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Segregation of Hourly Electricity Consumption: Quantification of Demand Types Using Fourier Transform

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ABSTRACT

Although aggregate electricity consumption provides valuable information for market analysis, it does not provide demand composition, including industrial, residential, illumination, and other uses. The information for subconsumptions is required for the reliable and cost-effective operation of the power system. As a first step towards the segregation of hourly total electricity consumption into its components, we use spectral analysis (Fast Fourier Transform) to determine relative strengths of the harmonics of annual, weekly and daily variations, to quantify the share of electricity consumption for heating, cooling, illumination and industrial activities. The method is applied to the data of France, Sweden, Finland, Norway, Turkiye, Italy, Spain, Greece, Germany, Great Britain, Poland and the Netherlands. Quantitative results obtained via spectral analysis are supplemented by qualitative features observed via time-domain analysis. The consumption ratios for each demand type are calculated using daily, weekly and annual harmonics and the results are presented.

Keywords: Electricity Consumption, Composition Analysis, Fast Fourier Transform, Fourier Series Expansion, Harmonics JEL Classifications: Q47, E17, Q40

1. INTRODUCTION

Electricity is a vital source of energy for daily life, the operation of industrial facilities, transportation, health and all other areas that can be considered basic needs. The disruption of electricity leads to possible financial losses, and reliable supply is vital at an affordable cost. Although electricity is a constantly consumed resource, consumption amounts vary according to consumers' working and living habits. Residential consumption shows cyclical characteristics according to weekday working and school hours, returning home, sleeping and waking times, and holiday times. Industrial consumption is formed in a parallel periodic structure shaped by working hours, shift situations and holiday times. In this type of consumption, weekend demand decreases significantly. Electricity demand for commercial units also increases outside working hours unlike residential and industrial units. Lighting and agricultural consumption can also be included in consumption types.

The determination of the amount of demand that may come from each type of consumption, recognizing customer, types and classifying them accordingly is very important information for the system operator, distribution company and manufacturers. Such information about the electricity consumption type is also important for the planning and operation of the system. Electricity saving measures and restrictions are planned for consumer types. Since invoicing is planned on a sectoral basis, it is not possible to separate the proportional value of electricity used for heating in residential use from the invoicing system. The separation of electricity into subconsumptions is also important for maintenance plans of electricity transmission lines, long-term generation sources and transmission line investment decisions. While a homogeneous use can be expected throughout the year in industrial use, residential use and use for heating and cooling may vary depending on the seasons.

Research on electricity demand and consumption is focused on forecasting methods in the literature and is quite extensive.

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Forecasting models can be classified into time series analysis, statistical methods, artificial neural networks and simulation, heuristic approaches, and temperature-based methods. Linear models and time series methods are widely used in the literature for demand forecasting. Anand and Suganti (2012) provides literature on prediction methods, including Artificial Neural Networks (ANN), Genetic Algorithms (GA), Support Vector Machines (SVM), and Particle Swarm Optimization (PSO), and other numerical methods. ARMA and ARIMA models are also used to incorporate stochastic effects for demand forecasting. Electricity demand forecasting methodologies reviewed in Lo and Wu (2003), Andersen et al., (2013), Niu et al., (2010) have shown that ARIMA models provide better results in making long-term trends and short-term forecasts such as weekly demand models and economic growth.

In the literature, time series methods are also combined with other heuristic approaches. Taylor and Buizza (2003)) examines electricity demand data for England and Wales and applies the Holt-Winters method for different periods with an AR model. In Wang et al. (2012), authors show that the "PSO optimal Fourier method" outperforms the seasonal ARIMA forecast results and applies it to the Northwest China power grid, where the forecast accuracy of the combined model is higher than the forecast accuracy of the single model. Similar studies such as (Ren et al., (2016), Vilar et al., (2012), Filik et al., (2011), Chakhchoukh et al., (2011)) show the effectiveness of the ARIMA method in demand forecasting.

The impact of temperature on electricity demand varies depending on infrastructure and heating sources, but temperature is used to improve forecast accuracy. Different aspects of the effect of temperature on electricity demand are analyzed in researches such as (Taylor, 2010), Taylor (2012), De Felice et al. (2013), De Felice et al. (2015), Lusis et al. (2017), Islam et al. (1995), Bašta and Helman (2013). Seasonal cycles determine the impact of temperature on electricity demand, especially if electricity is used for heating and cooling needs. The effect of temperature is more evident in residential and commercial consumption and is likely to affect total demand.

Studies on separating, classifying, or estimating electricity consumption or demand in subconsumptions are limited. Gajowniczek et al. (2017) present a segmentation approach to estimate electric load at individual household levels for smart meters. Ghofrani et al. (2011) proposed real-time metering data from smart meters to predict short-term demand for residential customers using Kalman filtering methodology. Arora and Taylor (2016) aim to estimate the conditional kernel density probability to predict the consumption of smart meters. Similar studies such as Wijaya et al. (2015), Hsiao (2014) and Taieb et al. (2016) provide methods to predict and analyze consumption drivers based on smart meter data. However, these studies are limited in scope and do not include the detailed analyses presented in this research. The inability to access hourly data in real-time has been documented in the literature. Valdes and Camargo (2021) proposed a segmentation method for Chile to produce synthetic data, as hourly consumption data cannot be obtained in real-time, specifically for the paper and food industry.

The authors have previously worked on electricity demand forecasting and published publications on the characteristics of total demand. In their latest research, they examined the hourly electricity demand of 38 European countries and carried out studies on the classification of countries from consumption data, determination of special days such as holidays, in which consumption class electricity is used and how it is affected by factors such as temperature (Yukseltan, 2024). Researchers have conducted studies on determining the residential, industrial and commercial consumption demand rate by using and comparing the consumption data of religious holidays, national holidays and other special days from the total electricity consumption data given in hourly resolution Yukseltan et al. (2017), Yukseltan et al. (2020), Yukseltan et al. (2022). They also carried out studies to determine the residential, industrial, and commercial consumption demand ratio by comparing weekday and weekend consumption. The studies were also carried out to determine the summer-winter residential, industrial and commercial consumption demand rate by using hourly consumption data. These studies were conducted using Fourier series expansion and qualitative results are given (Yukseltan et al., 2022).

In this work, the breakdown of electricity consumption, which was previously done qualitatively, into subconsumptions such as heating/cooling/lighting/industry will be obtained quantitatively. The analyzes were performed with the Fourier Series using hourly total electricity demand data of Turkiye and European countries and harmonics were determined with Fast Fourier Analysis. The proposed approach provides a framework for determining consumption patterns from data and helps decisionmakers with their market analysis and grid planning assignments. The models are applied at the country level in this work, but they are generic and can be used for operational planning and scheduling.

The novelty of this study is due to a comprehensive analysis made for the total electricity consumption of Turkiye and other countries while each country has a different population, industrialization level, culture, and electricity consumption means. The proposed methodology to determine the daily, weekly, and annual harmonics is unique and novel in the literature. The subconsumptions that are determined from the total and their comparisons with the total demand provide quite impressive results. The specific objectives of this paper further can be summarized as:

- To analyze the country specific total electricity consumption and determine daily, weekly, and annual harmonics that will provide information for consumption patterns
- To develop a methodology to analyze the ratio of electricity consumption of industrial, commercial, residential, and cooling/heating within the hourly total electricity demand

The remainder of the research is as follows. In Section 2, the problem background and methodological flow are presented. The methodologies and approaches are discussed in this section. The results for detection and consumption classification are presented in Section 3. The comparisons and the details for the harmonics and cycles are also presented. Finally, in Section 4, insights based on the computational results are discussed in the conclusion.

2. BACKGROUND AND METHODOLOGY

Separating the total electricity data into classes such as industrial, commercial, and residential will provide many benefits. The information for the hourly consumption of each consumption type will be useful in determining price policies and discount times. In addition, consumer classes and their consumption rates are useful to know in case of power outages due to possible electricity supply and transmission problems. On the other hand, the total consumption data is released hourly without any knowledge of subconsumptions. The subconsumptions are determined later at the end of each month after the billing period.

The residential component consists of consumption arising from household appliances, illumination, heating, and cooling needs. Appliances generally operate continuously, but illumination needs are determined by the daylight cycle, depending only on latitude and on work habits, and daily routines specific to communities. Consumption for heating or cooling needs is much more complex; it depends on weather conditions and social habits that determine comfortable temperatures and even memory effects caused by the heating of buildings.

Commercial use of electricity is mostly limited to daylight hours, although the effects may be different on weekends. On the other hand, offices are often closed on weekends. Industrial use of electricity is a significant component of non-household consumption and is difficult to predict as some factories may operate without interruption. Furthermore, in some countries there may be holidays or holiday periods when all (non-essential) facilities are closed. In such cases, it is possible to estimate the share of purely household consumption from the data. The relative proportions of weekday and weekend consumption are also an indicator of industrial activity.

The researchers have previously analyzed the electricity consumption data of the European Union countries and identified different consumption patterns (Yukseltan et al. (2022), Yukseltan et al. (2024). For example, Sweden's consumption was found to be high and erratic in winter, indicating that electricity is used for heating. For Croatia, increasing summer consumption indicates the need for cooling, and due to the same heating and cooling capacity, summer and winter consumption is higher than in spring and autumn. In Germany and Poland, the seasonality of electricity consumption is less dominant, indicating a higher weight of industrial consumption. Strong weekly changes are also indicators of industrialization. Consumption patterns for Germany and Poland are on different scales but follow a similar structure throughout the year. Within the scope of the research, lighting, heating and cooling rates for industrial and residential type consumption were determined monthly from total hourly electricity consumption and monthly sector reports.

In previous studies, electricity consumption was modeled with a Modulated Fourier Series expansion. In addition, meteorological data was also required in cases where the use of electricity for heating and cooling purposes was predominant. In this way, it was possible to make precise estimates of total consumption. In Turkiye, proportional values were obtained for consumer groups on special days and holidays when electricity consumption is predominantly residential (Yukseltan et al. (2020). In addition, in the study examining the total consumption of European countries, qualitative results were obtained on the distribution of electricity among consumer groups and the use of electricity for heating and cooling purposes (Yukseltan et al. (2022), Yukseltan et al. (2024).

In this work, the aim is to determine the previously obtained qualitative results as quantitative proportional values. For this purpose, in addition to the methods used before, Fast Fourier Transform (FFT) is used. The Fourier series expansion gives the weights of the periodic changes in the data. By adding seasonality modulation of daily change to the Fourier expansion, it was possible to make sound analyzes with a relatively small number of regressors. Fourier transform gives the weights of all periodic changes in the data, but it is necessary to make sense of many periodic components. The frequency spectrum obtained by FFT was used as an alternative to find possible unknown periods in the data and to determine the weights of known periods. In this way, consumption groups in the country or region examined and the consumptions are determined proportionally.

The subconsumptions are determined from the total following the data processing steps. First FFT applied to the annual or multi-year consumption of a selected country and the harmonic components of the 24-h, 1-week and 52-week periods were determined. Typical daily, weekly and annual consumption curves were obtained by using the Inverse Fourier transform. The change in the harmonics of a given period relative to each other determines the shape of that change as a time curve. The harmonic structure of the change with a period of 52 weeks is expected to be different for each country. The rates of consumption of electricity in heating/cooling will be determined from the ratios of the harmonic structure. Similarly, the change curve in the time zone of weekly change will provide quantitative information about weekday/weekend consumption and indirectly the industrialdomestic usage ratio.

As a second step, FFT is applied to the annual or long-term consumption of a selected country, and for each country, the proportional structure of the total amplitudes of all harmonics of 24-h, 1-week and 52-week changes is calculated as heating/ cooling/lighting/industrial. The results provide information that supports the rates obtained in previous analysis. The Fourier transform method and the description of spectral analysis using the Fast Fourier transform that are implemented in Section 2.3 provide a data-driven approach to demand segregation.

Finally, as a third step FFT, will be applied to determine the basic usage, lighting, heating and cooling breakdowns. In this context, the electricity loss breakdown should also be evaluated within the groups. On the other hand, since the use for lighting purposes is deterministic, it can be calculated from both the modulated Fourier series expansion and the Fourier transform. For the separation of holidays and Sundays, the FFT method will be supported with time zone profiles.

2.1. Overview of the Data

Data under consideration is obtained from ENTSO (ENTSOE, 2024) and from EPIAŞ (EPIAS, 2024) for Tukiye. Time domain of available data covering 3 years for Turkiye and about 13 years for the remaining countries are presented in Figures 1 and 2 displays 1-year data (2018) for all countries.

We note here qualitative features of the data. In the first column of the graph, France, Sweden, Finland and Norway are characterized by high consumption during winter, indicating the use of electricity for heating purposes. In the second column, data for Turkiye and Greece, in addition to the winter maxima, have a clear mid-year peak, indicating electricity consumption for cooling purposes. For the countries in the third column, average daily electricity consumption is more or less constant, except for the data of Great Britain, which displays a moderate winter peak.

As part of the overview of the data, we note, low consumption periods, that are clearly visible for Turkiye and Italy. The two, mid-year low consumption periods in Turkiye's data correspond to religious holidays during which almost all non-essential industrial activities are interrupted. In Italy, the mid-year low consumption period corresponds to annual shutdown of industrial plants. A precise knowledge of such industrial off-times is crucial for demand segregation and a method for the detection of these special days will be presented in a forthcoming paper. Here we note that such short duration disturbances are quite hard to detect in spectral analysis.

Holidays and special events can be country-specific or regional, like New Year's Eve, Christmas holidays, or changes to daylight savings time throughout Europe. In Muslim countries, the dates of the religious holidays are based on the lunar calendar, and they shift back by ten days each year relative to the Gregorian calendar. Also, countries with higher industrial activities have industrial shutdown periods in summer. Figure 3 shows a close-up of the data for Turkiye and Italy as typical examples. In Figure 3a, we display the low consumption period in Turkiye, corresponding to religious holidays. The decrease in consumption in Figure 3b corresponds to an industrial shutdown period in Italy.

2.2. Spectral Analysis of the Electricity Demand with Fast Fourier Transform

The Fourier transform of a function f(t), denoted as $F(\omega)$ is defined as

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$
(1)

provided that, f(t) satisfies certain conditions that would guarantee the existence of the integral. The variables t and ω are denoted as time and frequency domain variables and $F(\omega)$ is denoted the spectral representation of f(t). In fact, the function f(t) can be recovered by evaluating the inverse Fourier transform of the function $F(\omega)$.

We recall that the method of Fourier series expansion is applicable to periodic functions, while there is no such restriction for the Fourier transform. In fact, the Fourier transform of sinusoidal

Figure 1: Time domain data for electricity consumption of selected European countries



Figure 2: Time domain data for electricity consumption for 2018



functions is not defined in the classical sense, but in the sense of "distributions", i.e., if f(t) contains sinusoidal functions with period T, Dirac-delta functions emerge at corresponding frequencies in the Fourier transformation $F(\omega)$. The Fast Fourier Transform (FFT) is a method for calculating a discretized version of the Fourier transform and it is currently available on many computing platforms. F(ω) is a complex function and its absolute value $|F(\omega)|$, called the "frequency spectrum", provides information about periodic components of f(t).

We recall that if f(t) is a periodic function with period *T*, hence with angular frequency $\omega = 2\pi/T$ then it can be represented by its Fourier series

$$a_0 + a_1 \cos(\omega t) + b_1 \sin(\omega t) + \dots + a_n \cos(n\omega t) + b_n \sin(n\omega t)$$
 (2)

Sinusoidal functions with angular frequency $n\omega t$ are called the nth "harmonics" of f(t). These higher harmonics account for the

Figure 3: Holiday consumption changes. (a) Turkiye, low consumption periods coincide with religious holidays; (b) Italy, low consumption period corresponds to the shutdown of industrial facilities in mid-summer



deviation of the periodic function from a simple sinusoid of period T. To provide an example from the electricity consumption data, if electricity is used intensely for heating, there will be a predominant peak in winter, and the time variation would be represented by a sinusoid with period 1 year, with a vertical shift and an appropriate horizontal shift. In such cases, higher harmonics would be expected to be negligible. On the other hand, if electricity is used for heating and cooling, the time domain data would have two peaks, and the second harmonic would be expected to dominate.

Harmonics of a change with *m* periods in a major frequency component in the electricity consumption are 24-h, 1-week $(24 \times 7 \text{ h})$ and observation of length *N* will appear in the indexes as given below:

$$N/m+1, 2(N/m)+1, 3(N/m)+1, 4(N/m)+1, 5(N/m)+1$$
 (3)

In practical applications of the FFT for the evaluation of the Fourier transform, there are problems associated with the finiteness of the observation period and discretization of the data. The limitation associated with the discretization is mainly expressed by the Sampling theorem stating that the highest observable frequency component has twice the period of the sampling interval. In the case of hourly data, no variation of period <2 h can be observed. The finiteness of the observation period amounts to the multiplication of the infinite time signal by a "rectangular window" function. The multiplication of functions in the time domain amounts to the convolution of their frequency domain counterparts. For example, the FFT spectrum of a pure sinusoidal wave with period T should consist of a single value at the frequency $\omega = +/-2\pi/T$, but due to finite observation time, it leaks to side frequencies and masks neighboring smaller amplitude components. In earlier applications of FFT, the method required to use data of length 2^N, obtained by padding the original signal by zeros as necessary and "windowing" methods were needed to control leakage to side frequencies. At present, there are no restrictions on the length of the data and choosing the number of samples as an exact multiple of the dominant periods minimizes leakage. Since our basic periods in this work are 24 h and 1 week, the data to be used in the FFT analysis will be selected as multiples of 7×24 h.

The period corresponding to a frequency component at point k in the frequency spectrum is calculated as follows. Let the sampled data representing the time function be a vector of length N, taken in the time interval T. In this case, the frequency spectrum will also be a vector of length N. The first element of this vector is the average value of the time series as the average value is subtracted when calculating FFT, the first element of the frequency vector is 0. The n=2 element of the frequency vector will give the strength of change in period T. In general, point k of the frequency spectrum gives the power of change in the time domain with period T=N/(k-1). For example, if we take a 1-year observation period as 52 weeks (due to the constraint of being an exact multiple of the week), it will be $N = 24 \times 7 \times 52 = 8736$. We need to look for the 24-h period at $k_1 = N/24 + 1 = 24 \times 7 \times 52/24 + 1 = 365$. Similarly, the 12-h period, that is, the 2nd harmonic, should be expected at $k_2 = N/12 + 1 = 24 \times 7 \times 52/12 + 1 = 729$. By examining the frequency spectrum in this way, the relative weights of the frequencies we want to examine are determined.

We conclude this section by a comparison of the Fourier series and the Fourier transform methods. We recall that the Fourier series expansion involves harmonics of a known frequency, while the frequency spectrum obtained via the Fourier transform gives information about all periodic components. Thus, if the structure of the data is not well known it is advisable to start with the Fourier transform. In the case of the electricity consumption data, the FFT method didn't give hint for new, unexpected periodic variations. As studied in detail in our previous papers, electricity consumption data is characterized by strong modulation effects, that is, the amplitudes of the high frequency (24-h) period variations are not the same throughout the year. In the Fourier series expansion method modulation terms that are products of high and low frequency harmonics provided a quite faithful representation of the time domain function by using a reasonably low number of regressors. In the frequency domain approach, modulation effects would be the shift of the copies of the low frequency annual variation, to high frequencies, i.e., to the harmonics of the 24-h variation, by a convolution of the spectra of the components of the product. Although the convolution effect is observable, quantification is less precise compared to the modulated Fourier series expansion method. The advantage of the FFT method is to provide a precise measure of the strengths of higher harmonics of a given periodic variation, allowing to give quantitative criteria to describe consumption components.

In Fourier series expansion, as mentioned above, it is not possible to include all harmonics and all modulation terms in the model due to the limitations of matrix inversion. In contrast, Fast Fourier Transform (FFT) gives all harmonics and all modulation terms together. The frequency points corresponding to the modulation terms can be easily calculated from the relations of expressions such as sin(a+b), sin(a-b) and sin(a)sin(b). The disadvantage of the FFT method compared to the Fourier series, as mentioned above, is that the spectrum spreads to side frequencies, resulting from the finite observation time, but this spread is minimized by taking the observation time to be an exact multiple of the dominant frequencies.

3. CLASSIFICATION OF ELECTRICITY CONSUMPTION INTO SUBCONSUMPTIONS

In this section, we analyze how hourly electricity consumption data can be used to detect consumption for lighting, heating, cooling, residential needs and industrial production purposes.

Analysis of daily, weekly and annual changes in the data using Fourier transform and quantitative separation according to usage areas are given in Section 3.2. In Section 3.3, the qualitative characteristics of daily, weekly and annual changes were evaluated using the inverse Fourier transform.

3.1. Daily, Weekly and Annual Cycles in Electricity Consumption

The basic cycles in electricity consumption are daily, weekly and seasonal cycles. In terms of electricity use, the daily cycle is the use for lighting purposes, the day-night cycle of use of appliances used in daily life, and the cycle of uses arising from business life. Weekly change takes shape depending on social habits and the working principles of business life and industry. Seasonality is affected by differences in use for lighting purposes resulting from daylight, seasonal distribution of agricultural irrigation, seasonal economic activities and seasonal economic activities such as tourism, as well as the use of electricity for heating and cooling purposes. Additionally, linear trends resulting from increases in population growth and industrialization in general are also included in the analysis.

The changes mentioned above can be observed qualitatively in the graphs of hourly data. In models of data as Fourier series, the regression coefficients of the relevant periodic functions form a quantitative scale. On the other hand, the frequency spectrum obtained by the FTT method reflects all periodic changes in the data.

In this study, total consumption data of Turkiye, Germany, England, France, Italy, Spain, Poland, Greece, Sweden, Finland and Norway between 2006 and 2018 were analyzed. The data of the countries was cleaned, and the longest data was taken as multiples of 24, 7, and 52. Turkiye's data consists of N = 26203 elements, England, Sweden, Finland and Norway's data consists of 78624 elements, and other countries' data consists of 113568 elements. Due to different data lengths, the place of harmonics in the spectrum varies for each country. The location of the first 2 harmonics for each period group is given in Appendix Table 1.

Using FTT for each country, the daily changes with a 24-h period, weekly changes with a $24 \times 7 = 168$ -h periods, and annual changes with a $24 \times 7 \times 52 = 8736$ -h periods are listed. Since only the first 11 harmonics of the 24-h changes can be observed, only the first 11 harmonics were used for the others in the comparison.

On the other hand, the relative values of the first 11 harmonics are given in Table 1 below. The large proportional value of the annual change indicates that the seasonality effect is dominant in the data. The FFT spectrum contains both the total power and phase information of the relevant periodic transformation. However, only the total power is reflected in graphical representations and tables. The proportionally high annual change in the table below does not include information about whether electricity use is more dominant in summer or winter. This distinction can be seen from the phase information of the FFT spectrum. In all the samples analyzed within the scope of this study, phase information is not reflected in the tables because either winter use is dominant, or winter and summer usages have equal weight.

3.1.1. Annual change and use for heating-cooling purposes

The low frequency range of the power spectrum gives information of the harmonics of the annual variation, as displayed on Figure 4. Here we note that for countries that use electricity heavily for heating (i.e., France, Sweden, Finland, Norway), the shape of the average variation (as it can be seen from Figure 2) looks like a sinusoid with period 1 year, and accordingly, the first harmonic is dominant. For countries that use electricity for heating and cooling (i.e., Turkiye, Italy, Spain, Greece), average time domain variation looks like having 2 peaks over a year and accordingly the strength of the second harmonic with period 6 months is comparable to the strength of the first harmonic. For the countries in the third group (Germany, Great Britain, Poland, the Netherlands), the second harmonic does not play a special role but higher harmonics are observable. Relative powers of the harmonics are also presented on Figure 5.

Table 1: Ratios of the power of the first 11 harmonics of
the daily, weekly and annual variations of the countries

Country	Daily change	Weekly change	Yearly change
Türkiye	0.3507	0.3317	0.3176
Germany	0.3284	0.4079	0.2637
England	0.4066	0.2547	0.3387
France	0.2473	0.2579	0.4948
Italy	0.3574	0.3922	0.2504
Spain	0.3569	0.3794	0.2637
Poland	0.3541	0.3655	0.2804
Holland	0.3939	0.3819	0.2243
Greece	0.4236	0.4236	0.3367
Sweden	0.2487	0.2444	0.5069
Finland	0.2275	0.2464	0.5261
Norway	0.2329	0.1913	0.5758



Figure 4: Low frequency part of the power spectrum covering the range of variations form with periods ranging from 1 year to 3 months

Figure 5: Relative powers of the harmonics of annual variation



As can be seen from the Table 1, the weight of the annual change for France, Sweden, Finland and Norway is over 50%, indicating that the use of electricity for heating purposes is dominant. As typical examples, f(t) and $|F(\omega)|$ for the 1-year electricity consumption data of Germany and France are given in Figure 6. In these examples, 24-h variations and their harmonics modulate the weekly and annual variations and carry them to higher frequencies. Note that the first harmonic is dominant for France as expected.

As can be seen in Figure 6, the harmonic structure of the change with a period of 52 weeks is expected to be different for Germany and France. The rates at which electricity is used in heating/ cooling can be determined from the ratios of its harmonic structure. Similarly, the change curve in the time zone of weekly change can provide quantitative information about weekday/ weekend consumption and indirectly the industrial-domestic usage ratio. Figure 6: Change over time and frequency spectrum of 1-year electricity consumption data of Germany and France



Figure 6 shows that consumption is higher in France during the winter months than in Germany. This is a qualitative result and provides information of the use of electricity for heating purposes in France. Although the annual consumption curves for Sweden, Finland and Norway are not shown here, they are similar to the graph given for France.

3.1.2. Weekly change and residential-industrial/commercial use In the same lines as in the previous section, we present here the part of the power spectra covering the first 4 harmonics of weekly variations covering the periods ranging from 7 days to 7/4=1.75 days. As opposed to the annual variation, determination of the distinguishing characteristics of weekly variation is a more delicate process. In Section 3.2.3, we include a time domain analysis to supplement the frequency domain analysis given here. The main characteristic of the weekly variation is off days of the week, and the total power of the weekly variation is an indicator of the industrialization of a country, since, off days refer in general to the days during which industrial plants don't work. It can of course happen that industrial plants work by shifts all days during the week, and in that case, neither frequency nor the time domain analysis would reflect this feature. In such cases, information on country specific "special days" is crucial for estimating the share of industrial consumption. Among the countries that have been analyzed here, the consumption profiles for Turkiye and Italy are remarkable in this aspect. Electricity consumption for Turkiye displays low consumption periods corresponding to religious holidays, while the consumption for Italy reflects industrial shutdowns in summer. The power spectra and relative powers of the harmonics of the weekly variation are presented in Figures 7 and 8, respectively.

Extraordinary low consumption days and weekday-weekend distinction can be used as indicators in determining the distinction between residential and commercial/industrial electricity

consumption. However, in order to make this distinction properly, the social structure and habits must be known in detail. For example, it is known that in the first days of religious holidays in Turkiye, all commercial establishments and all industrial facilities except those that need to operate continuously are closed, especially until noon. Therefore, when the use for lighting and heating/cooling purposes is filtered, the consumption during these hours gives the consumption of residential and continuously operating industries.

Although special days are known for other countries, it is not always possible to obtain detailed information about social habits within the framework of a general study. For example, although commercial establishments in Turkiye can stay open until late hours and are also open on Saturdays, the situation may be different in some European countries.

The monthly consumption for each subconsumption is provided for Turkiye and the comparisons will be made on the monthly consumptions. For European countries, the contribution of the 168 h changes, which is an indicator of the weekday-weekend usage difference, to the FFT spectrum will be examined.

Table 2: After filtering the annual change, the proportional values of the first 11 harmonics of the countries' daily, weekly, and annual changes

Country	Daily change	Weekly change	Yearly change
Türkiye	0.5139	0.4861	0
Germany	0.4179	0.5191	0.0630
England	0.6149	0.3851	0
France	0.4562	0.4757	0.0681
Italy	0.4403	0.4832	0.0765
Spain	0.4578	0.4868	0.0554
Poland	0.4630	0.4778	0.0593
Holland	0.4888	0.4739	0.0373
Greece	0.6067	0.3432	0.0501
Sweden	0.5043	0.4957	0
Finland	0.4800	0.5200	0
Norway	0.5490	0.4510	0

Figure 7: Low frequency range of the power spectra covering the first 4 harmonics of the weekly variation



In the Table 1, the relative contribution of weekly change can be misleading, especially in cases where annual change, that is, seasonality, is dominant. Therefore, in order to determine the weight of the weekly change, that is, industrial/commercial consumption, the first and last 100 elements of the spectrum were reset to zero to filter the harmonics of the annual change and the weekly and daily change rates were calculated and are given in Table 2 below.

3.1.3. Daily change and social usage habits

Daily variation curves of electricity consumption which are given in Figure 9 reflect the lifestyle of a population. In the time domain, daily variation curves are characterized by a sharp rise in the morning and slower decay in the evening. Intraday variations are more country specific. It is hard to observe these features from the power spectra only, but the relative powers of the harmonics









as displayed on Figure 10 are more informative. We recall that stronger higher harmonics are indicators of intraday variations.



Figure 10: Relative strengths of the daily harmonics by country

Figure 11: Daily consumption profiles for weekdays, Saturdays and Sundays. The highest consumption (blue curve) corresponds to weekdays and the lowest consumption (yellow curve) to Sundays



Spectral analysis of weekly and daily variations is supplemented by a time domain analysis of the data as follows. 24-h variation cycles are grouped according to the days of the week and averages for each day as a 24-h variation are obtained. As for all the countries analyzed here the off days of the week were Saturdays and Sundays, no special treatment was needed to compare daily profiles of various countries. Daily consumption profiles are grouped as "weekdays", "Saturdays" and "Sundays", as displayed on Figure 11.

Although the analyses performed with the FTT method give different results for each country, they provide very successful results in determining the periodic structure. Daily change curves may differ depending on the seasons, whether that day is a weekday, weekend, or holiday, the characteristics of industrial electricity use, and the lifestyle of societies. Figures 12-14, below show daily consumption curves for Germany France, and Greece on selected days together with their frequency spectra.

3.2. Daily, Weekly, and Annual Changes with Fourier Transform and Inverse Fourier Transform Examination

Once the Fourier transform of any time series is taken, it is always possible to recover the time series by inverse Fourier transform. Separating certain frequency regions in the time series using the Fourier transform and then filtering certain frequencies from the time series by taking the inverse Fourier transform is defined as the filtering process. In this section, the changes in the time series with periods of 24 h, 1 week, and 1 year were selected from the FFT spectrum and inverse transformations were calculated for each. The method we used qualitatively reflected the consumption characteristics of the countries, but detailed quantitative information could not be obtained.

After applying the Fourier transform to 12 countries, the first 11 harmonics of daily, weekly, and monthly changes were selected, and these changes were separated in the time domain by calculating the inverse Fourier transform. Consumption data and annual, weekly, and daily changes in the time zone for Turkiye, Germany, France, and Greece are given in Figures 15-18 below.

A maximum of 11 harmonics can be selected for daily variation in the data. Therefore, the 24-h diurnal variation curve reflects all



Figure 12: (a) Daily consumption curves for Germany. (b) Frequency spectra for Germany



Figure 13: (a) Daily consumption curves for France. (b) Frequency spectra for France





Figure 15: First 11 harmonics of annual, weekly and daily changes extracted from the FFT spectrum, Turkiye



the information that can be obtained from the Fourier spectrum. On the other hand, since the period of weekly and annual changes are multiples of 24, only 11 harmonics were used because the use of a larger number of harmonics of these slow changes would approach the entire spectrum.

Although these graphs reflect the nature of the changes in different countries, they are far from providing information on a quantitative scale. For this reason, interpretations will be made

Figure 16: First 11 harmonics of annual, weekly and daily changes extracted from the FFT spectrum, Germany



based on the ratio of the harmonics of these periodic changes to each other.

The annual change comparisons provide interesting results for each country. For Turkiye, low consumption periods during holidays cause the annual change curve to differ from other countries and are not meaningful on their own. Similarly, irregular use for cooling purposes during summer periods, depending on air temperature, causes differences in the inverse Fourier transform and is not meaningful





Figure 18: First 11 harmonics of annual, weekly and daily changes extracted from the FFT spectrum, Greece



on its own. At this point, it is possible to get closer to the original data by increasing the number of harmonics, but it is not preferred because more harmonics will add weekly and daily changes to the annual change. For France, the inverse Fourier transform of the first 11 harmonics of the annual variation is significant.

The weekly change, which gives the difference in weekday/ weekend consumption, reflects the industrial and trade activity of the countries. For Greece, the differences between weekday and weekend consumption are smaller. More differences appear for Türkiye, Germany and France. In Germany and France, the weekly change is on a similar scale. When we look at the graphs in the first row of the figures, Germany's weekday-weekend usage difference seems more misleading and dominant. The reason for this situation is that the difference in summer and winter usage in France visually suppresses the weekly change, indicating that separating the different changes will be visually meaningful.

Daily changes in electricity consumption are caused by social habits as well as the daylight cycle. Since the countries we

examined are mid-latitude, their daylight cycles are not very different. However, the decrease in usage at noon hours may differ in each country. For example, while daily usage is almost constant for Turkiye, consumption at lunchtime decreases and oscillations during the day increase in Germany, France and Greece, respectively.

4. CONCLUSIONS

Electricity is a product that is consumed simultaneously at thousands of consumption points and it is still not possible to determine customer and consumption purpose instantly. However, pricing, transmission, distribution and expansion investments will vary according to the consumption amounts of each sector. In this work, methodologies are proposed to separate the total electricity demand published in hourly resolution into subconsumptions such as heating, cooling, lighting and industry. The ratio of components in the electrical data was analyzed using the FTT. Rates from total consumption were determined for industrial, heating, cooling and residential consumption and experiments were conducted on some European country data.

The daily, weekly and seasonal harmonics of the FTT and the ratios and order of the harmonics provide information about consumption habits. The ratios and order of harmonics showed which country used electricity for purposes such as heating and industry. In addition, those that are similar in terms of consumption behavior of the 12 European countries examined can be understood by using harmonics. Turkiye is similar to Germany in terms of consumption behavior. Sample consumption charts for Germany, France and Greece are also presented.

This study constitutes the first step towards the more comprehensive goal of decomposing a total hourly electricity consumption data into subconsumptions as an hourly time series. The methods presented within the scope of the paper have the potential to yield much more useful results if the data is more detailed. Methods can also produce interpretive outputs if regional and city-based consumption data are provided instead of whole country data. It is observed that the outputs of the model can provide results with higher accuracy, especially if an hourly data set is obtained. Future studies will include more detailed analysis for further classification of consumption in shorter periods. Hence, more detailed insights will be provided for market players and decision makers.

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APPENDIX

Values of daily, weekly and monthly harmonics on a country basis (n=1,...,11)

The values in the tables are obtained by dividing the relevant harmonic by the total power of all components in the spectrum and multiplying by 1000. The 7th Harmonic of the weekly change is equal to the 1st Harmonic of the daily change.

Annondiv Tabla 1. Valua	of daily wookly and monthly	y harmonics on a country basis
Appendix fable 1. values	s of ually, weekly and monthly	y hai monics on a countily pasis

Harmonic		Turkiye			Germany			England	
	Daily	Weekly	Yearly	Daily	Weekly	Yearly	Daily	Weekly	Yearly
1	15.91	7.0704	3,485	145.77	104,970	66,039	86,882	31,057	64,376
	6.2818	4.4214	8.3849	60.5865	57.9752	6.6649	33.9642	16.7256	5.9203
2 3	1.4708	2.8730	4.6447	4.0997	23.1297	21.5553	10.5140	4.7901	7.6902
4	1.5001	2.1652	1.8491	6.0102	17.5744	13.8061	4.9823	4.3066	5.3216
5	0.7459	1.8638	1.3677	5.9578	25.7151	18.7755	3.9906	7.3599	6.4718
6	0.1268	2.0391	1.5001	4.0529	29.2004	10.1812	0.2570	8.8311	6.1154
7	0.0995	15.9104	0.2178	2.2364	145.7722	15.1021	0.7119	86.8824	6.9163
8	0.1714	1.7390	0.7405	1.3097	17.4232	13.6239	0.5159	7.7692	5.1483
9	0.0501	1.2293	0.5791	1.5307	8.7004	9.3549	0.3290	4.8504	3.3364
10	0.0068	0.8201	0.2761	0.3219	1.8968	6.1151	0.3886	2.4147	4.3158
11	0.0085	0.7238	0.8367	1.3553	3.1433	6.0511	0.2440	1.3266	3.3369
Harmonic	0.0085	France	0.8307	1.5555	Italy	0.0311	0.2440	Spain	3.3309
marmonic	Deller			Deller			Deller		Vaada
1	Daily	Weekly	Yearly	Daily	Weekly	Yearly	Daily	Weekly	Yearly
1	73.7779	65.2103	205.9555	112.4901	70.0732	4.4538	66.7294	40.7180	14.8631
2 3	52.7966	33.5194	48.8976	52.6210	39.5348	21.7157	37.9520	23.2530	26.3638
	6.4716	12.1964	16.2789	11.4674	16.2155	25.2688	4.2601	9.6261	6.1533
4	12.5813	10.9750	10.5616	7.5143	11.7581	4.5909	3.1781	7.9948	3.9401
5	7.0377	17.5147	17.9615	1.0854	19.3476	21.3545	1.7028	11.6093	9.6779
6	7.1179	19.8415	8.8756	1.2012	23.7891	14.7520	0.1140	13.2768	5.7722
7	5.0286	73.7779	14.2073	1.4894	112.4901	8.4742	0.5568	66.7294	4.7336
8	1.9478	12.7782	12.5406	1.0829	15.9068	12.3254	0.5727	8.7248	5.4331
9	3.8557	5.7458	4.7808	0.7959	8.4212	5.9545	0.4083	4.4685	2.0385
10	1.9093	1.4746	5.9243	0.9256	3.3754	9.0399	0.0415	1.4332	3.1266
11	1.7921	2.5284	2.7840	0.2805	1.1036	5.8489	0.1922	1.9168	3.4091
Harmonic		Poland			Holland			Greece	
	Daily	Weekly	Yearly	Daily	Weekly	Yearly	Daily	Weekly	Yearly
1	38.4109	22.8197	22.5332	35.8322	16.1923	14.3339	14.6719	4.7720	2.9713
2	17.3362	13.7050	3.1490	12.3806	9.4919	3.1893	8.5835	2.7694	11.4383
3	1.1100	6.5785	4.8995	2.8634	3.5361	2.5382	2.1489	1.3842	2.9170
4	2.7948	4.3661	2.0347	1.2235	3.1822	1.9359	1.0649	1.3174	0.4609
5	1.3401	5.4433	2.3160	1.3905	5.1652	2.4097	0.4560	1.6086	0.7742
6	0.4220	5.6340	3.4569	0.2360	6.5619	0.9587	0.1422	1.9484	0.3717
7	0.4697	38.4109	3.4966	0.3467	35.8322	1.8291	0.1756	14.6719	0.9029
8	0.3877	2.8905	3.1720	0.4108	4.7044	1.3448	0.1782	1.0596	0.6231
9	0.2464	1.7024	2.2643	0.1735	2.7200	0.9018	0.2215	0.3827	0.1923
10	0.3561	0.9441	0.9182	0.0891	1.0715	1.0520	0.1695	0.1571	0.9715
11	0.2260	1.0355	1.7252	0.2154	0.8531	0.9139	0.0437	0.3600	0.5140
Harmonic		Sweden			Finland			Norway	
mannonic	Daily	Weekly	Yearly	Daily	Weekly	Yearly	Daily	Weekly	Yearly
1	19.8218	10.5117	43.1922	7.0927	4.0623	20.7164	13.4224	5.8400	44.3240
2	8.0569	5.5327	3.3922	4.1472	2.7055	2.0652	6.7513	3.6670	3.1587
3	1.2739	1.7160	2.0906	0.8945	0.9038	0.7266	1.1902	1.0174	1.2048
5	1.2/37			0.8945					
4		1 0724		11/1X11/	1.0362	1.2993	0.8191	1.2919	0.6256
4	0.8163	1.9724	2.3957		1 0/01	1.0264	0 55 4 1	2 0 4 9 1	1 1 2 0 0
5	0.8163 0.7380	2.7883	2.5955	0.2166	1.8681	1.0364	0.5541	2.0481	1.1200
5 6	0.8163 0.7380 0.2765	2.7883 3.1276	2.5955 1.1371	0.2166 0.3019	1.9072	0.9298	0.2668	2.0725	0.7606
5 6 7	0.8163 0.7380 0.2765 0.2163	2.7883 3.1276 19.8218	2.5955 1.1371 3.5119	0.2166 0.3019 0.1747	1.9072 7.0927	0.9298 1.0610	0.2668 0.1686	2.0725 13.4224	0.7606 2.6490
5 6 7 8	0.8163 0.7380 0.2765 0.2163 0.1087	2.7883 3.1276 19.8218 2.6348	2.5955 1.1371 3.5119 1.2525	0.2166 0.3019 0.1747 0.0512	1.9072 7.0927 1.1539	0.9298 1.0610 1.2624	$0.2668 \\ 0.1686 \\ 0.1650$	2.0725 13.4224 1.6089	0.7606 2.6490 0.9978
5 6 7 8 9	0.8163 0.7380 0.2765 0.2163 0.1087 0.0944	2.7883 3.1276 19.8218 2.6348 1.6232	2.5955 1.1371 3.5119 1.2525 1.6154	0.2166 0.3019 0.1747 0.0512 0.1599	1.9072 7.0927 1.1539 0.7406	0.9298 1.0610 1.2624 0.4432	0.2668 0.1686 0.1650 0.0686	2.0725 13.4224 1.6089 1.0212	0.7606 2.6490 0.9978 1.1065
5 6 7 8	0.8163 0.7380 0.2765 0.2163 0.1087	2.7883 3.1276 19.8218 2.6348	2.5955 1.1371 3.5119 1.2525	0.2166 0.3019 0.1747 0.0512	1.9072 7.0927 1.1539	0.9298 1.0610 1.2624	$0.2668 \\ 0.1686 \\ 0.1650$	2.0725 13.4224 1.6089	0.7606 2.6490 0.9978