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A Gas Cartel in the Global Market? Hype or Reality

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ABSTRACT

In late 2018, Qatar -as a key player in the global gas market-announced that it has decided to quit the Organisation of the Petroleum Exporting Countries (OPEC) and rather will be focussing on its gas industry. Such a decision, drew the attentions once again and after a decade to the issue of gas cartel formation. The basic question of this paper is under which conditions gas cartel formation is feasible in presence of an oil cartel. Using a game theory framework the paper models the interaction of countries that export both gas and oil and are member of OPEC with those that export only gas or oil. Results indicate that gas cartel formation is feasible but under very vulnerable conditions. It is also shown that active members of OPEC who are also key players in the gas market prefer to form the cartel, however, it may not be the case for countries with unbalanced portfolio of oil and gas export. The impact of gas cartel formation on the price and supply of both oil and gas is also discussed.

Keywords: Gas Market, Gas Cartel, Bargaining, Game Theory JEL Classifications: C78, D43, L13, Q41

1. INTRODUCTION

The long-lasting experience of collective actions by producers in oil market are administered through the Organisation of the Petroleum Exporting Countries (OPEC), formed in 1960. The stated mission of the organisation is to "coordinate and unify the petroleum policies of its member countries and ensure the stabilization of oil markets, in order to secure an efficient, economic and regular supply of petroleum to consumers, a steady income to producers, and a fair return on capital for those investing in the petroleum industry¹." The actions taken by OPEC in different periods of time can be classified differently, however, all of them more or less, actively affect the market price, consumer behaviour, and investment level in the industry². Perhaps the relative success of this organization in respecting the interests of members led to

2 Huppmann and Holz (2012) and Al-Qahtani, Balistreri & Dahl (2008) provide a comprehensive review on different research that model the OPEC behaviour. a kind of tendency for gas exporting countries to shape the same institution.

Considering important difference between physical characteristics of oil and gas in addition to two-sided mutual adaptation among oil and gas market, the similar performance of gas organization should be examined. Moreover, different types of issues seem to be considered from cartel formation point of view: feasibility of having a unique cartel in a geographically segmented market with long term contracts (Gabriel et al., 2012), the structure of the potential cartel and its market effect. In addition, a substantial question is to what extent the gas exporting countries are interested in being involved in the formation of a cartel.

In 2008, a group of countries³ signed the charter of Gas Exporting Countries Forum (GECF). Since then, the member states

3 Algeria, Bolivia, Egypt, Equatorial Guinea, Iran, Libya, Nigeria, Qatar, Russia, Trinidad and Tobago, United Arab Emirates, and Venezuela.

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¹ https://www.opec.org/opec_web/en/about_us/23.htm.

always denied the agenda of price setting, which is supported by some research works such as Hallouche (2006). Looking at technicalities, it is also possible to argue that the formation of a cartel in the gas market has some barriers compared to the one for oil market. Due to the relatively low cost of oil transportation, a cartel in oil market can simply affect the price by setting quota. But gas market is not as global as the oil market, which means gas should be consumed regionally (Gabriel et al., 2012). For instance, US cannot export gas to Europe unless the destination has already built a LNG terminal. Moreover, alike OPEC, the free riding is another barrier to overcome (Goldthau, 2016).

Notwithstanding this barrier, GECF expresses its function as a forum to collect energy data of member state and to produce analytical reports and promote the gas usage⁴. More importantly, energy ministers of members' states are supposed to attend periodic meeting to discuss market structure and to promote joint investment for the sake of deepening the gas market and adding more value to the gas (Ratner, 2018).

From 2008 till 2018, this forum had very smooth advancement in terms of playing a role in the gas market. However, when Qatar announced in December 2018 that it wants to quit the OPEC organizations, some analysts interpret it as the new decision to focus on collective action in gas market. By this decision, Qatar, with about 10% share of global gas export, will not be affected by commitments to OPEC resolution anymore. This withdrawal beside the declining cost of gas transportation technologies implies anticipated growing importance of GECF for stability of gas market. Considering evidences like decision to increase gas supply capacity after 12 years and new large-scale deal with China and Britain, the argument of "leaving OPEC to focus on GCEF" makes more sense for Qatar announcement. Last but not least, the rationale behind the announcement has been declared by the newly appointed minister of state for energy affairs and president and CEO of Qatar Petroleum, in which he states that Qatar's exit from OPEC "is not political, it was purely a business decision for Qatar's future strategy towards the energy sector⁵."

It should be noted that by using the term of "cartel" in this paper, we merely refer to a group of apparently independent producers who make decisions collectively. We don't take any position for analysing the logic behind their action whether it is market stabilization or profit maximization. Hence, as a disclaimer, the finding cannot be used in reading the intention of cartel members. This view is also supported by research conducted previously: Huppmann and Holz (2012) argues that OPEC ability to exert market power has been decreased and Böckem (2004) provides an assessment from oil market and shows that it is best described by a price-leader model.

To investigate different performance of GECF (as a gas organization) from OPEC (as an oil organization) and also to translate the recent behaviour of Qatar, this article is to model the interest of countries with gas export, oil export, or both. After a brief review of literature in section 2, the model will be articulated in section 3. Model is being solved and discussed in section 4. The last section concludes and provides some policy analysis.

2. BACKGROUND

Collusion in an oligopolistic market could be explicit or tacit, either price-based or quantity-based or both, one shot or repeated, single market or multi, local or global, in demand-related markets or in a single market. A cartel is the example for an explicit and repeated collusion which is formed by association of firms that set the price or hold back the output. Energy markets, especially natural gas export markets are a suitable case for studying collusive behavior due to the limited number of producers (more market concentration), relatively high demand elasticity and an impending cartel under process of forming by the GECF⁶.

On the one hand, the natural gas demand is rising (Egging et al., 2009) and (GECF, 2017) and global economy is becoming profoundly dependent on natural gas (Ehrman, 2006) and (IEA, 2017). Moreover, natural gas markets currently are evolving dramatically. Increasing shares of LNG⁷ in global natural gas trade, integration in the regional markets (Aune et al., 2009), and, last but not least a new potential gas cartel is among the most critical ones. The impending cartel of natural gas, which is referred to GECF is a crucial issue in global energy markets.

A wide range of studies dealt with cartel behavior in general or in specific goods and services. Harrington (2006) distils and organizes facts about 20 cartels in Europe including the properties of collusive outcome and the organizational structure of the cartels based on the European Commission decisions during the 1999-2004 period. Feuerstein (2005) investigates the implicit collusion and the factors which impact on collusion sustainability, both positively and negatively. Iwanari et al. (2007) focus on international cartels and antitrust enforcement implication. In their model, two different cartels are active in two different countries while the markets are identical. Escrihuela-Villar, (2004) demonstrates the relationship between the size of firms within a cartel and the sustainability of the cartel. Choi and Gerlach (2013) analyze dynamic cartel formation when firms operate in demand-related markets. In their paper two firms produce two demand-related products which are either substitute or complement.

As can be expected, given the importance of energy security, numerous studies have been conducted thus far to analyze the natural gas market trends and evolution. In particular there is a small but growing literature on natural gas cartelization and the effect

See GECF (2017) in which 5 objectives of the forum are discussed; including: sustainable maximization of the added value of gas for member states, Developing the GECF View on gas market developments, Cooperation, namely to develop effective ways and means for cooperation amongst GECF Member Countries in various areas of common interests, Promotion of natural gas and international positioning of the GECF as a globally recognized intergovernmental organization.

https://www.aljazeera.com/indepth/opinion/qatar-leftopec-181206102112634.html

⁶ The Gas Exporting Countries Forum includes alphabetically: Algeria, Bolivia, Egypt, Equatorial Guinea, Iran, Kazakhstan (observer), Libya, Netherland (observer), Nigeria, Norway (observer), Qatar, Russian Federation, Trinidad & Tobago and Venezuela. 7

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of the emerging cartel on the energy market⁸. So far, the analysis of natural gas cartelization has been mainly qualitative such as Ehrman (2006). She describes the GPEC formation, comparing to OPEC⁹ and its effect on the world gas market from an economic and political point of view. Jaffe and Soligo (2006) also investigate the feasibility of gas cartel formation and the incentives of the player in the market in joining the cartel applying a descriptive framework rather than a mathematical model. There have been a few attempts at forecasting the effects of a cartel behavior in the world gas market. For example, Egging et al. (2009) combine optimization methods and game theory analysis to simulate the effect of GPEC formation on the trade, consumption and production in the global gas market over the next 20 years. Gabriel et al. (2012) also goes further and analyzes the effect of a future potential gas cartel in different participant scenarios using the same method.

These research works focus on the post-cartel formation and skip the pre-cartel formation period in which a natural gas exporting country decides about joining the cartel or not.

As discussed earlier, there is a strong view which argues that GECF cannot be considered as a cartel (see [Hallouche, 2006]), however to investigate the potential for shaping a gas cartel, we need to situate it in the literature of multi-market colluders (Marx et al., 2015; Zhou, 2016) with the assumption of dynamism between the oil and gas market. A quite similar study has been conducted for environmental agreements by Chou and Sylla (2008), which describes the dynamism of direct transfer (money or taxation) or capacity transfer (cap and trade). Taking the concepts of multi-market colluders and distinct types of transfer, we consider the oil market as a substitute market for natural gas and analyze the interaction between three beneficial groups of countries. In the model we discuss in the next section, there are three agents, two substitute goods, one active and one potential cartel and finally two different scenarios.

3. THE MODEL

In this section, players, scenarios, functional forms and the game structure of the model are discussed.

3.1. Players

For modelling purposes three groups of countries are identified. Group A composed of countries which export only oil and are identified as a member of OPEC. Group B includes countries which export both oil and natural gas and are identified as a member of OPEC and finally group C pertains to countries which export only natural gas¹⁰.

3.2. Scenarios

Two different scenarios are analyzed. In the first, there is only an oil cartel active in the market, while in the second the new gas cartel is assumed to be formed and both B and C have to make decisions on forming the cartel or not. Accordingly, in scenario one there is only one cartel in the market which is an oil cartel, (current situation) while in scenario two there are two cartels both in the oil and natural gas markets, (potential condition). For simplicity we assume one country per group.

3.3. Profit Maximization Problem

To determine the profit maximization problem, we define the demand and cost functions as follows:

3.3.1. Demand function

We assume a linear demand function for both oil and gas market:

(1)
$$P_i(Q_i, Q_i) = \alpha_i - \beta_i Q_i - \theta_i Q_i$$

For $i, j \in \{O, G\}$ and $i \neq j$

- P_i : Price of product *i*
- Q_i : Quantity of product *j*
- α_i : Maximum price of product *i* (intercept)
- β_i : Effect of the quantity of product *i* (direct effect)
- θ_i : Effect of the quantity of product *j* (indirect effect)

Also, we assume that the direct effects always dominate the indirect effect, i.e. $\beta_i, \beta_j \ge \theta_i, \theta_j \ge 0$. It is notable that this condition is sufficient to guarantee positive equilibrium prices and quantities and to exclude any corner solutions¹¹.

3.3.2. Cost Function

We assume that the marginal cost of production is constant and the same for all producers of a given good. We use C_o and C_g to denote the marginal cost of oil and gas production respectively. This assumption is due to simplification and is not a technical one.

3.3.3. Cartel Profit Function

Using the definition of a cartel, the profits of a cartel is the summation of each members' profit function. Thus, if the cartel has n members:

$$\pi_{Cartel} = \pi_1 + \pi_2 + \ldots + \pi_n$$

If the cartel controls the production of good *i* one must maximize the above function subject to Q_i .

In this paper in both scenarios we have an oil cartel whose profit function is a combination of the profit functions of A and B and analogously in the second scenario the profit function of the gas cartel is a combination of the profit functions of B and C.

Let $\pi_i = Q_i (P_i (Q_i, Q_j) C_i)$ for $i \in [O, G]$ and $i \neq j$ represents the combined profits of the cartel of good *i* on the sales of its products.

⁸ It is notable that a relatively significant body of literature was developed aftermath of GECF formation in the years of 2008-2012. However, tracking the literature reveals that there have been few attention to this topic over the past recent years. Authors expect to see more academic and policy research sine Qatar exit from OPEC and its announcement.

⁹ Organization of Oil Exporting Countries includes alphabetically: Algeria, Angola, Ecuador, Iran, Iraq, Kuwait, Libya, Nigeria, Qatar, Saudi Arabia, Nigeria, United Arab Emirate and Venezuela.

¹⁰ For simplicity, countries which export only oil and are not identified as a member of OPEC and countries which export both oil and natural gas and are not identified as a member of OPEC are not included in the modelling.

¹¹ We also need that α_i be greater than the marginal cost of production.

We suppose that a cartel maximizes the sum of profits of its members. Since country B will consider the cross-effects between the two markets, we assume that this sum includes the profits of country B in the substitute market.

Members of cartel battle for power within the cartel. In our model, λ_A represents the power of country A in the oil cartel, where country A receives $\lambda_A \pi_0$, while country B receives $(1 - \lambda_A)\pi_0$. It is assumed that λ_A is determined by the political and economic power of A in the oil cartel. If the gas cartel forms, λ_C represents the power of the country C within the cartel. It receives $\lambda_A \pi_G$ while country B receives $(1 - \lambda_C)\pi_G$.

One may ask about the rationale of considering B's production of gas in the oil cartel maximization problem as well as B's production of oil in the gas cartel maximization problem. The underlying reason is the important link between the two markets. As mentioned before, θ_i shows the cross-effect between the oil and gas markets. Any decision making of oil cartel members affect the price of gas through θ_G . Likewise, the gas cartel board negotiation would affect the oil market evolution, through θ_O . Since country B is involved in both markets, it will take it into consideration when making decisions.

Regarding this compendium, the scenario's problems are depicted below.

3.4. Scenario One Maximization Problem

As discussed earlier, within the first scenario, there is only one cartel in the market which is OPEC. Again, the player in the market are A (produces only oil and a member of oil cartel), B (produces both gas and oil and a member of oil cartel) and C (produces only gas). Accordingly, we have to solve three different maximization problems. Let $\pi_{G,B}$ and $\pi_{G,C}$ be the profits of countries B and C coming from the gas market and $q_{G,B}$ and $q_{G,B}$ are non-maximized production of gas by B and C respectively, such that $Q_G = q_{G,B} + q_{G,C}$.

Profits of oil cartel:

(2)
$$\pi_{OC}^{1} = \lambda_{A}\pi_{0} + (1 - \lambda_{A})\pi_{0} + \pi_{G,B}$$
$$= [Q_{O}(a_{O} - \beta_{O}Q_{O} - \theta_{O}Q_{G} - C_{O}]$$
$$+ q_{G,B} \Big[a_{G} - \beta_{G} (q_{G,B} + q_{G,c}) - \theta_{G}Q_{O} - C_{G} \Big]$$

Profits of B:

(3)
$$\pi_{\rm B}^{\rm l} = (1 - \lambda_{A})\pi_{0} + \pi_{G,B} = (1 - \lambda_{A})$$
$$\begin{bmatrix} Q_{\rm o} (a_{\rm o} - \beta_{\rm o}Q_{\rm o} - \theta_{\rm o}(q_{\rm G,B} + q_{\rm G,C}) - C_{\rm o} \end{bmatrix} + q_{\rm G,B}$$
$$\begin{bmatrix} a_{\rm G} - \beta_{\rm G} (q_{\rm G,B} + q_{\rm G,C}) - \theta_{\rm G}Q_{\rm o} - C_{\rm G} \end{bmatrix}$$

Profits of C:

(4)
$$\pi_{G,C}^{1} = q_{G,C} \Big[(a_{G} - \beta_{G}(q_{G,B} + q_{G,C}) - \theta_{G}Q_{O} - C_{G}) \Big]$$

Taking derivative subject to $Q_O, q_{G,B}$ and $q_{G,C}$ respectively, results in reaction functions for each cartel/country. The next step is to

solve the system of equations which includes three equations and find the equilibrium prices and profits.

We define $Q_0^1, Q_{G,B}^1, Q_{G,C}^1, P_0^1, P_G^1$ to be the optimal values found after optimization and $Q_G^1 = Q_{G,B}^1 + Q_{G,C}^1$. The corresponding profits are π_i^1 for $i \{A, B, C\}$.

3.5. Scenario Two Maximization Problem

In this scenario, both cartels are active.

Profits of oil cartel:

$$\pi_{\rm OC}^2 = \lambda_A \pi_0 + (1 - \lambda_A) \pi_0 + (1 - \lambda_A) \pi_0 + (1 - \lambda_C) \pi_G$$
$$= \pi_0 + (1 - \lambda_C) \pi_G$$

(5)
$$\begin{bmatrix} Q_o (a_o - \beta_o Q_o - \theta_o Q_G - C_o) \end{bmatrix} + (1 - \lambda_c) [Q_G (a_G - \beta_G Q_G - \theta_G Q_o - C_G)]$$

Profits of gas cartel:

(6)
$$\pi_{GC}^{2} = \lambda_{C}\pi_{G} + (1 - \lambda_{A})\pi_{0} + (1 - \lambda_{C})\pi_{G} = \pi_{G} + (1 - \lambda_{A})\pi_{0} = \left[Q_{G}\left(a_{G} - \beta_{G}Q_{G} - \theta_{G}Q_{O} - C_{G}\right)\right] + (1 - \lambda_{A})\left[Q_{O}\left(a_{O} - \beta_{O}Q_{O} - \theta_{O}Q_{O} - C_{O}\right)\right]$$

Taking derivatives subject to Q_o and Q_G respectively results in reaction function for each cartel. Next step is to solve the system of equations which includes two equations and find the equilibrium prices and profits.

We define Q_0^2 , Q_G^2 , P_0^2 , P_G^2 to be the optimal value found after optimization. The corresponding profits are π_i^2 for $i \in \{A, B, C\}$.

3.6. Stability of the Cartel and Bargaining Factors

Finding a mutually beneficial agreement is akin to finding a core allocation, i.e. an allocation such that no agent or group of agents would prefer to leave the coalition. The notion of the core goes back to Gillies (1959). Since we have two players (B and C) in the problem, the core restrictions are nothing more than those of individual rationality: agents must be better off than if they acted on their own. Then a cartel gets implemented only if its members are better off in it than outside. Then in our model, B prefers the cartel if $\pi_B^1 \le \pi_B^2$ and C prefers cartel if $\pi_C^1 \le \pi_C^2$. We define T as the transfer from B to C with T possibly negative. After transfer, profits are: $\pi_B^{2,T} = \pi_B^2 - T$ and $\pi_C^{2,T} = \pi_C^2 + T$. It is possible to consider three different ways of bargaining between countries B and C, which are as follows:

Definition 1: *Full Agreement*: Countries can negotiate both money transfer from B to C or C to B and the distribution of the power in the cartel board and decision-making process. (T and λ_c are the bargaining factors).

Definition 2: Monetary Agreement: Countries can negotiate only money transfer from B to C or vice-versa. Here, the power of each country is fixed and non-negotiable. (*T* is the only bargaining factor).

Definition 3: Political Agreement: In this case money transfer is either impossible or illegal. Countries can negotiate only the power in the cartel board. In other words cartel decides only for export capacity and there is no money transfer. (λ_c is the only bargaining factor).

Supposing that we can have full agreements as defined above is similar to supposing that the corresponding cooperative game has fully transferable utility among agents. The political agreement is nothing more than a restriction on the transferable utility assumption. It limits the maximum amount of money (utility) that can be transferred between B and C. The monetary agreement does assume transferable utility, but the fact that agents cannot bargain on the control of the production decisions within the cartel modifies the game, as it limits the potential gains of B and C. See Myerson (2013) for a discussion of bargaining problems with and without transferable utility.

4. RESULTS AND DISCUSSION

4.1. Solving the Model

Since the fully asymmetric version of the model proves to be too complicated to efficiently solve analytically, we simplify it by assuming a symmetric demand and cost functions, i.e., $a_0 = a_G = a$, $\beta_0 = \beta_G = \beta$, $\theta_0 = \theta_G = \theta$ and $C_0 = C_G = C$.

4.2. The Impact on Prices and Quantities

We first look at the effect of the gas cartel on prices and quantities. (Equilibrium prices, quantities and profits are presented in the appendix)

Theorem 1: Following the formation of the gas cartel, for any set of parameters:

i) Q_o increases,

ii) Q_{c} decreases,

iii) \mathbf{P}_{G} increases.

Proof:

We have to show $Q_0^2 \ge Q_0^1$. To do this, we demonstrate that the expression $Q_0^2 - Q_0^1 = \frac{(a-C)\beta\theta(2(\beta-\theta)+\theta\lambda_A)\lambda_C}{(2(\beta^2-\theta^2)+\lambda_A\theta^2)(4\beta^2-\theta^2(\lambda_A-2)(\lambda_C-2))}$

is positive. Numerator is positive as we have assumed that $\alpha > C$ and $\beta \ge \theta$. As a matter of denominator, $(2(\beta^2 - \theta^2) + \lambda_A \theta^2)$ is positive as $\alpha > C$. Also $(4\beta^2 - \theta^2 (\lambda_A - 2) (\lambda_C - 2))$ is always positive since at the maximum value of $\theta^2 (\lambda_A - 2) (\lambda_C - 2)$ (which is $4\theta^2$ when λ_A and $\lambda_C = 0$) we have $4\beta^2 > \theta^2 (\lambda_A - 2) (\lambda_C - 2)$. Therefore $Q_o^2 - Q_o^1$ is always positive.

ii) We have to show that $Q_G^2 \leq Q_G^1$. We can show that $\frac{\partial (Q_G^2 - Q_G^1)}{\partial \lambda_C}$ ≤ 0 . Consequently, $Q_G^2 - Q_G^1$ is at its maximum when $\lambda_C = 0$. We can also show that $\frac{\partial (Q_G^2 - Q_G^1)}{\partial \lambda_A}|_{\lambda_{C=0}} \geq 0$ and thus $(Q_G^2 - Q_G^1)|_{\lambda_{C=0}}$ is at

its maximum when $\lambda_A = 1$ Plugging back $\lambda_C = 0$ and $\lambda_A = 1$ in the

initial expression of $Q_G^2 - Q_G^1$ results in $-\frac{(a-C)(2\beta-\theta)}{12\beta^2-\theta^2}$ which is always negative. Since $Q_G^2 - Q_G^1$ at its maximum amount (with respect to λ_A and λ_C) is negative, obviously it will be negative for any other sets of parameters.

iii) Applying the same method, we have to show that $P_G^2 > P_G^1$. We can show that $\frac{\partial (P_G^2 - P_G^1)}{\partial \lambda_C} \le 0$ and $\frac{\partial (P_G^2 - P_G^1)}{\partial \lambda_A} \le 0$. Consequently, $P_G^2 - P_G^1$ is minimum when $\lambda_C = 1$ and $\lambda_A = 1$. Plugging back $\lambda_C = 1$ and $\lambda_A = 1$ in the initial expression of $P_G^2 - P_G^1$ results in $\frac{2(a-C)\beta(\beta^2-\theta^2)}{3(2\beta+\theta)(2\beta^2-\theta^2)}$ which is always positive. Since $P_G^2 - P_G^1$ at its

minimum amount (with respect to λ_A and λ_C) is positive, obviously it will be positive for any other sets of parameters.

Theorem 2: After the cartel formation the changes direction of P_o is ambiguous. Figure 1 shows those sets of parameters in which P_o increases after the cartel formation. Obviously, the empty part of the cube shows those sets of parameters in which P_o decreases after the cartel formation.

The ambiguity mostly comes from the different effects of substitutability between oil and gas as well as power of C and A in the gas or oil market and their impact on the internalizing the markets externalities. As can be driven from the relation (1), demand function of oil, we have:

$$(7) \Delta P_O = -\beta \Delta Q_O \quad \theta \ \Delta Q_G$$

Based on the Theorem 1, $\Delta Q_O \ge 0$ and $\Delta Q_G \le 0$. Also based on the model assumptions β and $\theta \ge 0$. Therefore, the first expression, $-\beta \Delta Q_O$, is always negative, while the second one $-\theta$ is positive. When the second expression dominates the first, it means that the oil price increases and vice-versa.

In general, B aims to internalize the maximum amount of market externalities by playing with oil or gas production where possible.



Figure 1: Values of θ , λ_A and λ_C for which the oil price increases after the formation of the gas cartel with ($\alpha = 1, \beta = 2, C = 0$).

It is worth mentioning that based on the relations (5) and (6)- the oil and gas cartels' profit functions after the gas cartel formation, $-\lambda_A$ affects the gas cartel decision making and consequently ΔQ_G , while λ_C affects the oil cartel decision making and consequently ΔQ_Q .

With a low λ_{A^2} B controls a large part of the oil production. When maximising the gas production, more weight will be given to the cross-effect on the oil market.

To do this, in the gas market it prefers to decrease the gas production to increase the price as much as possible. Then in low values of λ_A , as B can control the rival market (oil market), the decrease would be profound. Then low values of λ_A results in high decrease in Q_G which means that $-\theta \Delta Q_G$ is a big positive number. It will result in positive ΔP_O . While in high values of λ_A , using the same method, the decrease in gas production would not be too much. Consequently, ΔP_O could be negative.

But in the oil market, B tries to absorb more profit of the market by increasing the oil production. Then in high value of λ_C , as it has no control on the rival market (gas market) to internalize it's externalities, the increase will be much higher rather than the case in which λ_C is low. Then ΔQ_O is a big positive number and consequently $-\beta \Delta Q_O$ will be very small negative number which results in positive ΔP_O .

Additionally, based on the relation (7), the θ also impacts the price change. In high values of θ which shows high substitutability, any small increase in the gas price will increase the oil price as well.

According to the above analysis, we can show that in the graph, when λ_c and λ_A are high and θ is low, the oil price decreases after the gas cartel formation. However, when λ_c and λ_A are low and θ is high the oil price increases after the gas cartel formation.

4.3. Gas Cartel Formation Feasibility

In this section, we claim that there are some set of parameters assuming which cartel formation is not always desirable. In Theorem 3, we support this claim by comparing the profit of each member by itself (no money transfer) before and after cartel formation. The assumption of no money transfer is released in Theorem 4 to show that even with mutually beneficial monetary agreement, the cartel is not feasible. Then we discuss some specific parameters to shed lights on policy making issues¹².

Theorem 3: There are some set of parameters such that there are no mutually beneficial political agreements.

In this case we apply the graphical method. The graph (Figure 2) shows all points where B and C prefer the gas cartel $(\pi_B^2 > \pi_B^1 \text{ and } \pi_C^2 > \pi_C^1)$ when the bargaining factor is only λ_C . The empty part of the cube clearly shows those set of parameters, in which there are no

mutually beneficial political agreements. As can be seen, this accounts for a big portion of the cube. In some cases, monetary agreement is illegal or impossible, then B and C can only negotiate power within the cartel, which is coincides with gas production. Considering the figure, as an example when $\lambda_c = 0.5$ and $\theta = 0$, whatever the λ_A is, mutually beneficial political agreement is possible. It means that when markets are completely independent, C does not have any concern about the rival product, oil, and they can find an agreement.

Theorem 4: There are some set of parameters such that there are no mutually beneficial monetary agreements.

To prove this Theorem, the related graph is drawn (Figure 3). This graph shows all points where combined profits of B and C is greater in case of cartel implementation when the bargaining factor is only the transfer payment. At any given $\lambda_c \neq 0$ the empty part of the cube represents those points in which there are no mutually beneficial monetary agreements. To be more obvious, one should imagine a plane which cuts the graph perpendicular to λ_c and parallel to λ_4 axis.

Figure 2: Values of θ , λ_A and λ_C for which the profit of B and C (independently) is grater after the formation of the gas cartel with



Figure 3: Values of θ , λ_A and λ_C for which the aggregate profit of B and C is grater after the formation of the gas cartel with ($\alpha = 1, \beta = 2$,



¹² In Theorems 4 and 7, we compare the aggregate profit of B and C with cartel and without cartel while in Theorems 4, 5, and 6 this comparison is made individually with assumption of no money transfer between B and C.

As an example, suppose a point such that $\lambda_C = 1$, $\lambda_A = 1$ and $\theta = 2$. In this case, B has no power in both cartels. Then it has no revenue to be negotiated but C could pay him.

However, based on the hidden side of the graph, when $\lambda_c = 0$, for any set of parameters, there is always a mutually beneficial monetary agreement. We see further details in Theorem 7.

There are also some extreme cases of previous Theorems. First, we examine the effects of the gas cartel in the extreme case of $\lambda_c = 0$.

Theorem 5: Without money transfers, if $\lambda_c = 0$, then B always and C never wants to join the gas cartel.

Proof: Plugging back
$$\lambda_c = 0$$
 in the $\pi_B^2 - \pi_B^1$ results in

$$\frac{5(C-a)^2 \left(2\beta^2 + 2\theta^2 (\lambda_A - 1) - \beta \theta \lambda_A\right)^2}{36\beta (2\beta^2 - \theta^2) + \lambda_A \theta^2)^2} > 0.$$
 Likewise back
 $\lambda_c = 0$ in the $\pi_c^2 - \pi_c^1$ results in $-\frac{(C-a)^2 \left(2\beta^2 + 2\theta^2 (\lambda_A - 1) - \beta \theta \lambda_A\right)^2}{9\beta (2(\beta^2 - \theta^2) + \lambda_A \theta^2)^2} < 0.$

When all the power within the gas cartel goes to the B, i.e. B only exports gas, forming a cartel is always desirable for B and but obviously C refuse it as it has no revenue.

Secondly, we consider the gas cartel effect in case of $\lambda_c = 1$.

Theorem 6: Without money transfers, if $\lambda_c = 1$, there exists points such that B prefers the cartel and C does not.

Proof: In both cases there are at least one set of parameters that satisfy the condition. For example: if $\left\{a=1, \beta=\frac{9}{8}, C=0, \theta=1, \lambda_A=\frac{1}{4}\right\}$ then profit of B increases (from 0998 to 120) after joining the cartel and profit of C will

decrease (from.0912 to.0683) after joining the cartel and profit of C will words, B prefers and C refuses the cartel formation.

It seems irrational for B to want to join the cartel while it has no power in it. According to the above example, once cartel is formed, all power goes to C and it will act as a monopoly and cut the gas production. Due to the relatively high θ , this leads to a high price of oil which is beneficial for B. But it still considers the effect on the profits of B on oil.

Theorem 7: For any set of parameters, there is a mutually beneficial full agreement such that $\lambda_c = 0$.

Proof: To Prove this, simply we plugged back $\lambda_c = 0$ into the $(\pi_B^2 + \pi_c^2)$ $(\pi_B^1 + \pi_c^1)$ which results in

$$\frac{\left(C-a\right)^{2}\left(2\beta^{2}+2\theta^{2}\left(-1+\lambda_{A}\right)-\beta\theta\lambda_{A}\right)^{2}}{9\beta\left(2\left(\beta^{2}-\theta^{2}\right)+\lambda_{A}\theta^{2}\right)^{2}}\geq0$$

Theoretically, B and C tries to internalize all externalities (opportunities) in the both markets. When they can bargain on

more bargaining factors, (both money transfer and production power) it enables them to control the markets more and internalize more externalities in the market.

The Theorem goes beyond and asserts that if they agree on $\lambda_c = 0$, then they can set a mutually beneficial agreement, whatever the other parameters are. In this case, finding a core allocation is always possible such a way that no agent wants to leave the cartel, and both are better off within the cartel. In other word, when $\lambda_c = 0$, all power in the gas market goes to B and it can control fully the gas market. It means that it can internalize the externalities in the gas market as much as possible and also compensate lost profit of C by paying.

4.4. Extension

As mentioned before, the fully asymmetric version of the model proves to be too complicated to efficiently solve analytically. We simplified it by assuming symmetry of the demand and cost functions. Then, up to now, all results have been obtained in the symmetric model (level 1 below). However, in this section we present some of the results in case of asymmetric parameters. The following four levels of symmetry have been distinguished:

$$a_o = a_G = a, \beta_o = \beta_G = \beta, \theta_o = \theta_G = \theta$$
 and $C_o = C_G = C$.

- Level 1: all parameters are symmetric except λ , i.e. $a_0 = a_G = a$, $\beta_0 = \beta_G = \beta$, $\theta_0 = \theta_G = \theta$ and $C_0 = C_G = C$.
- Level 2: only θ is asymmetric, i.e. $a_0 = a_G = a$, $\beta_0 = \beta_G = \beta$ and $C_0 = C_G = C$.

Level 3: β and θ are asymmetric, i.e. $a_0 = a_G = a$, and $C_0 = C_G = C$.

Level 4: All parameters are asymmetric.

Obviously, the negative results of Theorem 1, 4, and 5 hold for all levels. The positive results of Theorem 2 and 3 hold for level 2. While we were not be able to fully prove it, we have found no counter example for level 3 and 4.

5. CONCLUSIONS AND POLICY IMPLICATIONS

In this paper, we considered three groups of gas and oil exporting countries; group A, as a member of the oil cartel which exports only oil, B, as a member of the oil cartel which exports both gas and oil and lastly, C which exports merely gas. In addition, we defined two different scenarios; base scenario, in which there is only the oil cartel in the market and second scenario which demonstrates the gas cartel formation in presence of the oil cartel.

Where all parameters are symmetric and $\lambda_c = 0$, then B always and C never wants to join the gas cartel. However, when $\lambda_c = 1$, there exists points such that B prefers the cartel and C does not. Results show that after the cartel formation, price of gas increases and its quantity decreases. Also, the quantity of oil increases while its price change is ambiguous. Regarding the current political and legal situation, no money transfer is possible. Hence the cartel formation is feasible only in narrow region of λ_c and θ which makes it vulnerable (Figure 2). We may look at Figure 2 from three different cross sections: low, medium and high θ . In low θ , we can think of gas cartel only for λ_c around 0.5 but as θ goes up we see lower λ_A and higher λ_c as feasibility requirements of formation of a cartel. This finding implicitly indicates that λ_A is not a crucial factor impacting the formation of gas cartel. In other words, no matter what portion of oil market is exported by a member of group B, the incentive of that specific member to form a gas cartel is identified by λ_c and θ .

Translating it into policy words, countries who play key roles in both cartels have incentives to be involved in them and have a type of integrated energy policy. However, important players of gas market that cannot affect on the quota of oil market (either due to political reason or market share) may prefer to stick to gas market and literally disregard the activity in the oil market. In other words, such countries would prefer to move from group B to group C. Making a linkage between this finding and the dichotomy of oil-gas market, powerful and active members of OPEC who are also key players of gas market prefer a joint strategy for both oil and gas. But countries that have unbalanced portfolio of oil and gas supply may not be interested to remain in both if there is an efficient gas cartel. Qatar is a fit example as its minister for energy affairs indicated that "Achieving our ambitious strategy will undoubtedly require focused efforts, commitment and dedication to maintain and strengthen Qatar's position as the leading LNG producer." Technically, for countries like Qatar, this means optimizing λ_c in gas market is more feasible than having constraints for both λ_c and λ_A . To give a sense for this statement, we may argue that moving from group B to group C releases some capacity for oil market active members to set a λ_{4} that is desirable for group C at some points.

Looking at the parameters discussed above, that is highlighted by Qatar exit, GECF may underpin the importance of GECF view on gas market developments through just analysis and forecasting, based on some exclusive access on data and information. This transform GECF to an umbrella the countries under which receive a type of tacit transfer, which is encouraging for them to join the GECF and empower it. Moreover, to emphasize on the importance of gas, GECF may make some joint investment to make gas as the fuel of choice and to increase added value for exporting countries (e.g through developing the downstream).

Future studies on gas markets and cartelization could include all five mentioned country groups and modelling their willingness for joining the cartel and the effects of new gas cartel on each group countries' profits.

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APPENDIX

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Equilibrium Prices, Quantities and Profits

Scenario One

The equilibrium prices and quantities are as follows:

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$$\begin{aligned} \mathcal{Q}_o^{\mathrm{l}} &= \frac{\left(\propto -C \right) \left(\beta - \theta \right)}{2 \left(\beta^2 - \theta^2 \right) + \lambda_A \theta^2} \\ \mathcal{Q}_{G,B}^{\mathrm{l}} &= \frac{\left(\propto -C \right) \left(2\beta - \theta \right) \left(\beta + \theta \left(-1 + \lambda_A \right) \right)}{3\beta (2 \left(\beta^2 - \theta^2 \right) + \lambda_A \theta^2)} \\ \mathcal{Q}_{G,C}^{\mathrm{l}} &= \frac{\left(\propto -C \right) \left(2\beta - \theta \right) \left(\beta + \theta \left(-1 + \lambda_A \right) \right)}{3\beta (2 \left(\beta^2 - \theta^2 \right) + \lambda_A \theta^2)} \\ \\ \mathcal{P}_o^{\mathrm{l}} &= \frac{C \left(\beta + \theta \left(3\beta^2 - 2\beta\theta + \theta^2 \left(-1 + \lambda_A \right) \right) \right)}{3\beta (2 \left(\beta^2 - \theta^2 \right) + \lambda_A \theta^2)} \\ \\ &+ \frac{a \left(3\beta^3 - \beta^2 \theta - \theta^3 \left(-1 + \lambda_A \right) + \beta \theta^2 \left(-3 + 2\lambda_A \right) \right)}{3\beta (2 \left(\beta^2 - \theta^2 \right) + \lambda_A \theta^2)} \\ \\ \mathcal{P}_G^{\mathrm{l}} &= \frac{C \left(\beta + \theta \right) \left(4\beta + \theta \left(-4 + \lambda_A \right) \right) + \alpha \left(2\beta^2 + 2\theta^2 \left(-1 + \lambda_A \right) - \beta \theta \lambda_A \right)}{6\beta^2 + 3\theta^2 \left(-2 + \lambda_A \right)} \end{aligned}$$

The profit functions of each country is calculated as following:

$$\pi_{A}^{1} = \frac{(a-C)^{2} (\beta-\theta) \lambda_{A} (3\beta^{3} - \beta^{2}\theta - \theta^{3} (-1+\lambda_{A}) + \beta\theta^{2} (-3+2\lambda_{A}))}{3\beta (2(\beta^{2} - \theta^{2}) + \lambda_{A}\theta^{2})^{2}}$$

$$\pi_{B}^{1} = \frac{-(a-C)^{2} (2\beta^{3}\theta (9-7\lambda_{A}) + 5\theta^{4} (-1+\lambda_{A})^{2} + \beta^{4} (-13+9\lambda_{A}))}{9\beta (2(\beta^{2} - \theta^{2}) + \lambda_{A}\theta^{2})^{2}}$$

$$+ \frac{-(a-C)^{2} (\beta^{2}\theta^{2} (8+\lambda_{A} (-17+8\lambda_{A})) - 2\beta^{3} (-1+\lambda_{A}) (-9+7\lambda_{A}))}{9\beta (2(\beta^{2} - \theta^{2}) + \lambda_{A}\theta^{2})^{2}}$$

$$\pi_{C}^{1} = \frac{(a-C)^{2} (2\beta^{2} + 2\theta^{2} (-1+\lambda_{A}) - \beta\theta\lambda_{A})^{2}}{9\beta (2(\beta^{2} - \theta^{2}) + \lambda_{A}\theta^{2})^{2}}$$

Scenario Two:

The equilibrium prices and quantities are as follows:

$$Q_O^2 = \frac{(\alpha - C)(2\beta + \theta(-2 + \lambda_A))}{4\beta^2 - \theta^2(-2 + \lambda_A)(-2 + \lambda_C)}$$

$$Q_G^2 = \frac{(\alpha - C)(2\beta + \theta(-2 + \lambda_c))}{4\beta^2 - \theta^2(-2 + \lambda_A)(-2 + \lambda_c)}$$

$$P_O^2 = \frac{-C(2\beta^2 + \theta^2(-2 + \lambda_A) + \beta\theta\lambda_c)}{-4\beta^2 + \theta^2(-2 + \lambda_A)(-2 + \lambda_c)} + \frac{\alpha(-2\beta^2 + \theta^2(-2 + \lambda_A)(-1 + \lambda_c) + \beta\theta\lambda_c)}{-4\beta^2 + \theta^2(-2 + \lambda_A)(-2 + \lambda_c)}$$

$$P_G^2 = \frac{-C(2\beta^2 + \theta^2(-2 + \lambda_A)(-2 + \lambda_c))}{-4\beta^2 + \theta^2(-2 + \lambda_A)(-2 + \lambda_c)} + \frac{\alpha(-2\beta^2 + \theta^2(-2 + \lambda_c)(-1 + \lambda_A) + \beta\theta\lambda_A)}{-4\beta^2 + \theta^2(-2 + \lambda_A)(-2 + \lambda_c)}$$

The profit functions of each country in the second scenario is calculated as following:

$$\begin{aligned} \lambda_{A} \left(a-C\right)^{2} \left(2\beta-\theta\right) + \theta\lambda_{C}\right)) \\ \pi_{A}^{2} &= \frac{\left(2\beta^{2}-\theta^{2}\left(-2+\lambda_{A}\right)\left(-1+\lambda_{C}\right)-\beta\theta\lambda_{C}\right)}{\left(-4\beta^{2}+\theta^{2}\left(-2+\lambda_{A}\right)\left(-2+\lambda_{C}\right)\right)^{2}} \\ \\ \pi_{B}^{2} &= \frac{\left(a-C\right)^{2} \left(\frac{2\beta^{3}\left(-2+\lambda_{A}\right)\left(-1+\lambda_{A}\right)}{\left(-2+\lambda_{C}\right)\left(-1+\lambda_{C}\right)}\right)}{\left(-4\beta^{2}+\theta^{2}\left(-2+\lambda_{A}\right)\left(-2+\lambda_{C}\right)\right)^{2}} - \\ \\ &= \frac{\left(a-C\right)^{2} \left(\frac{4\beta^{3}\left(-2+\lambda_{A}+\lambda_{C}\right)}{\left(+4\beta^{2}\theta\left(-2+\lambda_{A}+\lambda_{C}\right)\right)}\right)}{\left(-4\beta^{2}+\theta^{2}\left(-2+\lambda_{A}\right)\left(-2+\lambda_{C}\right)\right)^{2}} \\ + \frac{\left(a-C\right)^{2} \beta\theta^{2} \left(-8+3\lambda_{A}^{2}\left(-1+\lambda_{C}\right)+-3\left(-4+\lambda_{C}\right)\lambda_{C}\right)}{\left(-4\beta^{2}+\theta^{2}\left(-2+\lambda_{A}\right)\left(-2+\lambda_{C}\right)\right)^{2}} \\ \\ &= \frac{\left(a-C\right)^{2} \beta\theta^{2} \left(\lambda_{A}\left(12+\lambda_{C}\left(-16+3\lambda_{C}\right)\right)\right)}{\left(-4\beta^{2}+\theta^{2}\left(-2+\lambda_{A}\right)\left(-2+\lambda_{C}\right)\right)^{2}} \\ \\ &= \frac{\lambda_{C} \left(a-C\right)^{2} \left(2(\beta-\theta)+\theta\lambda_{A}\right)}{\left(-4\beta^{2}+\theta^{2}\left(-2+\lambda_{C}\right)\left(-1+\lambda_{A}\right)-\beta\theta\lambda_{A}\right)} \\ \\ &= \frac{\left(2\beta^{2}-\theta^{2}\left(-2+\lambda_{C}\right)\left(-1+\lambda_{A}\right)-\beta\theta\lambda_{A}\right)}{\left(-4\beta^{2}+\theta^{2}\left(-2+\lambda_{A}\right)\left(-2+\lambda_{C}\right)\right)^{2}} \end{aligned}$$