inputs or outputs and implies constant marginal products throughout. At best, these Laspeyres or Paasche indices provide bounds for the true underlying volume or price indices (see Schreyer, 2001).

On the other hand, a strong argument has been made in favor of the Törnqvist index, which is one of the 'superlative' index numbers (see Diewert, 1976). The reason is that the Törnqvist index can be directly derived from the translog flexible functional form, while this functional form is a widely used specification in empirical economics.[10] Thus, if one accepts a translog functional form as a reasonable approximation to a production function, and uses standard assumptions about producer behavior, the Törnqvist volume index is supposed to provide an exact formulation for inputs and outputs (Caves *et al*., 1982).

Currently at Statistics Norway, annual chain-linking of Laspeyres volume and Paasche price indexes is applied, which is considered a good approximation of superlative index numbers, and it is recommended by Eurostat. However, given that several world leading national statistical institutes (such as the Bureau of Economic Analysis (BEA) in the United States, Statistics Canada) have already changed their index number formulas to the superlative indexes (such as Fisher index) for compiling productivity indicators (see Landefeld and Parker, 1998; Baldwin *et al*., 2007), it is worth experimenting such compilations by applying the superlative index number formulas at Statistics Norway.

Moreover, the results from compiling productivity indicators by applying the superlative index number formulas can at least serve as a check to see whether or not the currently applied annual chain-linking of Laspeyres volume and Paasche price indexes is indeed a good approximation of superlative index numbers.

The Tornquist volume index for gross output and intermediate input can be defined as the following weighted geometric average of gross output and intermediate input relatives:

(10)
$$\frac{Y_{j,t}}{Y_{j,t-1}} = \prod_i \left(\frac{Y_{i,j}^t}{Y_{i,j}^{t-1}}\right)^{\bar{v}_{i,j}^Y},$$

$$\frac{X_{j,t}}{X_{j,t-1}} = \prod_x \left(\frac{X_{x,j}^t}{X_{x,j}^{t-1}}\right)^{\bar{v}_{x,j}^X},$$

where $\bar{v}_{i,j}^Y$ is the arithmetic average share of product *i* in the total nominal value of output in industry *j* over the two adjacent period, *t-1* and *t*, and similarly, $\bar{v}_{x,j}^X$ is the arithmetic average share of intermediate input *x* in the total nominal value of intermediate inputs in industry *j* over the same period.

[10] 'Superlative' index numbers are those that can be directly derived from functional forms that provide a second-order approximation to an arbitrary, twice differentiable linear homogenous function, covering a wide range of utility, production, distance, cost or revenue functions. A 'superlative' index is called 'exact' if it can be directly derived from a particular functional form. For example, Törnqvist index is exact for the translog flexible functional form, and Fisher index is exact for a quadratic functional form.

The value share $v_{i,j}^{Y}$ and $v_{x,j}^{X}$ are defined respectively as follows:

(11) $$v_{i,j}^{Y} = \frac{P_{i,j}^{Y} Y_{i,j}}{P_{j}^{Y} Y_{j}} = \frac{P_{i,j}^{Y} Y_{i,j}}{\sum_i P_{i,j}^{Y} Y_{i,j}},$$

$$v_{x,j}^{X} = \frac{P_{x,j}^{X} X_{x,j}}{P_{j}^{X} X_{j}} = \frac{P_{x,j}^{X} X_{x,j}}{\sum_i P_{x,j}^{X} X_{x,j}}.$$

With the volume index being ready, the corresponding price index for either gross output or intermediate input of industry *j* can be derived simply by using the nominal value of gross output or intermediate input of industry *j* divided by the respective Törnqvist volume index as defined in (10).

Finally, taking the logarithm on both sides of (10) yields another frequently-used form of Törnqvist volume index:

(12) $$\Delta \ln Y_j = \sum_i \bar{v}_{i,j}^{Y} \Delta \ln Y_{i,j},$$

$$\Delta \ln X_j = \sum_x \bar{v}_{x,j}^{X} \Delta \ln X_{x,j}.$$

Value added

Independent of the way how economic units (such as establishments, firms, industries etc.) are organized, current-price values of value added can simply be summed up across different units, without the risk of double-counting. However, this convenience also raises challenges for the index construction for the volume of value added.

As mentioned, both economic and index number theories can provide guidance on the choice of index number formula. Though not observable in quantitative form, it is always possible to construct a volume index of value added, which, conceptually, serves as another measure of output. Depending on the form of the underlying production function, this output index may or may not be independent of primary inputs (see Goldman and Uzawa, 1964).

For instance, the volume index of value added can be defined as a continuous-time Divisia index in a theoretical construct (see e.g. Basu and Fernald, 1995, 1997):

(13) $$\frac{d \ln Z_j}{dt} = \frac{1}{v_{Z,j}^{Y}} \left(\frac{d \ln Y_j}{dt} - v_{X,j}^{Y} \cdot \frac{d \ln X_j}{dt} \right),$$

where

(14) $$v_{Z,j}^{Y} = \frac{P_j^{Z} Z_j}{P_j^{Y} Y_j},$$

$$v_{X,j}^{Y} = \frac{P_j^{X} X_j}{P_j^{Y} Y_j}.$$

As shown, the (percentage) volume change in value added ($\frac{d \ln Z_j}{dt}$) can be defined as an average of the (percentage) volume change of gross output ($\frac{d \ln Y_j}{dt}$) and the (percentage) volume change of intermediate inputs ($\frac{d \ln X_j}{dt}$). The volume change of intermediate inputs is weighted by the share of intermediate inputs in gross output ($v_{X,j}^{Y}$) and the entire expression is multiplied by the inverted share of value added in gross output ($v^{Y}_{Z,j}$). Because the volume change for value added combines the volume change for gross output and intermediate inputs, it constitutes a general-form double deflation.

As shown in (4), the volume measure of value added in an industry *j* is currently obtained by subtracting a constant-price intermediate inputs from a constant-price gross output at Statistics Norway. It can be shown that (4) leads to the following equation:

(15) $$\frac{\Delta Z_{j,t}}{Z_{j,t-1}} = \frac{Y_{j,t-1}}{Z_{j,t-1}}\left(\frac{\Delta Y_{j,t}}{Y_{j,t-1}} - \frac{X_{j,t-1}}{Y_{j,t-1}} \cdot \frac{\Delta X_{j,t}}{X_{j,t-1}}\right),$$

which corresponds to a discrete-time approximation of the continuous-time Divisia index by a fixed-weight Laspeyres volume index, because $\frac{d \ln Z_j}{dt} \approx \frac{\Delta Z_{j,t}}{Z_{j,t-1}}$, $\frac{d \ln Y_j}{dt} \approx \frac{\Delta Y_{j,t}}{Y_{j,t-1}}$, and $\frac{d \ln X_j}{dt} \approx \frac{\Delta X_{j,t}}{X_{j,t-1}}$.

As mentioned, the use of fixed-weight Laspeyres volume index, however, raises a number of problems, and implicitly poses restrictive assumptions on the underlying production technology. The situation is different when the empirical discrete-time approximation to the continuous-time Divisia index is based on the ‘superlative’ index numbers (such as the Törnqvist index).

The Törnqvist volume index of value added in an industry *j* can be defined as:

(16) $$\Delta \ln Z_j = \frac{1}{\bar{v}^{Y}_{Z,j}}\left(\Delta \ln Y_j - \bar{v}^{Y}_{X,j} \cdot \Delta \ln X_j\right),$$

where $\bar{v}^{Y}_{Z,j}$ is the period average of the share of value added in gross output, and $\bar{v}^{Y}_{X,j}$ is the period average of the share of intermediate inputs in gross output. The two value shares $v^{Y}_{Z,j}$ and $v^{Y}_{X,j}$ are defined as follows:

(17) $$v^{Y}_{Z,j} = \frac{P^{Z}_{j} Z_j}{P^{Y}_{j} Y_j},$$

$$v^{Y}_{X,j} = \frac{P^{X}_{j} X_j}{P^{Y}_{j} Y_j}.$$

Labor input

For the computation of labor productivity measures, hours actually worked are used as labor input within the Norwegian national accounts compilation system, which is in accordance with the recommendations by current international standards for national accounts, such as SNA 2008 (United Nations, 2009) and ESA 2010 (Eurostat, 2013).

Note that hours worked as currently applied at Statistics Norway do not adjust for differences in the qualifications, skill levels and composition of labor. Although such quality adjustment does not have impact on the calculation of apparent labor productivity, it does have impact on the interpretation of labor productivity growth and, in particular, on both the interpretation and the estimation of the total factor productivity growth. More discussions will be given later.

4. Industry total factor productivity growth

4.1. Current method

Like labor productivity, the growth of total factor productivity can also be measured as either gross output-based or value added-based. This document will focus on the value added-based total factor productivity growth since it is this measure that is currently published at Statistics Norway.

The growth of the value added-based total factor productivity of industry j (TFP_j^Z) is constructed as:

(18) $$\Delta \ln TFP_j^Z = \Delta \ln Z_j - \bar{v}_{K,j}^Z \Delta \ln K_j - \bar{v}_{L,j}^Z \Delta \ln H_j,$$

where Z_j is the volume of industry j's value added, $\bar{v}_{K,j}^Z$, and $\bar{v}_{L,j}^Z$ are the period average share of capital (K_j) and labor (H_j) input in the nominal value added of industry j, respectively. The value share of capital and labor input is defined as:

(19) $$v_{K,j}^Z = \frac{P_j^K K_j}{P_j^Z Z_j},$$

$$v_{L,j}^Z = \frac{P_j^L H_j}{P_j^Z Z_j},$$

where P_j^K and P_j^L are the price indexes of capital (K_j) and labor (H_j) input in industry j, respectively.

By invoking the standard assumption of constant returns to scale, the total compensation of labor and capital will then exhaust the value added of industry j, i.e.

(20) $$P_j^Z Z_j = P_j^K K_j + P_j^L H_j,$$

then the two shares in (19) sum to unity, and (18) can then be reorganized as:

(21) $$\Delta \ln TFP_j^Z = \Delta \ln LP_j^Z - \bar{v}_{K,j}^Z \Delta \ln KI_j,$$

where LP_j^Z is defined as shown by (8), and $KI_j = \frac{K_j}{H_j}$ is capital intensity (capital services per hour) in industry j.

Formally, the first item on the right-hand side of (18), $\Delta \ln Z_j$, is estimated by following (4) as outlined in Section 3.1. Labor compensation ($P_j^L H_j$) is estimated first from labor accounts in the Norwegian national accounts, then capital compensation ($P_j^K K_j$) is estimated residually by following (20), given the value added in an industry j ($P_j^Z Z_j$). The growth of labor input in industry j ($\Delta \ln H_j$) is simply estimated as the logarithmic growth rate of the total hours worked in industry j.

Therefore, once the growth of capital input in industry j ($\Delta \ln K_j$) is ready, the value added-based total factor productivity growth of industry j ($\Delta \ln TFP_j^Z$) can be estimated residually by following (18). However, the estimation of capital input growth in industry j ($\Delta \ln K_j$) merits more introduction here, since it is of crucial importance for a better understanding of my comments later.

Because various types of capital in an industry have different productivities when used in production, to account for this heterogeneity, the user-cost approach is employed, based on which capital input is measured as capital services, rather than as capital stock as used for growth analysis in earlier (e.g. Solow, 1970; Kuznets, 1971) and even in recent years (e.g. Feenstra *et al*., 2015; Timmer *et al*., 2015).

To measure capital services that are not directly observable, estimates of the capital stock and the shares of capital remuneration are needed for detailed assets. The capital stock is measured by the Perpetual Inventory Method (PIM), according to which, capital stock is defined as a weighted sum of past investments with weights given by the relative efficiencies of capital goods at different ages.

By further applying geometric pattern (implying that a given vintage of investment loses a fixed percentage of its productive capacity each year) for all assets in an industry, the capital stock of an asset *k* in industry *j* at time *t*, $S_{k,j,t}$, is given by the following equation:

(22) $$S_{k,j,t} = S_{k,j,t-1}\left(1 - \delta_{k,j}\right) + I_{k,j,t} \ ,$$

where $\delta_{k,j}$ is the rate of depreciation, and $I_{k,j,t}$ is the investment of asset *k* in industry *j* during the period *t*.

For the aggregation of capital services over different asset types, it is assumed that aggregate services are a translog function of the services of individual assets. It is further assumed that the flow of capital services by each asset type *k*, $K_{k,j}$ is proportional to its stock $S_{k,j}$. Note that time subscript *t* is again suppressed here for convenience.

Then the growth of capital input in industry *j* is estimated as follows:

(23) $$\Delta \ln K_j = \sum_k \bar{v}_{k,j}^K \Delta \ln K_{k,j} \ ,$$

where $\bar{v}_{k,j}^K$ is the period average share of each asset component in the value of total capital compensation in industry *j*, and

(24) $$v_{k,j}^K = \frac{P_{k,j}^K K_{k,j}}{\sum_k P_{k,j}^K K_{k,j}} \ ,$$

with $P_{k,j}^K$ being the price of capital services from asset type *k*, which is also called the rental price, or the user cost of capital.

The rental price is determined by the following cost-of-capital equation:

(25) $$P_{k,j,t}^K = i_{j,t} P_{k,j,t-1}^I + \delta_{k,j} P_{k,j,t}^I - \left(P_{k,j,t}^I - P_{k,j,t-1}^I\right) \ ,$$

where $i_{j,t}$ is the nominal rate of return, and $P_{k,j,t}^I$ is investment price of asset *k* at time *t*.

Since the detailed investment price for each asset *k* ($P_{k,j,t}^I$) is available in the Norwegian national accounts compilation system, and the rate of depreciation ($\delta_{k,j}$) is identical to the rate used in the construction of the capital stock estimates in (22), to calculate the rental price, the only unknown variable in (25) is the nominal rate of return, $i_{j,t}$.

The nominal rate of return ($i_{j,t}$) can be estimated in two different approaches. The first one is the residual, or ex-post approach, which estimates the rate of return as a residual, given the value of capital compensation from the national accounts, depreciation and the capital gains. The second one is the ex-ante approach, which is based on some exogenous value for the rate of return, for example interest rates on government bonds.

The advantages of the ex-post approach are that it ensures complete consistency between income and production accounts by assuming that the total value of capital services for each industry equals its compensation for all assets, thus generating an internal rate of return that exhausts capital income and is consistent with constant returns to scale assumption.

Nonetheless, the ex-post approach also has some disadvantages. For instance, the gross operating surplus contains compensation for all assets, including those not covered in the current national accounts, leading to an overestimated rate of return. In addition, the assumptions, such as equalization of rates of return across all assets in an industry, are rather strong. Furthermore, such endogenously calculated rate of return is volatile and can result in negative rental prices.

With all the considerations given, the ex-post approach is finally chosen in the current Norwegian national accounts compilation system for calculating capital services for published statistics.

4.2. Comments

Value added

The volume of industry *j*'s value added (Z_j), as applied in (18) for making estimation of the total factor productivity growth, is currently measured by the difference between two Laspeyres volume indexes as shown in (4). However, as pointed out in Section 3.2, it is a better choice to apply the superlative index such as Törnqvist index instead.

It is interesting to know that the second and the third items on the right-hand side of (18) has already been constructed together as a Törnqvist volume input index. If the current difference of Laspeyres volume indexes for output as applied in (18) is substituted by a Törnqvist volume index, Equation (18) can be interpreted as that the growth of total factor productivity in industry *j* is estimated by the difference between two Törnqvist indexes, one for output and the other for input.

Labor input

The labor input of industry *j* (H_j), as applied in (18) for making estimation of the total factor productivity growth, is currently measured as the sum of total hours worked in the industry, regardless of the quality differences between hours worked by people with high and those with low education or skills. This does not make sense.

In theory, labor input can be regarded as labor services generated by human capital embodied in labor forces working in industry. Since human capital developed varies across different types of labor, the productivity of various types of labor (such as low- versus high-skilled) will differ. [11] This heterogeneity must be taken on board when analyzing the productivity and the actual contribution of labor input to output growth.

[11] For more general discussions on how human capital is developed, composed and what kind of benefits, including improved labor productivity, can be generated through human capital investment, please refer to Liu and Fraumeni (2014, 2016).

Suppose that the labor force can be divided into different types by characteristics considered to be important factors determining the labor productivity, such as age, gender, educational attainment, and occupation, etc. Further assume that aggregate labor services are a translog function of the services delivered by individual types, that the flow of labor services for each type is proportional to hours worked, and that workers are paid their marginal productivities, then, the aggregate labor input L in industry j, L_j, can be measured by a Törnqvist volume index of individual types, indexed by *l*:

(26) $$\Delta \ln L_j = \sum_l \bar{v}^L_{l,j} \Delta \ln H_{l,j},$$

where $\bar{v}^L_{l,j}$ is the period average share of each labor type *l* in the value of total labor compensation in industry j, such that the sum of shares over all labor types within the industry j is unity. The term $\Delta \ln H_{l,j}$ indicates the growth of actual hours worked by labor type *l* in industry *j* over the period.

The value share of each individual labor type *l* is defined as:

(27) $$v^L_{l,j} = \frac{P^L_{l,j} H_{l,j}}{P^L_j L_j} = \frac{P^L_{l,j} H_{l,j}}{\sum_l P^L_{l,j} H_{l,j}},$$

where $P^L_{l,j}$ is the price of one hour worked received by labor type *l* in industry *j*.

As we assume that marginal revenues are equal to marginal costs, the weighting procedure as shown in (27) ensures that an individual labor type which has a higher price also has a larger influence in the labor input index. For example, a doubling of hours worked by a high-skilled worker gets a bigger weight than a doubling of hours worked by a low-skilled worker.

Let H_j indicate total hours worked by all types of labor in industry j, i.e. $H_j = \sum_l H_{l,j}$, then we can further decompose the change in labor inputs as shown in (26) as follows:

(28) $$\Delta \ln L_j = \sum_l \bar{v}^L_{l,j} \Delta \ln \frac{H_{l,j}}{H_j} + \Delta \ln H_j = \Delta \ln LC_j + \Delta \ln H_j.$$

The first item on the right-hand side of (28) is defined as the change in labor composition, and the second item is the change in total hours worked in an industry j. It can be seen that if only proportions of each labor type change, while keeping total hours worked unchanged, then the impact on the growth of labor input will be reflected only by the change of labor composition.

If $\Delta \ln H_j$ in (18) is replaced by $\Delta \ln L_j$ as shown in (28), and that the two shares in (19) sum to unity still holds, then (21) becomes

(29) $$\Delta \ln TFP^Z_j = \Delta \ln LP^Z_j - \bar{v}^Z_{K,j} \Delta \ln KI_j - \bar{v}^Z_{L,j} \Delta \ln LC_j.$$

Comparing (21) with (29) shows immediately that the current estimate of the total factor productivity growth by (21) is biased owing to that the third item on the right-hand side of (29) is missing.

The biasedness to the estimate of total factor productivity growth due to the absence of the change in labor composition could be either upward or downward, depending on whether $\Delta \ln LC_j$ is positive or negative. As shown by recent research, the change of labor composition in Norwegian industries over the period

1997-2014 varies across industries and had both positive and negative values over the period (see Liu, 2017, 2018).

By considering the quality differences of labor input, the so-called quality adjusted labor inputs are important statistics *per se* that can be used for addressing a number of interesting issues, such as new job creation related to digital economy along with the process of globalization. Recently, with the purpose of improving the conventional measure of labor input for comparative analysis, the European Commission launched a QALI (Quality Adjusted Labor Inputs) project and the results were already published as experimental statistics at Eurostat.[12]

Adjusting labor inputs by gender and educational attainment in the Norwegian national accounts was once implemented for the period 2008-2014 and published in online databank as Table 10585 (Compensation and employees, by industry, education and sex 2008 – 2014)[13]. Further, the one-time adjusted statistics were used as data inputs for compiling a Norwegian KLEMS database (see Liu, 2017). For quality adjusted labor inputs data beyond the period 2008-2014, more work needs to be done.

Since 2015, a new data reporting system has been introduced in Norway,[14] which offers a good opportunity for compiling high quality labor statistics in the Norwegian national accounts compilation system. In particular, there exists possibility for constructing the statistics of labor inputs that could be cross-classified not only by gender and educational attainment, but also by age and occupation.

Age is frequently used as a proxy indicator reflecting working experiences due to e.g. on-the-job training while the latter is one of the most important channels for human capital development (see e.g. Liu and Fraumeni, 2014, 2016). Moreover, occupation is often employed to directly identify new job creation, which is important information for analysis related to digitalization and globalization.

In short, information from this new data source should be exploited as much as possible, and the work for incorporating quality adjusted labor inputs cross-classified by age, gender, educational attainment, and occupation into the labor accounts of the Norwegian national accounts should be placed on the priority agenda in the future.

Capital services

Following the ex-post approach for calculating the rate of return, the nominal rate of return is assumed to be the same for all assets in an industry *j* but is allowed to vary across industries. According to the current method (see Todsen, 2019), the nominal rate of return for industry *j* is calculated as follows:

(30) $$i_{j,t} = \frac{P^K_{j,t}K_{j,t} - \sum_k P^I_{k,j,t}\delta_{k,j}S_{k,j,t}}{\sum_k P^I_{k,j,t-1}S_{k,j,t}} + \left(\frac{P^I_{j,t}}{P^I_{j,t-1}} - 1\right),$$

where the first term in the numerator of the first item, $P^K_{j,t}K_{j,t}$, is the total capital compensation in industry j, which under constant returns to scale can be derived as value added minus labor compensation, i.e. as gross operating surplus; $(\frac{P^I_{j,t}}{P^I_{j,t-1}})$ is the price change of total capital investment of all asset types in industry *j*.

[12] See https://ec.europa.eu/eurostat/web/experimental-statistics/qali.

[13] See https://www.ssb.no/en/statbank/table/10585/.

[14] A-ordning in Norwegian. See https://www.skatteetaten.no/en/business-and-organisation/employer/the-a-melding/about-the-a-ordning/about-a-ordningen/.

As such, the nominal rate of return in each industry ($i_{j,t}$) can be determined by using (30), and subsequently, this estimated rate is used to calculate the capital service prices as shown in (25).

Unfortunately, it is not correct to calculate the nominal rate of return by using (30) by following the ex-post approach. The correct formula ought to be:

$$(31) \qquad \widehat{\iota_{J,t}} = \frac{P^K_{j,t} K_{j,t} + \sum_k \left(P^I_{k,j,t} - P^I_{k,j,t-1} \right) S_{k,j,t} - \sum_k P^I_{k,j,t} \delta_{k,j} S_{k,j,t}}{\sum_k P^I_{k,j,t-1} S_{k,j,t}}.$$

The reason why (30) is not correct is that in the current method as applied at Statistics Norway, the real rate of return is defined as:

$$(32) \qquad r_{j,t} = i_{j,t} - \left(\frac{P^I_{j,t}}{P^I_{j,t-1}} - 1 \right),$$

which is independent of asset type. This is equivalently to say that there is only one real rate of return for each industry.

However, the correct real rate of return ought to be defined as:

$$(33) \qquad \widehat{r_{k,J,t}} = \widehat{\iota_{J,t}} - \left(\frac{P^I_{k,j,t}}{P^I_{k,j,t-1}} - 1 \right),$$

which is asset-specific, and the correct real rate of return is interpreted as the nominal rate of return adjusted for asset-specific capital gains.

By comparing (30) with (31), it can be shown that if and only if the following identity holds, can it follow that $\widehat{\iota_{J,t}} = i_{j,t}$:

$$(34) \qquad \frac{P^I_{j,t}}{P^I_{j,t-1}} = \sum_k \left(\frac{P^I_{k,j,t}}{P^I_{k,j,t-1}} \right) v^I_{k,j,t-1},$$

where $v^I_{k,j,t-1} = \frac{P^I_{k,j,t-1} S_{k,j,t}}{\sum_k P^I_{k,j,t-1} S_{k,j,t}}$, and is the value share of asset type *k* in total capital stock of industry *j*, measured in investment prices of period *t-1*.

As such, equation (34) implies that if and only if the price index of total capital investment in industry *j* is constructed as exactly defined by (34), can it follow that $\widehat{\iota_{J,t}} = i_{j,t}$. Clearly, this 'sufficient and necessary' condition does not hold in reality, simply because the industry price index as currently applied is a Paasche index.

Under an extremely special circumstance where, for each asset type *k*, the price change of asset *k* in industry *j* ($\frac{P^I_{k,j,t}}{P^I_{k,j,t-1}}$) is the same and equal to the price change of total capital investment in industry *j* ($\frac{P^I_{j,t}}{P^I_{j,t-1}}$), then it follows that $\widehat{\iota_{J,t}} = i_{j,t}$. However, there is also no guarantee for this 'if' to hold necessarily.

To sum up, following the ex-post approach, the nominal rate of return ($\widehat{\iota_{J,t}}$) is assumed to be the same for all asset types in one industry, but there exists no one and the same real rate of return for industry *j*, simply because the correct real rate of return ($\widehat{r_{k,J,t}}$) is asset-dependent. This is in contrast with the interpretation that has been made in e.g. internal annual reports, such as *Øknomisk Utsyn* (see e.g. Statistics Norway, 2019).

One of the consequences by applying the current method is that the capital services both for each asset and for the total in an industry are wrongly calculated, leading in the end to the biased estimate of the total factor productivity growth at the industry level. Furthermore, the biasedness at the industry level will carry over through aggregation to the corresponding estimates at aggregate level.

In Appendix A, an updated estimation program for calculating capital services is attached where we have made necessary corrections for calculating the capital services by following the ex-post approach (i.e. by means of (31) and (33) instead of (30) and (32)). The results from the updated method are compared with those from the current (and not correct) method in Appendix B, based on the simple criterion that the sum of labor compensation and estimated capital services should be equal to the value added in an industry.

As shown in Table B1 in Appendix B, the difference between value added and the sum of estimated labor and capital services for some selected industries are almost zero by means of the updated program (see Appendix A). On the contrary, the difference by means of the current method differs significantly from zero.

Based on preliminary investigation, it has also been found that the differences of the results between the current estimation method and the updated method seem to be small at the aggregate level, they are, however, significantly large at the industry level.

The updated method as shown in Appendix A has become a common practice adopted by other statistical agencies including Statistics Canada (Gu, 2012) and the Australia Bureau of Statistics (Voskoboynikov *et al*. 2020). The practice has also been adopted by the EU-KLEMS project (Timmer *et al*., 2007, 2010), following the pioneering work of Jorgenson and his co-authors in developing growth accounts for the United States (Jorgensen *et al*., 1987, 2005).

Despite the wide application of the ex-post approach, there are also reasons to opt for the ex-ante measure. For instance, the rate of return by the ex-ante approach is much less volatile and does not need strong assumptions. On the other hand, the main problem with the ex-ante approach is what to be chosen as the exogenous rate of return. Moreover, resorting to information outside the national accounts system is usually needed. Nonetheless, the ex-ante approach should also be tested in the future research.

5. Aggregation

5.1. Current method

Labor productivity growth as published in Table 09174

As mentioned in Section 2, the estimates of labor productivity growth, both gross output-based and value added-based, are published in Table 09174 in the online databank, where a simple aggregation method is taken for aggregating estimates at the annual-national-accounts industry level up to the A38, A64 and A88 industry levels. The method for calculating labor productivity of an aggregate industry/sector is to use the summed gross output or value added in constant prices across the annual-national-accounts industries to be divided by the summed hours worked across the same industries.

Let *S* stand for either an aggregate industry or sector which is an aggregate of lower-level industries, then *S* could refer to any of the aggregate industry within A38, or A64, or A88, and it could also refer to any sector as listed in Table 1.

Using value added-based labor productivity as an example, the volume of value added in an aggregate industry/sector *S*, Z_S, is defined as:

(35) $$Z_S = \sum_{j\in S} Z_j,$$

Then the corresponding labor productivity in *S*, LP_S^Z, is defined as:

(36) $$LP_S^Z = \frac{Z_S}{H_S} = \frac{\sum_{j\in S} Z_j}{H_S},$$

where $H_S = \sum_{j\in S} H_j$, the sum of hours worked across industries in the aggregate industry/sector *S*.

Labor and total factor productivity growth for various sectors

As for the elsewhere-published growth of labor and total factor productivity for those sectors that are aggregate of various industries dealing with market activities, aggregation is based on valued added concept, and volume indices of value added are aggregated by forming weighted averages, with weights adding to unity. Moreover, value added-based productivity measures of aggregates are also weighted averages of their components and can therefore be compared across levels of aggregation.

Different from (35), the volume of value added in sector *S* is now defined as:

(37) $$\Delta \ln Z_S = \sum_{j\in S} \bar{v}_{Z,j}^S \Delta \ln Z_j,$$

where $\bar{v}_{Z,j}^S$ is period average share of value added of industry *j* in the nominal valued added in sector S, and

(38) $$v_{Z,j}^S = \frac{P_j^Z Z_j}{\sum_{j\in S} P_j^Z Z_j}.$$

The volume of capital services in sector *S* is defined as:

(39) $$\Delta \ln K_S = \sum_{j\in S} \bar{v}_{K,j}^S \Delta \ln K_j,$$

where $\bar{v}_{K,j}^S$ is period average share of capital compensation of industry *j* in the total capital compensation in sector *S*, and

(40) $$v_{K,j}^S = \frac{P_j^K K_j}{\sum_{j\in S} P_j^K K_j}.$$

Similarly, the volume of labor services in sector *S* is defined as:

(41) $$\Delta \ln L_S = \sum_{j\in S} \bar{v}_{L,j}^S \Delta \ln H_j,$$

where $\bar{v}_{L,j}^S$ is period average share of labor compensation of industry *j* in the total labor compensation in sector *S*, and

(42) $$v_{L,j}^S = \frac{P_j^L H_j}{\sum_{j \in S} P_j^L H_j}.$$

Finally, the total factor productivity growth in sector *S* is constructed as:

(43) $$\Delta \ln TFP_S^Z = \Delta \ln Z_S - \bar{v}_K^S \Delta \ln K_S - \bar{v}_L^S \Delta \ln L_S$$

where $\bar{v}_K^S$ is period average share of total capital compensation of sector *S* in the nominal value added in sector *S*, and similarly, $\bar{v}_L^S$ is period average share of total labor compensation of sector *S* in the nominal value added in sector *S*. In addition, we have:

(44) $$v_K^S = \frac{\sum_{j \in S} P_j^K K_j}{\sum_{j \in S} P_j^K K_j + \sum_{j \in S} P_j^L H_j} = \frac{\sum_{j \in S} P_j^K K_j}{\sum_{j \in S} P_j^Z Z_j},$$

$$v_L^S = \frac{\sum_{j \in S} P_j^L H_j}{\sum_{j \in S} P_j^K K_j + \sum_{j \in S} P_j^L H_j} = \frac{\sum_{j \in S} P_j^L H_j}{\sum_{j \in S} P_j^Z Z_j}.$$

Note that in (44) the following identity has been used:

(45) $$\sum_{j \in S} P_j^Z Z_j = \sum_{j \in S} P_j^K K_j + \sum_{j \in S} P_j^L H_j,$$

which indicates that the total nominal value added in sector *S* is not only the sum of nominal value added, but also the sum of total labor and capital compensations across all industries within the sector *S*.

According to the current method, the sector labor productivity (LP_S^Z) is defined as:

(46) $$LP_S^Z = \frac{Z_S}{L_S},$$

which is (weighted) value added divided by weighted hours worked. Note that L_S is an index of labor input, rather than hours worked, cf. (41). As the index is weighted by labor compensation, hours worked by workers with high wages count more the hours worked by workers with low wages. The weighting scheme aims to capture labor heterogeneity via the wage differentials, in order to compensate for lack of data on employment by skill level, experience, etc, cf. the discussion of QALI above.

Then (43) can be rewritten as:

(47) $$\Delta \ln TFP_S^Z = \Delta \ln LP_S^Z - \bar{v}_K^S \Delta \ln KI_S,$$

where $KI_S = \frac{K_S}{L_S}$ can be defined as capital intensity in the sector *S*.

5.2. Comments

Labor productivity growth as published in Table 09174
Note that (36) can be further written as:

(48) $$LP_S^Z = \frac{\sum_{j \in S} Z_j}{H_S} = \frac{\sum_{j \in S}\left(H_j LP_j^Z\right)}{H_S} = \sum_{j \in S}\left(\frac{H_j}{H_S}\right) LP_j^Z,$$

where the use is made of the definition of industry labor productivity as defined in (8).

Equation (48) indicates that the aggregate labor productivity is a weighted sum of individual industry labor productivity, where the weight is the industry labor input share. As a traditional decomposition formula, equation (48) is widely used as a departure to measure the contribution of different industry to aggregate productivity growth (see e.g. Denison, 1962; Dekle and Vandenbroucke, 2006; IMF, 2006; and Usui, 2011).

However, the underlying assumption of this formula is that output (here, value added) in constant prices is calculated using fixed-base Laspeyres volume and Paasche price indexes at both the aggregate and industry levels, which guarantees that the aggregate output (value added) is equal to the sum of output (valued added) of constituent industries.

Since annually chained Laspeyres volume and Paasche price indexes are used at Statistics Norway, equation (35) and accordingly (48) do not hold anymore. Because when chained index is applied, the aggregate output (value added) is equal to the sum of its individual components only for the chosen reference year (when output (value added) in constant prices is also equal to its nominal value), and the difference between the two increases as one moves away from the reference year.

Recognizing this unfortunately notorious non-additivity characteristic related to the use of chained index, other (than equation (48)) decomposition methods should be applied. A number of alternative methods have been suggested in the literature (see e.g. Nordhaus, 2002; Reinsdorf, *et al*., 2002; Tang and Wang, 2004; Reinsdorf and Yuskavage, 2010; De Avillez, 2012; Dumagan, 2013). For a rather general treatment on this issue and on the decomposition of aggregate MFP growth as well, please refer to Diewert (2013). Given limited space, we will not dwell on these issues in the following.

Labor and total factor productivity growth for various sectors

By inserting (37), (39) and (41) into (43), we have:

(49)
$$\Delta \ln TFP_S^Z = \sum_{j\in S} \bar{v}_{Z,j}^S \Delta \ln Z_j$$

$$-\bar{v}_K^S\left(\sum_{j\in S} \bar{v}_{K,j}^S \Delta \ln K_j\right) - \bar{v}_L^S\left(\sum_{j\in S} \bar{v}_{L,j}^S \Delta \ln H_j\right)$$

$$= \sum_{j\in S} \bar{v}_{Z,j}^S\left(\Delta \ln Z_j - \bar{v}_{K,j}^Z \Delta \ln K_j - \bar{v}_{L,j}^Z \Delta \ln H_j\right)$$

$$= \sum_{j\in S} \bar{v}_{Z,j}^S \, \Delta \ln TFP_j^Z \ ,$$

where the use is made of (18) and the following identities:[15]

(50)
$$v_K^S * v_{K,j}^S = \frac{\sum_{j\in S} P_j^K K_j}{\sum_{j\in S} P_j^Z Z_j} * \frac{P_j^K K_j}{\sum_{j\in S} P_j^K K_j} = \frac{P_j^Z Z_j}{\sum_{j\in S} P_j^Z Z_j} * \frac{P_j^K K_j}{P_j^Z Z_j}$$

$$= v_{Z,j}^S * v_{K,j}^Z,$$

$$v_L^S * v_{L,j}^S = \frac{\sum_{j\in S} P_j^L H_j}{\sum_{j\in S} P_j^Z Z_j} * \frac{P_j^L H_j}{\sum_{j\in S} P_j^L H_j} = \frac{P_j^Z Z_j}{\sum_{j\in S} P_j^Z Z_j} * \frac{P_j^L H_j}{P_j^Z Z_j}$$

[15] An assumption is made here that the identities hold also for period average shares.

$$= v_{Z,j}^{S} * v_{L,j}^{Z} \,.$$

Equation (49) indicates that similar with that the growth of sector value added volume is weighted (by value added share of industry in sector) growth of industry value added volume, the growth of sector total factor productivity is weighted (by value added share of industry in sector) growth of industry total factor productivity. This is a nice construction.

However, the sector labor productivity as currently defined in (46) is not consistent across levels of aggregation, because the conventional way, also applied at the lower-industry level (see (8)), is to define labor productivity in an economic production unit as the volume of output divided by the corresponding *hours worked* in the unit, regardless of the quality differences from different types of labor.

Following the convention and most important, to be consistent across levels of aggregation, the sector labor productivity ought to be defined as:

(51) $$\widehat{LP}_S^Z = \frac{Z_S}{H_S},$$

where $H_S = \sum_{j \in S} H_j$, i.e. the sum of hours worked across industries in sector *S*. Then we have:

(52) $$\Delta \ln \widehat{LP}_S^Z = \Delta \ln Z_S - \Delta \ln H_S$$

$$= \sum_{j \in S} \bar{v}_{Z,j}^{S} \Delta \ln Z_j - \sum_{j \in S} \bar{v}_{Z,j}^{S} \Delta \ln H_j + \sum_{j \in S} \bar{v}_{Z,j}^{S} \Delta \ln H_j - \Delta \ln H_S$$

$$= \sum_{j \in S} \bar{v}_{Z,j}^{S} \Delta \ln LP_j^Z + \left(\sum_{j \in S} \bar{v}_{Z,j}^{S} \Delta \ln H_j - \Delta \ln H_S \right)$$

$$= \sum_{j \in S} \bar{v}_{Z,j}^{S} \Delta \ln LP_j^Z + R \,,$$

The term in brackets in (52) is the reallocation of hours (R) and reflects differences in the share of an industry in aggregate value added and its share in aggregate hours worked. This term will be positive when industries with an above-average labor productivity level show positive employment growth or when industries with below-average labor productivity have declining employment shares.

The decomposition approach taken by (52) follows the suggestion by Stiroh (2002). This approach has been applied by economic analysis based on the EU KLEMS database (see Timmer *et al*., 2010), and also used by those based on the Norwegian KLEMS database (see Liu, 2019, 2020).

By inserting (21) into (52), various contributions to the growth of sector labor productivity can be defined:

(53) $$\Delta \ln \widehat{LP}_S^Z = \sum_{j \in S} \bar{v}_{Z,j}^{S} \left(\bar{v}_{K,j}^{Z} \Delta \ln KI_j + \Delta \ln TFP_j^Z \right) + R.$$

In this way, the contribution of various inputs and total factor productivity growth from each industry *j* to aggregate sector labor productivity growth can be calculated.

We define the contribution of capital intensity in industry *j* to aggregate sector labor productivity growth as:

(54) $$CTLP_{K,j} = \bar{v}_{Z,j}^{S} * \left(\bar{v}_{K,j}^{Z} \Delta \ln KI_j \right) = \bar{v}_{K,j}^{S,Z} \Delta \ln KI_j,$$

which is the growth of capital services per hour worked in industry *j* weighted by the average share of capital compensation in industry *j* in aggregate sector nominal value added ($\bar{v}_{K,j}^{S,Z}$). The weight itself is the product of the average share of industry *j* in aggregate sector value added ($\bar{v}_{Z,j}^{S}$) and the average share of capital compensation in industry *j*'s value added ($\bar{v}_{K,j}^{Z}$).

In addition, the contribution to aggregate sector labor productivity growth from industry *j*'s total factor productivity growth is defined as:

(55) $$CTLP_{TFP,j} = \bar{v}_{Z,j}^{S} \Delta \ln TFP_j^Z,$$

which is the growth of total factor productivity in industry *j* weighted by the average share of industry *j* in aggregate sector value added.

Since the contribution of total factor productivity to labor productivity growth is one for one point, then we can define the growth of sector total factor productivity as the sum of this contribution as shown in (55) across all industries in the sector:

(56) $$\Delta \ln \widehat{TFP_S^Z} = \sum_{j \in S} \bar{v}_{Z,j}^{S} \Delta \ln TFP_j^Z = \Delta \ln TFP_S^Z,$$

which indicates that the sector total factor productivity growth as defined in the updated method ($\Delta \ln \widehat{TFP_S^Z}$) is the same as that based on the current method as shown by (49) ($\Delta \ln TFP_S^Z$).

Compared to the current method as applied at Statistics Norway, the updated method ensures that not only labor productivity growth but also capital services intensity are defined and estimated consistently across the level of aggregation.

At higher aggregation levels, the differences between the two methods are large. Appendix C shows the differences between estimated key growth rates based on the current method as applied at Statistics Norway and the updated method (as suggested in this paper). Figure C1 refers to the market economy of mainland Norway, and Figure C2 to the Norwegian industries. As shown, except for value added for which the difference of estimated growth rates is zero between the two methods, the differences for all other key indicators are significant. For example, the growth rate of hours worked in the mainland market economy is revised upward by 0.1 – 0.4 percentage points in most years in the period 1972 – 2018.

6. Average over time

6.1. Current method

Arithmetic average is used for calculating the average growth rate, g_t, over time in the current method at Statistics Norway.

Suppose that $\Delta \ln q$ indicates the logarithmic growth rate of variable *q* (such as labor or total factor productivity) between *t-1* and *t*, then one has:

(57) $$g_t = \ln q_t - \ln q_{t-1},\ t = 1, 2, \ldots, T.$$

The arithmetic average growth rate over the period of *t = 1, 2, …, T* is:

(58) $$\overline{g_t^A} = \frac{\sum_{t=1}^{T} g_t}{T}.$$

6.2. Comments

Following the definition, (58) can be rewritten as:

$$(59) \qquad \overline{g_t^A} = \frac{\sum_{t=1}^{T} g_t}{T} = \frac{\ln q_1 - \ln q_0 + \ln q_2 - \ln q_1 + \cdots + \ln q_T - \ln q_{T-1}}{T} = \frac{\ln q_T - \ln q_0}{T},$$

which indicates that the arithmetic average growth rate over the period can also be calculated by using the values only at the beginning and the end time point.

In our opinion, a conceptually more correct average over time is the geometric average as:

$$(60) \qquad \overline{g_t^G} = \left(\prod_{t=1}^{T}(1 + g_t)\right)^{\frac{1}{T}} - 1$$

$$= \big((1 + \ln q_1 - \ln q_0)(1 + \ln q_2 - \ln q_1) \ldots$$

$$\ldots (1 + \ln q_T - \ln q_{T-1})\big)^{1/T} - 1 \ .$$

However, under some circumstances, the growth rate for some time could be such that $(1 + g_t) < 0$, leading in worst situations to that the formula (60) breaks down. On the contrary, the simple arithmetic average as shown in (58) does not have such weakness.

Based on the consideration, a suggestion is made that both the arithmetic average as shown in (58) and the geometric average as shown in (60) be calculated at the same time for further comparison.

7. Conclusions

This document reviews the methodologies currently applied for compiling various measures of productivity growth at Statistics Norway. Based on detailed data, productivity growth at annual-national-accounts industry level (with around 150 industries) is estimated first and then aggregated up to higher industry or sector level for publishing at Statistics Norway.

Generally speaking, the applied methodologies are sound. However, different measures of labor and total factor productivities are produced is a way that is not always internally consistent. Moreover, some of the methods applied should be updated, to be more in line with the modern framework of growth accounting.

Under the current method, the concept of sectoral output/input which excludes intra-industry deliveries has not been adopted for measuring gross output for an industry. Although the bias introduce by this omission may be small, further research is needed for making better estimates of productivity based on gross output measures.

Annually chained Laspeyres volume and Paasche price indexes are widely used for constructing output (both gross output and value added) and input indexes at industry level at Statistics Norway. Although being in conformity with the current international standards, these indexes are theoretically inferior to other superlative indexes, such as Törnqvist or Fisher indexes. Research results have demonstrated the feasibility of compiling superlative index based on the Norwegian national accounts data (see Liu, 2017).

In an effort to estimate both labor and total factor productivity growth within the same framework, a Törnqvist index was actually applied for compiling volume index for capital services at industry level at Statistics Norway. However, there is an error by following the ex-post approach for calculating the rate of return, leading to the estimated growth of both capita services and total factor productivity biased.

Another potential bias for the estimated total factor productivity growth is due to that the current labor input is not quality-adjusted so that the change of labor composition, one of the increasingly important elements, is missing in the current method, resulting in biased, either upward or downward, estimates of total factor productivity growth at industry level. In respect of this, a newly introduced data source for labor statistics (A-ordning) at Statistics Norway could be of valuable use for constructing quality adjusted labor inputs that are cross-classified, at minimum, by age, gender, educational attainment, and occupation.

Currently, only a simple arithmetic average is applied for calculating the average growth rate of productivity over time. However, a conceptually more correct way is to use geometric average instead. However, since the geometric average has other disadvantages, an alternative way to the current method is to calculate both arithmetic and geometric averages over time for consideration of comparison.

Following the bottom-up approach, measures of productivity growth at industry level should be aggregated up to higher industry, sector or even the economy-wide level in a consistent way. Unfortunately, there exists inconsistencies in the current aggregation process. For instance, labor productivity is not consistently defined both in different published tables at the online StatBank and across the aggregation levels. For instance, the current method as applied at Statistics Norway for calculating the labor productivity at higher level (aggregated groups of KNR industries, sectors) is to use output (e.g. value added) divided by an index of labor, while at the KNR industry level, the labor productivity is calculated as output (e.g. value added) divided by hours worked, which is not consistent with each other.

Based on the review results, we conclude that to improve productivity and growth accounts at Statistics Norway, we could:

1. Assess the feasibility of applying the concept of sectoral output to measuring gross output;

2. Choose better index number formula, such as Törnqvist volume index;

3. Correct the calculation error of capital services in the current program;

4. Compile quality adjusted labor inputs that are cross-classified by age, gender, educational attainment, and occupation;

5. Calculate both arithmetic and geometric averages over time for comparison;

6. Remove the inconsistencies as regards labor productivity in different tables and across the level of aggregation;

7. Replace the term of total factor productivity (TFP) by that of multi-factor productivity (MFP).

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Vedlegg A: Updated FAME program for calculating capital services: kap_endo_updated.inp

```
-- Updated by LIU 201908

-- Programmed by STO 8. august 2017
-- Beregner kapitaltjenester til TFP beregninger
-- med endogen kapitalavkastning
-- Arbeidskraftkostnader lages med arb.inp eller arb_utd.inp
-- Kjøres fra FAME med kommandoen: inp kap_endo_updated

block
ignore addition on
ignore function on
over on
glue dot
deci 6
lprefix ";"
column score null
replace decimals ","
width 1000
length full
IMAGE DATE  "; <year>"
replace ND ";ND"
replace NC ";NC"

date 1971 to *

close all

load <channel warning none> "$NASJREGN/tidsserier/meta/lag_formel2" -- to
lister, f.eks. nær*art

open <access over>
"$NASJREGN/rea_hr2014/produktivitet/famedb/kap_endo.db" as kap
open <access read> "$NASJREGN/rea_hr2014/produktivitet/famedb/arb.db" as
arb  -- Arbeidskraftkostnader
open <access read> "$NASJREGN/rea_hr2014/kjed/famedb/knrpub16" as knrpub
open <access read> "$NASJREGN/knr_hr2014/struktur/famedb/kontoplan" as kp


loop for naer in NRLKNRPS -- Alle KNR næringer

try

-- Prisendring på kapital etter næring (vpkapn)

vpkapn&naer = 0.01*YTYPCT(kapn&naer&vr/kapn&naer&vl)

-- Kapitalserier på næring og art

  loop for art in nrlknrjr

 try
```

```
-- Prisendring på kapitalart (NB! IKKE nødvendigvis lik vpkapn)
vpkapn&naer&art = 0.01*YTYPCT(kapn&naer&art&vr/kapn&naer&art&vl)

diff&naer&art&vr = vpkapn&naer&art * kapn&naer&art&vr[t-1]

end try

  end loop  -- Art


-- Summere opp diff etter art (sum over art)

$lag_formel2 {diff}, name(naer), {naer}, nrlknrjr,  {vr}

-- Nominell nettoavkastningsrate etter næring (nar)

nar&naer = (bnpb&naer&vr - arbkost&naer&vr - dep&naer&vr + diff&naer&vr)
/ kapn&naer&vr[t-1] -- Endogen


  loop for art in nrlknrjr

  try

-- Kapitaltjenester (kaptj) i verdi, negative verdier settes lik 0

kaptj&naer&art&vr = if (dep&naer&art&vr + ((nar&naer - vpkapn&naer&art) *
kapn&naer&art&vr[t-1])) le 0 &&
                    then 0 &&
                             else (dep&naer&art&vr + ((nar&naer - vpkapn&naer&art) *
kapn&naer&art&vr[t-1]))

otherwise

  type lasterror

end try

  end loop  -- Art


-- Kapitaltjenester i verdi for næring (sum over art)

$lag_formel2 {kaptj}, name(naer), {naer}, nrlknrjr,  {vr}

-- Sammenveid Tornquist volumvekst for kapitaltjenester


 loop for art in nrlknrjr

  try

    vkaptj&naer&art = LOG(kapn&naer&art&vl / kapn&naer&art&vl[t-1])
    akaptj&naer&art = MAVE(kaptj&naer&art&vr / kaptj&naer&vr ,2)
    dkaptj&naer&art&d = akaptj&naer&art * vkaptj&naer&art
```

```
 otherwise
 type lasterror

   end try

  end loop  -- Art


$lag_formel2 {dkaptj}, name(naer), {naer}, nrlknrjr,  {d}

vkaptj&naer = dkaptj&naer&d

vkapn&naer = LOG(kapn&naer&vl / kapn&naer&vl[t-1])

 otherwise

    type lasterror

   end try

end loop  -- Næring


-- Tabell på KNR næring

date 1972 to *
deci 1

output <acc over>
"$NASJREGN/rea_hr2014/produktivitet/wk24/kap_endo_tabell.txt"

loop for naer in NRLKNRPS

type name(naer)

try

repo 100*vkaptj&naer as "Vekst i kapitaltjenester", &&
     100*nar&naer as "Nominell kapitalavkastning, endogen", &&
     dep&naer&vr as "Kapitalslit løpende pris", &&
     kapn&naer&vr as "Kapitalbeholdning løpende pris", &&
     kaptj&naer&vr as "Kapitaltjenester løpende pris", &&
     100*vpkapn&naer as "Prisendring på kapitalbeholdning", &&
     100*vkapn&naer as "Vekst i kapitalbeholdning"

otherwise

 type  lasterror

end try

end loop

output terminal

end block
```

Vedlegg B: Comparison between results from current and updated methods as regards the calculation of capital services

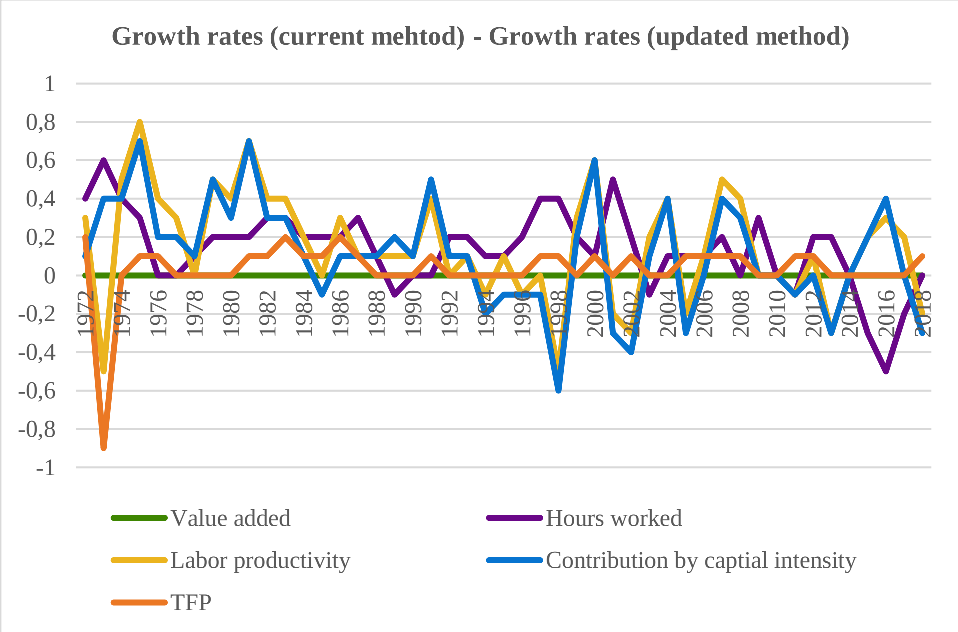
Table B1. Differences between value added and the sum of estimated labor and capital services in selected industries (NOK Millions, current prices)

	Based on current method					Based on updated method				
Year	**2323**	**2335**	**2344**	**2361**	**2377**	**2323**	**2335**	**2344**	**2361**	**2377**
1972	-0.4	-0.5	3	0.3	0.2	0	0	0	0	0
1973	-1.9	-4.4	8.9	4.4	0.1	0	0	0	0	0
1974	-1.1	4.3	31.2	1.3	2.4	0	0	0	0	0
1975	0.5	0	10.1	0.4	0.1	0	0	0	0	0
1976	-0.2	7.6	1.2	-1.4	-0.4	0	0	0	0	0
1977	0.6	0.3	22.7	2.5	1.9	0	0	0	0	0
1978	-0.4	2.9	-46.9	-2.5	-1.4	0	0	0	0	0
1979	0	8.9	19.8	2.8	3.3	0	0	0	0	0
1980	-0.4	-2	-36.2	-0.8	-1.6	0	0	0	0	0
1981	1.1	3	11.4	1	1.1	0	0	-0.1	0	0
1982	0.1	13.5	5.4	0.6	2.5	0	0	0	0	0
1983	1.1	-7.6	11	-0.6	0.3	0	0	0	0	0
1984	-1.2	1.3	24.6	-1.3	-2.8	-1	0	0	0	0
1985	0.3	6	-3.7	-1.4	-3.2	0	0	0	0	0
1986	0.4	5.7	-11.3	-0.8	-15.2	0	0	0	0	0
1987	0.3	1.9	64.4	4	-10.7	0	0	0	0	0
1988	-4.1	4.2	-124.2	-4.8	-3.1	0	0	0	0	0
1989	4.3	-5.1	175.8	8.1	10.8	-1	0	0	0	0
1990	3.6	2.7	177	10	13.1	0	0	0	0	0
1991	1.5	-7	137.2	18.6	6.3	0	0	0	0	0
1992	-3.4	-1.3	-117.2	-1.8	-2.6	-1	0	0	0	0
1993	5.8	-2.8	77.4	20.5	0.1	0	0	0	0	0
1994	-4.3	1.8	-60.1	-16.6	-5.5	0	0	0	0	0
1995	-3.5	22.8	-59	-26.6	-0.2	-1	0	0	0	0
1996	-0.8	11.2	-5.8	-23.4	12.8	0	0	0	0	0
1997	1.5	32.3	64.5	25.5	24.9	0	0	0	0	0
1998	-0.1	-0.6	3.9	33.6	-75.7	-1	0	0	0	-3
1999	11.4	27.7	106.5	-23.7	-1	0	0	0	0	-4
2000	9.7	24	37.3	5.6	-4.9	0	0	-0.1	0	-5
2001	-3.9	14.8	58.6	-31.7	-8.3	0	0	0	0	-19
2002	-2	51.1	41.2	15.6	0.8	0	0	0	0	-14
2003	-6.9	-12.5	126.7	-60.4	14.2	0	0	0	0	0
2004	7.4	1.8	70.7	-26.4	60.3	0	0	0	0	0
2005	1.4	15.8	259.2	28.5	28.8	0	0	0	0	0
2006	19.2	8.6	190.1	-145.4	-47.7	0	0	0	0	0
2007	-12	27.4	99.5	-16	225.6	0	0	0	0	0
2008	8.9	58.2	24.8	50.2	19	0	0	0	0	0
2009	0	-0.8	19	-11.7	-4.6	-0.1	0	0	0	0
2010	-15.4	28.9	95.5	16.5	40.8	0	0	0	0	0
2011	-6.6	10.9	65.1	0.1	48.5	0	0	0	0	0
2012	-2.3	6	26.3	16.3	2.1	0	0	0	0	0
2013	-1.1	63.5	64.2	29.9	66.9	0	0	0	0	0
2014	3	-5.9	10.6	-35.6	14.8	0	0	0	0	0
2015	4.2	-6.7	24.7	30	58.4	0	0	0	0	0
2016	1.2	-8	90.6	15.1	-175.4	0	0	0	0	0
2017	-6.5	-7.6	66.3	3.2	56.9	0	0	0	0	0.1
2018	-1.2	-0.2	0.8	2.1	37.9	0	0	0	0	-0.1

Note: Negative values in in some industries over some years as shown in Panel 'Based on updated method' are due to that negative capital services are forced to be zero.
Source: Author's calculation based on data in March 2019.

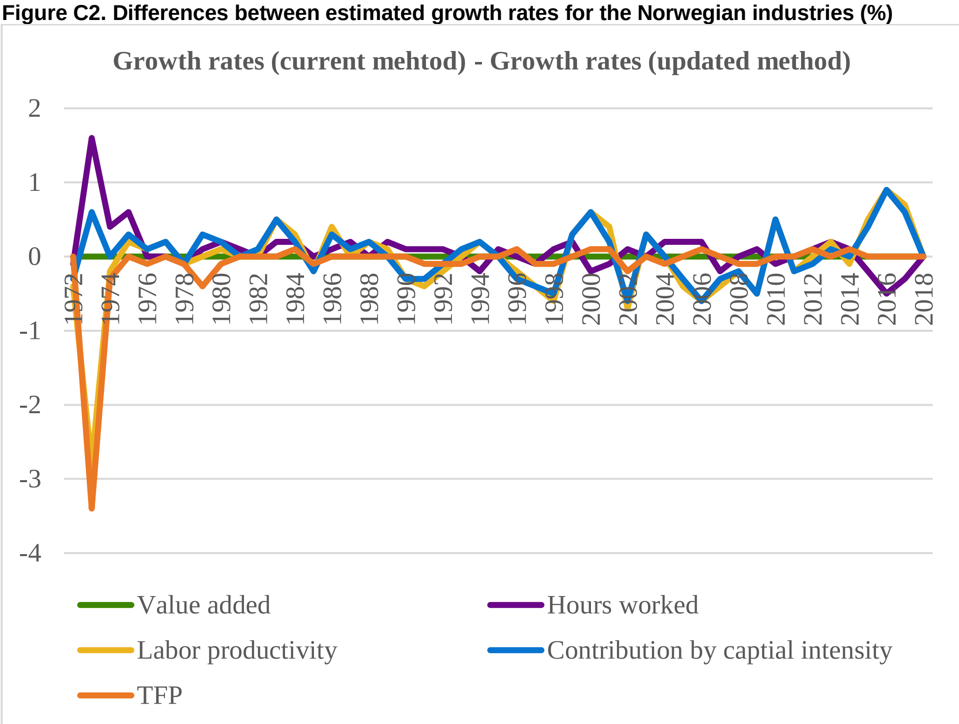
Vedlegg C: Comparison between results from current and updated methods as regards the aggregation method

Figure C1. Differences between estimated growth rates for the market economy of mainland Norway (%)



Source: Author's calculation based on data in March 2019.

Figure C2. Differences between estimated growth rates for the Norwegian industries (%)



Source: Author's calculation based on data in March 2019.