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## Article

# Effect of vibration dampers on the dynamic state of a drill string

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## EFFECT OF VIBRATION DAMPERS ON THE DYNAMIC STATE OF A DRILL STRING

*The object of research is the drill string bottom structure using the correcting devices of drilling modes to control the dynamics of the drill string. The work is aimed at the study of longitudinal, torsional and transverse oscillations when adjusting the axial load on the bit by using a drilling shock absorber with a two-link characteristic.*

*A dynamic model of the drill string during the drilling process is presented, which is a concentrated-continuous nonlinear system that interacts with the ball bit and the downhole engine as a source of energy. It is shown that elastic oscillations of all types affect the interaction of the bit with the outcrop rock. In order to evaluate drilling performance, it is necessary to take into account not only longitudinal, torsional and transverse oscillations, but also parametric oscillations, which are associated with the deformation of the weighted drill pipe under axial loads and during its rotation in the process of deepening the hole. It was found that transverse vibrations of weighted drill pipes, as a flexible element, during the drilling process are, as a rule, parametric in nature. They lead to the appearance of additional stresses in the elements of the column and, as a result, to the acceleration of their destruction and accelerated wear of the bit arms and rolling bearings. It is proved that in the future, for the development of dynamic models, it is necessary to take into account their hydrodynamics and the type, design and parameters of the applied punching elements.*

*The obtained research results can be applied in practice in the process of designing the structure of the drill string bottom structure (DSBS) using correcting devices of drilling modes to control the dynamics of the drill string, by using a drilling shock absorber with a two-link characteristic in order to correct the axial load on the bit.*

**Keywords:** drill string, dynamic state, anti-vibration shock absorbers, pitting, pitting engine, dynamic model.

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## 1. Introduction

Great attention is always paid to issues of drill string dynamics when drilling oil and gas wells. Therefore, the technical and economic indicators are determined, for the most part, by the efficiency of the rock-crushing tool – mechanical speed and penetration on the bit [1–3].

The use of drill string bottom structure (DSBS) with the use of drilling mode correction devices is the most realistic way of controlling the dynamics of the drill string [1, 4, 5].

In order to improve the performance of drilling wells, the use of DSBS corrective devices is of great importance. Their tests in industrial conditions showed sufficiently high efficiency [6–8].

Therefore, the study of the processes occurring in the drill string during well drilling is quite relevant and timely.

Thus, *the object of the research* is the drill string bottom structure (DSBS) with the use of corrective devices of drilling modes to control the dynamics of the drill string.

And *the aim of research* is to study the longitudinal, torsional and transverse vibrations when adjusting the axial load on the bit by using a drilling shock absorber with a two-link characteristic.

## 2. Materials and Methods

The method of mathematical modeling of a mechanical system was used. The influence of longitudinal, torsional and transverse vibrations during the correction of the axial load on the bit by using a correction device was investigated.

For research, a corrective device was used, as a drilling shock absorber with a two-link characteristic.

## 3. Results and Discussion

The effective deepening of the wellbore depends on the applied parameters (modes) of drilling, the conditions for cleaning the borehole from mud, rock properties, and other geological and technical drilling conditions. Nevertheless, insufficient attention is paid to the influence of vibration loads and the use of anti-vibration devices in the practice of three-roller blasting. In a number of works, for example, options for using protective vibration devices installed above the chisel with pseudo-linear and linear characteristics are considered [1, 6].

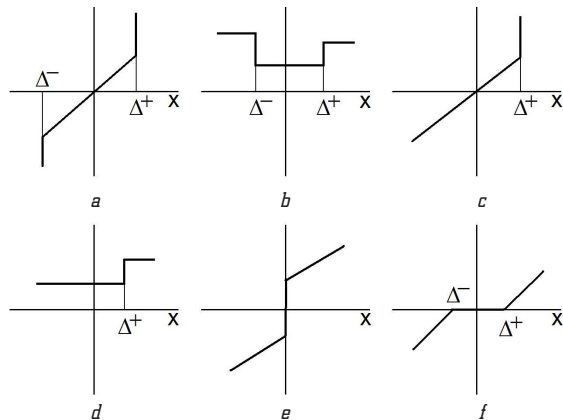
At the same time, all proposed drilling shock absorbers have complex characteristics. Mathematically, piecewise-linear elastic and dissipative characteristics (with a num-

ber of no more than three) and are described by certain functions [7].

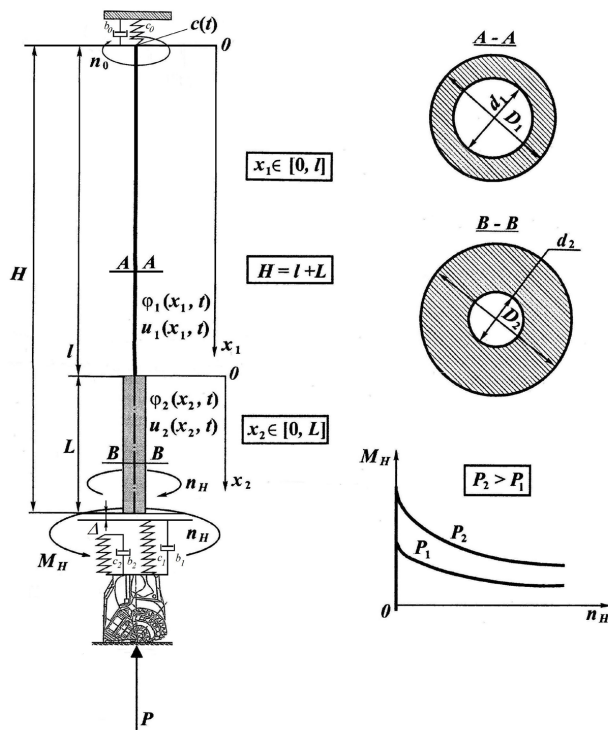
Such typical piecewise linear functions include the characteristics presented in Fig. 1 [7].

The original system consists, during rotary drilling, of three autonomous systems – a string of drill pipes, a column of weighted drill pipes (WDP), a bit with an over-bit kit, when using a downhole motor (drilling with a downhole motor), which produces driving torque and generates forcing forces factors (torsional and longitudinal vibrations).

Fig. 2 shows the calculation scheme for analyzing the drill string dynamics.



**Fig. 1.** Typical piecewise dissipative functions:  
a – elastic; b – two-link; c – dissipative; d – preliminary tension;  
e, f – elastic characteristics with a gap

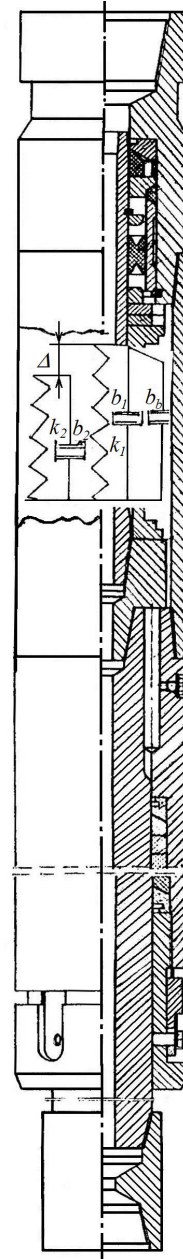


**Fig. 2.** Calculation scheme for analyzing the drill string dynamics

This approach to modeling makes it possible to approach the structure of the drill string as a machine with a complex rod structure, the elements of which (rods) can carry out bending-longitudinal-torsional oscillations. The proposed methods of mathematical modeling of the

rod system make it possible to simultaneously take into account the inertia of rotation, the influence of chain forces, physical and geometric nonlinearity, the transverse displacement of the cross-sections of the column and WDP elements, the elastic dissipative properties of the collapsing rock, internal and external friction.

As the main elastic system, let's offer a drilling shock absorber with a two-link characteristic. The construction of such a shock absorber is shown in Fig. 3.



**Fig. 3.** Drill shock absorber with two link characteristics:  
 $k_1, k_2, b_1$  – stiffness coefficients of the main elastic elements and elastic movement limiters and coefficient of internal friction in the main elastic elements;  $b_2$  – damping coefficient, which takes into account the inelastic resistance during movement of the teeth of the shell in the rock;  $\Delta$  – gap between the end of the WDP and the complementary elastic element in the position of static equilibrium;  $b_2$  – complementary resistance

The conditions for the generation of subharmonic oscillations, as shown in [9], largely depend on the symmetry or non-symmetry of the elastic characteristics of the system.

When developing a calculation scheme of a drill string, it is necessary to take into account not only its movement, but also elastic vibrations and their specificity, as well as the nature of torsional, bending and longitudinal vibrations.

Let's conduct an analysis of the drill string stability, using for this equation and boundary conditions for torsional vibrations of the string [10]:

$$\frac{\partial^2 \varphi_1}{\partial t^2} + f_{T1} \left( \frac{\partial \varphi_1}{\partial t} \right) = \lambda_{T1}^2 \frac{\partial^2 \varphi_1}{\partial z_1^2}, \quad z_1 \in [0, l]; \quad (1)$$

$$\frac{\partial^2 \varphi_2}{\partial t^2} + f_{T2} \left( \frac{\partial \varphi_2}{\partial t} \right) = \lambda_{T2}^2 \frac{\partial^2 \varphi_2}{\partial z_2^2}, \quad z_2 \in [0, L]. \quad (2)$$

Boundary conditions without an elastic shock absorber above the bit:

- 1)  $z_1 = 0; \varphi_1 = n_0 t;$
- 2)  $z_1 = l; z_2 = 0; G_1 J_{P1} \frac{\partial \varphi_1}{\partial z_1} = G_2 J_{P2} \frac{\partial \varphi_2}{\partial z_2};$
- 3)  $z_1 = l; z_2 = 0; \varphi_1 = \varphi_2;$
- 4)  $z_2 = L; G_2 J_{P2} \frac{\partial \varphi_2}{\partial z_2} = -M_N \left( P, \frac{\partial \varphi_2}{\partial t} \right).$

Boundary conditions 2 and 3 show the equality of moments at the junction of the WDP and the drill string. The calculation scheme for analyzing the behavior of the drilling tool is presented in Fig. 2.

When torsional self-oscillations occur during drilling, regardless of the geological properties of the rocks being drilled, there is a pulsation of the axial load on the bit, which amplifies the parametric oscillations associated with the WDP bending deformation.

The movement of the drill string and its elements, taking into account the forces of dry friction, elastic vibrations and friction of its elements in the washing fluid, can be described by a system of differential equations of the form:

$$m_d \ddot{z} = F_d(\dot{\varphi}_r) - F_r(\dot{\varphi}_r) = P(\varphi, \ddot{\varphi}, \ddot{\varphi}), \quad (3)$$

where  $m_d$  – reduced mass of the bit and vibrating block;  $\varphi$  – angle of rotation of the ball bit and the shaft of the punching motor.

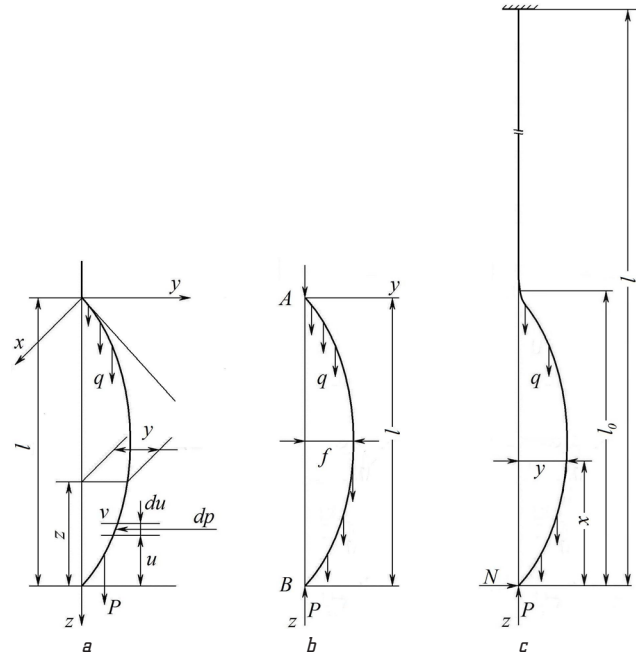
When drilling with the use of a downhole motor, the equation of its operation will be written:

$$J \ddot{\varphi}_d = M_d - \vartheta \dot{\varphi}_m - i_p M_c, \quad (4)$$

where  $J$  – moment of inertia of the shaft and rotating parts of the engine and the bit weight;  $M_d$  – driving moment;  $M_c$  – moment of resistance during rotation of the bit and frictional forces between the pipes and the wall of the well;  $\vartheta$  is the coefficient of friction in the supports of the knock-out engine.

During the drill string rotation, the WDP in the drilling process changes due to deflection. Deflection also varies with axial load during tool feed. A periodic change in the axial load when feeding the tool and rotating the WDP leads to perturbation of parametric oscillations, while kinematic oscillations can also be perturbed (Fig. 4) [11].

Thus, taking into account the bending stiffness of the form and the multipoint character of the external disturbance allows to determine the intensity of the elastic vibrations of the chip bit, which determines the force effect on its interaction with the rock of the pit.



**Fig. 4.** WDP transverse deformation: *a* – during rotation; *b* – WDP curvature under the action of its own weight and axial load; *c* – curvature of the column on part of its length [11]

The WDP movement and taking into account the forces of dry friction, elastic vibrations, etc., can be described by a system of differential equations.

The WDP motion is described by the following differential equation:

$$EJ \frac{\partial^4 u}{\partial z_2^4} + \lambda \frac{\partial^2 u}{\partial t^2} + b_8 EJ \frac{\partial^5 u}{\partial t \partial z_2^4} = 0, \quad (5)$$

where  $EJ, b_8, \lambda$  – bending stiffness, coefficient of internal friction and linear mass (taking into account the characteristics of the rocks being drilled).

Equation (5) is supplemented by boundary conditions:

$$\begin{aligned} \frac{\partial^2 u}{\partial z_2^2}(t) = 0; \quad \frac{\partial^3 u}{\partial z_2^3}(t) = 0; \\ [L=0] \quad \frac{\partial^2 u}{\partial z_2^2}(t) = 0; \quad \frac{\partial^3 u}{\partial z_2^3}(t) = 0; \quad \left[ z = \frac{L}{2} \right]. \end{aligned}$$

The action of the vibrating block on the drill string is taken into account by the condition of installing an elastic element (shock absorber) between the bit and the WDP:

$$\begin{aligned} EJ \frac{\partial^3 u}{\partial z_2^3}(t) = P_0 \sin \omega t + P_d \sin(\omega_d t + \varphi_d) - \\ - F_r - m_b \frac{\partial^2 u}{\partial t^2} - (b_1 + b_b + F_d) \frac{\partial u}{\partial t}(t). \end{aligned} \quad (6)$$

The functions  $F_d$  and  $F_r$  in equation (6) are determined by the expressions:

$$F_d = \begin{cases} 0, & u(t) \leq \Delta; \\ \frac{k_2}{b_1 \frac{k_1}{k_1}}, & u(t) > \Delta; \end{cases} \quad (7)$$

$$F_r = \begin{cases} k_1 u(t), & u(t) \leq \Delta; \\ (k_1 + k_2) u(t) - k_2 \Delta, & u(t) > \Delta, \end{cases} \quad (8)$$

where  $L$  – WDP length;  $m_d$  – mass of the bit and the bit system (vibrating block);  $k_1, k_2, b_1$  – stiffness coefficients of elastic elements and elastic travel limiter;  $b_b$  – damping coefficient, which takes into account the damping of elastic waves in the rock during its drilling;  $\Delta$  – gap between the WDP end and the additional elastic element.

The reaction of the rock being drilled is determined by the nonlinear elastic force  $F_r$  and dissipative forces [7].

The analysis of the scheme showed that the longitudinal and WDP bending oscillations are related to each other, which is explained by the fact that the forces of its own weight cause its deflections.

The presence of such deflections leads to the fact that time variables act along the WDP (frictional forces, bit oscillation forces; wave processes, etc.). When feeding the tool, the axial load on the bit increases, which leads to a change in frictional forces, obviously, which leads to a change in the longitudinal and torsional vibrations of the WDP and drill pipes – the entire string. Longitudinal and bending vibrations can be described by a system of differential equations:

$$\lambda \frac{\partial^2 u}{\partial t^2} = EF \frac{\partial^2 u}{\partial z_2^2} + f_z, \quad (9)$$

$$EJ_\alpha \frac{\partial^4 y}{\partial z_2^4} + \lambda \frac{\partial^2 y}{\partial t^2} + \frac{\partial}{\partial z_2} \left[ N(z, t) \frac{\partial u}{\partial z_2} \right] = f_y. \quad (10)$$

Under boundary conditions:

$$\left. \frac{\partial u}{\partial z_2} EF \right|_{z=0} = F_r(u_r) + k_{on} u|_{z=0} + F_d(\dot{u}_r) + b_{on} \dot{u}|_{z=0}, \quad (11)$$

$$\left. \frac{\partial u}{\partial z_2} EF \right|_{z=l} = -k_{on} u|_{z=l} - b_{on} \dot{u}|_{z=l}, \quad (12)$$

$$\left. \frac{\partial^2 u}{\partial z_2^2} \right|_{z=0} = \left. \frac{\partial^2 u}{\partial z_2^2} \right|_{z=l} = 0, \quad (13)$$

where

$$b\ddot{u}_r \text{ at } |u_r| \leq \Delta; u_r = u|_{z=0} - u_1;$$

$$F_d(\dot{u}_r) = k' b \ddot{u}_r \text{ at } |u_r| \leq \Delta; k' = \frac{k_2 + k_1}{k_1};$$

$$F_r(u_r) = \begin{cases} k_1 u_r & \text{at } |u_r| > \Delta, \\ (k_2 + k_1) u_r - \Delta k_2 \operatorname{sgn} u_r & \text{at } |u_r| > \Delta. \end{cases}$$

Mathematically, piecewise linear elastic characteristics (when the number of sections does not exceed three) are described by functions taken from the work [7].

The drill string consists of WDP, a guide, a guide pipe connected to the bit and, together with it, is a shaft that rotates during rotary drilling and transmits energy to the hole.

When drilling a given interval, the length of the lower part of the drill string remains constant, and the upper part, as a result of the increase as the well deepens, changes in size.

Drill pipes, due to the possibility of their breakage, must withstand increased and complex loads, which become more difficult with depth.

Drill pipes are subjected to a complex load, which occurs as a result of the simultaneous action of the main stresses of tension, torsion, bending and parametric oscillatory processes. Some of these loads are constant and some vary, resulting in fluctuating stresses. In the drill string, when drilling wells

and the interaction of the bit arms with the hole, wave processes occur [12], the analysis of which makes it possible to detect longitudinal resonance zones, based on Fig. 2, which shows the calculation scheme of the drill string. According to it, the problem for the longitudinal movements of the cross sections of a complex drill string will be written [12]:

$$\frac{\partial^2 u_1}{\partial t^2} + \mu \left( \frac{\partial u_1}{\partial t} \right) = \chi^2 \frac{\partial^2 u_1}{\partial z_1^2} + g, \quad u_1 \in [0, l]; \quad (14)$$

$$\frac{\partial^2 u_2}{\partial t^2} + \mu \left( \frac{\partial u_2}{\partial t} \right) = \chi^2 \frac{\partial^2 u_2}{\partial z_2^2} + g, \quad u_2 \in [0, L]. \quad (15)$$

Boundary conditions:

$$1) \quad z_1 = 0; E_1 F_1 \frac{\partial u_1}{\partial z_1} = cu;$$

$$2) \quad z_1 = l; z_2 = 0; E_1 F_1 \frac{\partial u_1}{\partial z_1} = E_2 F_2 \frac{\partial u_2}{\partial z_2};$$

$$3) \quad z_1 = l; z_2 = 0; u_1 = u_2;$$

$$4) \quad z_2 = L; E_2 F_2 \frac{\partial u_2}{\partial z_2} = -P,$$

where  $\mu$  – damping coefficient of wave processes in the drilling fluid;  $\chi$  – propagation speed of longitudinal elastic waves in the drill string.

The force  $P$  is periodic depending on the period of torsional self-oscillations  $T_0$ .

Drill pipes withstand complex loads that arise due to the simultaneous action of main stresses, stretching, torsion and bending.

Some of these loads act constantly, some are constantly changing, due to which oscillating stresses arise in the column. The drill string is subjected to maximum stresses in case of resonant oscillations. During normal drilling, those that operate throughout the drill string are taken into account.

These main stresses are: flexural oscillations; torque fluctuations; static and dynamic loads that act in the process of deepening the well, etc. However, in deep drilling, one cannot neglect the loads arising under the action of the pressure of the drilling fluid in the pipes.

If to imagine a string of drill pipes as an elastic rod that is stretched under the weight of its own weight, then the differential equation will be written in the form:

$$EF \left( 1 + b_1 \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial z_1^2} + q(z, t) = \rho F \frac{\partial^2 u}{\partial t^2} + b_2 \frac{\partial u}{\partial t} + F_r(u). \quad (16)$$

Equation (16) is supplemented by boundary conditions in the case of fixing the rod at the ends. Other boundary conditions are given in works [7, 13]:

$$u(0, t) = 0; u(l, t) = 0; N(0, t) = N_{01}; N(l, t) = N_{02}.$$

The longitudinal force  $N$  at the intersection with the coordinate  $z_1$  is obtained by the formula:

$$N = EF \left( 1 + b_1 \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial z_1^2}, \quad (17)$$

where  $E$  – elasticity modulus of the column material;  $F$  – cross-sectional area;  $b_1$  – coefficient of internal friction; reaction of



the rock as an external medium of nonlinear elasticity  $F_r(u)$  and dissipative forces  $F_d = b_2 \frac{\partial u}{\partial t}$ .

The action on the upper part of the column from the WDP side, as an external forcing force of the form  $P(t)$ , is taken into account by the condition:

$$EF \left( 1 + b_1 \frac{\partial}{\partial t} \right) \frac{\partial^2 u_{i0}}{\partial z_i^2} = P(t). \quad (18)$$

The symbol  $i0$  in equation (18) denotes the cross-section of the column at infinitesimally small distances from this cross-section to the point of application of the concentrated load (the junction of the WDP and the pipe column)  $P(t)$ .

The obtained research results can be applied in practice in the process of designing the drill string bottom structure (DSBS) using correcting devices of drilling modes to control the dynamics of the drill string, by using a drilling shock absorber with a two-link characteristic in order to correct the axial load on the bit.

There are no restrictions for the implementation of research results, to implement the obtained results in practice.

In the future, it is necessary to take into account the hydrodynamics and type, as well as the design and parameters of the applied drilling elements for the development of their dynamic models.

## 4. Conclusions

A dynamic model of the drill string in the drilling process has been obtained, which is a concentrated-continuous nonlinear system that interacts with the ball bit and the downhole engine as a source of energy.

It has been shown that elastic oscillations of all types affect the interaction of the bit with the outcrop rock. In order to evaluate drilling performance, it is necessary to take into account not only longitudinal, torsional and transverse vibrations, but also parametric vibrations, which are associated with the WDP deformation under axial loads and during its rotation in the process of deepening the hole.

It has been found that WDP transverse vibrations, as a flexible element, during the drilling process are, as a rule, parametric in nature. They lead to the appearance of additional stresses in the elements of the column and, as a result, to the acceleration of their destruction and accelerated wear of the bit arms and rolling bearings.

In the future, for the development of dynamic models, it is necessary to take into account their hydrodynamics and type, as well as the design and parameters of the applied drilling elements.

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## Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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## Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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