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Electric Vehicle Battery Supply Strategy Considering Brand Spillover Effect

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ABSTRACT

Electric vehicle manufacturers ensure power battery supply through either direct sourcing or technology cooperation. This paper examines the battery supply strategy of an entrant manufacturer (m2), in a two-tier supply chain that includes a battery supplier (s), an incumbent manufacturer (m1), and the aforementioned entrant. It investigates these strategies under the influence of the brand spillover effect. It reveals that when m1 opts for direct battery procurement, if m2 boasts higher after-sales service standards, adopting a technology cooperation strategy becomes more advantageous. A sweet spot exists wherein, given m2's brand power and after-sales service level fall within a certain range, all supply chain participants can achieve mutual benefits. In scenarios where m1 depends on technology cooperation for its battery supply, it is advisable for m2 to mirror this approach by also engaging in technology cooperation.

KEYWORDS

Brand Spillover Effect, Technology Cooperation, Direct Sourcing, After-Sales Service Level

INTRODUCTION

The electric vehicle market continues to attract a diverse array of players, from traditional electric vehicle manufacturers (EVMs) to newcomers, driven by the growing demand for sustainable transportation solutions (Fang et al., 2024). Traditional EVMs, such as Mercedes-Benz, have established advantages in after-sales service and sales channels, facilitating smoother market penetration (Jin & Guo, 2018; Jin & Wu, 2021). Mercedes-Benz's 2023 financial report indicates that pure electric vehicles constituted 12% of total sales, marking a 61% increase from 2022 figures and reaching 240,600 units sold (Mercedes-Benz Group, 2023).

Brand power plays an important role in product marketing (Song et al., 2022). Entrant EVMs, such as Seres-Huawei and Xiaomi, face initial resource constraints, but leverage strong brand identity to gain market traction. The launch of the AITO M5 by Seres-Huawei in March 2022 set a new benchmark for rapid sales achievement, surpassing 10,000 units in a single model (Ke, 2023). Xiaomi's SU7, introduced in March 2024, outperformed Huawei's model in sales volume (Science and Technology Geek, 2024). These examples underscore the pivotal roles of brand power in market entry and growth. These factors significantly impact sales of emerging brands.

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Power batteries stand as the heart of electric vehicles, and their procurement strategy is a critical concern for all EVMs (Cui, 2011; Fan et al., 2022). There are two main battery supply models: direct sourcing and technology cooperation. Emerging manufacturers often lack the technological expertise for in-house battery production, leading them to rely on direct sourcing from specialized suppliers (Fan et al., 2022). This approach reduces research and development (R&D) expenditures and streamlines operations, as seen in the partnerships between Contemporary Amperex Technology Co., Limited (CATL) and EVMs, such as Ford, Huawei, NIO, and IDEAL. Additionally, Tesla models S and X rely on battery supply from LG Energy Solution (Yiche Encyclopedia, 2024). In direct sourcing, the battery supply becomes more convenient. EVMs reduce expenses related to R&D and operational costs. However, battery production technology faces constraints. As companies encounter supplier monopolies and potential weaknesses in the supply chain, their acquisition costs increase (Chen et al., 2023).

In technology-intensive sectors, technology cooperation is a prevalent strategy (Zhou et al., 2021). New entrants in the electric vehicle sector face barriers to entry, supply chain uncertainties, and market risks. By engaging in technology cooperation with battery suppliers, these manufacturers can license technology, thereby mitigating risks and ensuring supply stability (Yan et al., 2023). Tesla exemplifies this approach by licensing its battery technology to peers such as Panasonic, Toyota, GM, and Volkswagen, thus enhancing its market presence and fostering collaborative networks (MOTO, 2020). Similarly, Wide Temp's technology licensing to Xinke New Materials in 2022 facilitated the production of innovative aluminum-based battery cells and PACK products, expanding market applications (Luo, 2022). Technology cooperation mitigates battery supply risks and lowers procurement expenses for producers. Nevertheless, the production line setup at battery manufacturing plants consumes considerable time. Manufacturers must demonstrate strong management capabilities while facing elevated operational costs.

Enterprises with established brand reputations tend to enjoy higher consumer loyalty (Long et al., 2019). When both renowned and emerging manufacturers source batteries from the same supplier, the latter can leverage this association to boost product visibility and attract potential buyers (Liu, Z. et al., 2023). Brand reputation of famous brand manufacturers spills over to competitors because they acquire products from a common supplier. We refer to such a phenomenon as brand spillover (Wu et al., 2021). Analogous to how Chery Automobile Co. Ltd. promoted its Flag Cloud model by highlighting the use of a BMW engine, or how Netease Yanxuan and MINISO capitalized on their shared suppliers with global brands, emerging EVMs can benefit from the brand spillover effect (Yiche, 2023). Xiaomi's electric vehicles share battery components with BYD Company Limited, yet consumers may prefer BYD's offerings owing to brand perception. This scenario raises intriguing questions: Is the brand spillover effect always advantageous for new entrants? How does it shape their battery procurement strategy?

Building our research for this study upon empirical observations and existing literature, we explore the optimal battery procurement strategy for entrant manufacturers, considering the brand spillover effect. Factors such as brand power and after-sales service level significantly influence decision-making, yet their collective impact on battery procurement choices and profitability remains underexplored. Our investigation focuses on the strategic landscape for entrant manufacturers in a competitive market, accounting for these critical elements.

We propose the following questions:

- What are the implications of direct sourcing versus technology cooperation for entrant manufacturers? What are the optimal battery supply decisions and expected profits under each scenario? What are the primary outcomes?
- Does the brand spillover effect universally benefit entrant manufacturers in battery procurement? How does it affect their supply strategy?

 What is the impact of the entrant manufacturer's battery supply strategy on the profits of battery supplier and incumbent manufacturer, and can a triple win situation for all supply chain members be achieved?

To address these questions, we designed a two-tier supply chain model featuring a battery supplier, an incumbent manufacturer, and an entrant manufacturer. We formulated four game models to examine four distinct scenarios: Model SS for direct sourcing by both parties, Model ST for direct sourcing by the entrant and technology cooperation by the incumbent, Model TS for technology cooperation by the entrant and direct sourcing by the incumbent, and Model TT for technology cooperation by both parties. These models allowed us to analyze pricing strategies and profit distributions across the supply chain. Moreover, we evaluated the influence of brand power and after-sales service levels on decision variables and profitability. The study culminates with an in-depth exploration of the brand spillover effect on entrant manufacturers' battery supply strategy.

LITERATURE REVIEW

In this paper we explore the optimal battery sourcing strategy for the entrant manufacturer considering brand spillover effect—whether it is direct sourcing or technology cooperation. We therefore address three streams of literature, including key component procurement, technology cooperation, and brand spillover effect.

Key Component Procurement

Procuring critical components is a strategic decision that can significantly impact a manufacturer's operational efficiency and financial performance. Scholars have extensively studied component procurement strategies (Arya et al., 2008; Hou et al., 2023; Huang et al., 2013; Wang et al., 2013). Dong et al. (2021) observed that original equipment manufacturers (OEMs) might shift from single to dual sourcing in response to increased competition and contract manufacturer (CM) encroachment. Jung (2020) analyzed procurement under supply and demand uncertainties and found that finding that high correlations in yield between offshore and onshore sources lead to Pareto-efficient outcomes under single-sourcing options. Zhao et al. (2024) investigated the pricing decisions of power batteries in the closed-loop supply chain for power battery recycling. Chen et al. (2023) explored three procurement strategies—single sourcing, dual sourcing, and partial outsourcing—within a supply chain featuring a common supplier and rival manufacturers. Liu et al. (2023) investigated the impact of consumers' anticipated regret on the price and quality decisions of both battery-switching vehicle and battery-charging vehicle manufacturers. Yang and Chen (2022) highlighted the value of partial sourcing over complete sourcing for start-up suppliers under conditions of high-capacity shortage risk and unlimited order induction effects. Arbabian (2022) examined whether retailers should adopt direct sourcing or 3D printing technology to fulfill their component supply needs. Although these studies offer valuable insights into diverse procurement strategies for essential components, this literature contains a notable gap regarding battery supply strategies within the electric vehicle supply chain. In practice, battery procurement is a critical issue for both traditional and internet companies entering the electric vehicle market.

Many scholars examine strategic decision-making using enhanced Fuzzy Scoring Function, Group Decision and genetic algorithm-based (Baranidharan et al., 2022; Baranidharan et al., 2024; Changdar et al., 2016, Changdar et al., 2017). Balasundaram et al. (2023) found that utilizing multi-criteria decision-making methods allows for the selection of the most suitable key material, specifically phase change material, in thermal energy storage systems. In this paper we diverge from previous articles by examining the pricing decision problem using a game-theoretic framework.

EVMs opt for direct battery purchases owing to the lack of in-house battery production capabilities. To gain control over the battery supply chain, some EVMs pursue technology cooperation

to establish their battery facilities. In this paper we aim to explore two key battery supply strategies for EVMs—direct sourcing and technology cooperation—and examine the pricing decision problem using a game-theoretic framework.

Technology Cooperation

Technology cooperation is a strategic tool for managing risks associated with entry barriers, supply disruptions, and market uncertainties(Bhavani et al., 2023; Li et al., 2023; Rau et al., 2019; Ritala, 2012; Yu et al., 2021). Amoozad Mahdiraji et al. (2024) found that cooperation between small and medium-sized enterprises will increase the efficiency and effectiveness of production and transportation; this literature solely examines production collaboration among companies, neglecting the aspect of technology cooperation between those companies. Zhou et al. (2021) noted a significant short-term enhancement in autonomous technological capabilities through dual-sourcing procurement contracts and technology licensing contracts in complex product systems. Ebrahimi Bajgani et al. (2023) investigated the technological collaboration between OEMs and remanufacturers considering information leakage. Chen et al. (2019) found that technology cooperation between innovators and licensees can force nonlicensed manufacturers out of the market. Chai et al. (2023) concluded that technology licensing hinders remanufacturing development, regardless of government subsidy or carbon quota policies. Yan, Chen, and Yang (2022) explored the impact of production efficiency and brand power on enterprises' preferences for technology collaboration and authorization modes.

The majority of existing literature focuses on technology cooperation among competing manufacturers, OEMs, and independent remanufacturers. However, there is a scarcity of research integrating these insights into the context of the electric vehicle supply chain. This study contributes to the field by investigating technology cooperation between EVMs and battery suppliers as a long-term battery supply strategy.

Brand Spillover Effect

The concept of brand spillover effect has been widely studied in the literature. Wu et al. (2022) investigated the influence of brand differentiation and brand spillover levels on enterprise procurement structure selection. Ke et al. (2024) suggested that entrants should adopt an outsourcing strategy when the brand spillover effect is significant. Wu et al. (2021) questioned whether companies with weaker proprietary brands should leverage brand spillover as a marketing strategy. Liu et al. (2023) found that free brand spillover enhances the attractiveness of retailers' proprietary brands. Quamina et al. (2023) discovered that utilizing the brand spillover effect of luxury brands to create co-brands is beneficial for a luxury brand to attract new customers, enhance brand image, and boost sales. Quamina & Singh (2023) showed that a crisis within a brand alliance not only generates unfavorable perceptions among consumers of the primary brand but also adversely affects the perception of the co-branded product, resulting in a negative brand spillover effect. Empirical studies by Klostermann et al. (2024) and Chang (2023) further validated the impact of brand spillover effects.

Previous research has primarily examined the role of brand spillover effect on strategic choices within supply chains when used as a marketing strategy. However, there is a lack of attention to the interplay between brand spillover effect and battery supply strategy. We address this gap by considering two battery supply strategies—direct sourcing and technology cooperation—and analyzing their impact on the optimal battery supply strategy for entrant manufacturers in light of the brand spillover effect.

In summary, battery acquisition poses a significant challenge for traditional businesses and tech enterprises venturing into the electric vehicle sector. The literature exhibits notable gaps regarding battery supply strategies and the technical partnerships between battery suppliers and automotive manufacturers across the electric vehicle supply chain. In contrast to earlier studies, we examine the technical collaboration between electric vehicle producers and battery suppliers as a strategic approach to long-term battery supply. We analyze two primary battery supply strategies employed by electric

Table 1. Notations

	Symbol	Symbol Definition		
Decision variables	w	Wholesale price of batteries from supplier s to manufacturers		
	W _{m1}	Wholesale price of batteries from supplier s to m_1		
	W _{m2}	Wholesale price of batteries from supplier s to m_2		
	r _{m,}	Licensing fee from supplier s to m ₁		
	r _{m2}	Licensing fee from supplier s to m ₂		
	P _{m1}	Retail price of m ₁		
	P _{m2}	Retail price of m ₂		
	q _{m,}	Selling quantity of m ₁		
	q _{m₂}	Selling quantity of m ₂		
Parameters	$U_{_1}$	Utility of consumers buying electric vehicle m ₁		
	U_2	Utility of consumers buying electric vehicle m ₂		
	v	Perceived value of consumers to electric vehicle		
	θ	Brand power of m ₂		
	S ₁	After-sales service level of m ₁		
	S ₂	After-sales service level of m ₂		
	α	Brand spillover effect		
	С	Battery production cost of supplier s		
	λ_1	Negotiation power of m ₁		
	λ_2	Negotiation power of m ₂		
	$\pi_{_{_{S}}}$	Profit of supplier s		
	$\pi_{_{\mathrm{m}_{_{1}}}}$	Profit of m ₁		
	$\pi_{_{\mathrm{m}_{_{2}}}}$	Profit of m ₂		

vehicle manufacturers: direct procurement and technical collaboration. Furthermore, we evaluate the influence of brand spillover effects on the optimal battery supply strategy for emerging manufacturers.

Model Establishment and Solution

In this section, we develop a mathematical model to analyze the optimal battery supply strategy for entrant manufacturers, considering the brand spillover effect. The model involves a battery supplier, an incumbent manufacturer (m_1), and an entrant manufacturer (m_2). We build on previous research by assuming that the battery supplier plays a dominant role in the two-tier supply chain, a reflection of actual industry practices (Fan et al., 2022; Wang, 2022). Unlike previous studies, our model assumes that licensing fees are determined endogenously through negotiations between the supplier and the collaborating manufacturers (X. Chen et al., 2019b; Mondal et al., 2022; C. H. Wu, 2018). The parameters and decision variables are defined in Table 1.

Consumer utility for purchasing electric vehicles from m1 and m2 is represented by $U_1 = \theta_1 v - p_{m1} + s_1$ and $U_2 = \theta_2 v - p_{m_2} + s_2$, respectively. We assume consumers are heterogeneous, with their perceived value v to electric vehicles uniformly distributed in (0, 1) (Chiang et al., 2003; Gao & Su, 2017; Ke & Zhou, 2024). Brand power $\theta(i = 1, 2)$ and after-sales service levels $s_1(i = 1, 2)$

are critical factors influencing consumer utility. Specifically, $\theta_1 > \theta_2$, so let $\theta_1 = 1$, and $\theta_2 = \theta$, indicating higher brand loyalty toward m1 owing to its established market presence (Christopher, 1996; Matthews et al., 2017). $s_i(i = 1, 2)$ represents the after-sales service level of manufacturer m_i , depicting whether the EVM can quickly respond to consumers, provide professional technical support, efficient maintenance, and improve the consumer's post-purchase experience (Liu, 2024).

The demand functions for m, and m, are given as follows:

For m_1 , use the formula shown in equation (1).

$$\mathbf{q}_{\mathbf{m}_{1}} = 1 - \frac{p_{\mathbf{m}_{1}} - s_{1} - p_{\mathbf{m}_{2}} + s_{2}}{1 - \theta} \tag{1}$$

For m_2 , use the formula shown in equation (2).

$$\mathbf{q}_{\mathbf{m}_{2}} = \frac{\mathbf{p}_{\mathbf{m}_{1}} - \mathbf{s}_{1} - \mathbf{p}_{\mathbf{m}_{2}} + \mathbf{s}_{2}}{1 - \theta} - \frac{\mathbf{p}_{\mathbf{m}_{2}} - \mathbf{s}_{2}}{\theta} \tag{2}$$

In these equations, θ denotes the brand power. It is assumed that m_2 can enter the electric vehicle market only if its brand power θ reaches a sufficient level, leading to a competitive scenario with m_1 . Thus, the domain of θ is constrained to

$$\frac{p_{m_2} - s_2}{p_{m_1} - s_1} < \theta < 1 - p_{m_1} + s_1 + p_{m_2} - s_2$$

We considered four scenarios for battery supply strategies, denoted by the combinations of direct sourcing (S) and technology cooperation (T):

- Direct sourcing—Direct sourcing (Model SS): Both m₁ and m₂ source batteries directly from s (Figure 1).
- Direct sourcing—Technology cooperation (Model ST): m₁ sources batteries directly from s, while m, engages in technology cooperation (Figure 2).
- Technology cooperation—Direct sourcing (Model TS): m₂ sources batteries directly from s, while m₁ engages in technology cooperation (Figure 3).
- Technology cooperation—Technology cooperation (Model TT): Both m₁ and m₂ engage in technology cooperation with s (Figure 4).

Direct Sourcing—Direct Sourcing (Model SS)

In this section, we analyze the case where both incumbent manufacturer m_1 and entrant manufacturer m_2 adopt a direct sourcing strategy from the same battery supplier s. The analysis focuses on the impacts of brand power $\theta + \alpha(1-\theta)$ and brand spillover level α on pricing decisions and market outcomes.

Given the brand spillover effect, the demand functions for the two manufacturers are formulated as shown in equations (3) and (4):

$$q_{m_1} = 1 - \frac{p_{m_1} - s_1 - p_{m_2} + s_2}{1 - (\theta + \alpha(1 - \theta))}$$
(3)

Figure 1. Direct sourcing—Direct sourcing (Model SS)

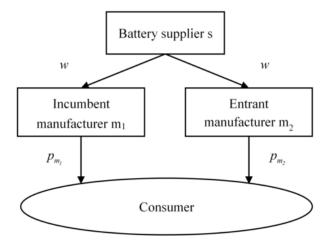
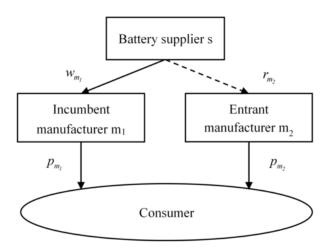


Figure 2. Direct sourcing—Technology cooperation (Model ST)



$$q_{\rm m_2} = \frac{p_{\rm m_1} - s_1 - p_{\rm m_2} + s_2}{1 - (\theta + \alpha(1 - \theta))} - \frac{p_{\rm m_2} - s_2}{(\theta + \alpha(1 - \theta))}$$
(4)

To ensure the presence of two manufacturers in the market, we assume the following:

$$\frac{\frac{p_{\rm{m_1}}-s_{\rm{2}}}{p_{\rm{m_1}}-s_{\rm{1}}}-\alpha}{1-\alpha} \ < \ \theta \ < \ \frac{1-p_{\rm{m_1}}+s_{\rm{1}}+p_{\rm{m_2}}-s_{\rm{2}}-\alpha}{1-\alpha}$$

Figure 3. Technology cooperation—Direct sourcing (Model TS)

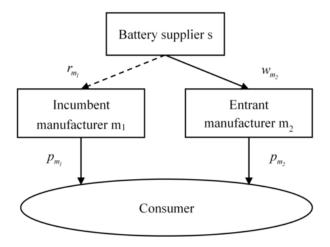
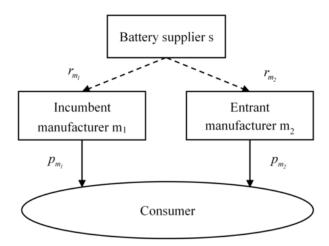


Figure 4. Technology cooperation—Technology cooperation (Model TT)



Decision sequence: Battery supplier s determines the wholesale price w, followed by simultaneous decisions by m_1 and m_2 regarding their retail prices p_m and p_m .

The profit functions are calculated using the formulas shown in equations (5)-(7).

$$\pi_s(\mathbf{w}) = (\mathbf{w} - c)(\mathbf{q}_{\mathbf{m}_1} + \mathbf{q}_{\mathbf{m}_2})$$
 (5)

$$\pi_{m_{i}}(p_{m_{i}}) = (p_{m_{i}} - w) q_{m_{i}}$$
(6)

$$\pi_{m_2}(p_{m_2}) = (p_{m_2} - w) q_{m_2}$$
 (7)

	1 2 1 2		
	θ	α	
w ^{SS}	P	P	
$p_{m_1}^{SS}$	P	P	
$p_{m_2}^{SS}$	P	P	
$q_{m_{_{1}}}^{SS} \\$	$s_1 < s_{1, N_1}^0, s_1 > s_{1, P_2}^0$	$s_1 < s_{1, N_1}^0, s_1 > s_{1, P_2}^0$	
$q_{m_2}^{SS}$	$s_1 < s_{1}^1, p; s_1 > s_{1}^1, N.$	$s_1 < s_1^1, p; s_1 > s_{1,N}^1$	

Table 2. The effect of θ and α on w^{ss}, $p_{m,}^{ss}$, $p_{m,}^{ss}$, $q_{m,}^{ss}$ and $q_{m,}^{ss}$

According to the backward induction method, equations (6) and (7) are differentiated regarding p_{m_1} and p_{m_2} , resulting in the reaction functions $p_{m_1}(w)$ and $p_{m_2}(w)$. By inserting $p_{m_2}(w)$ and $p_{m_2}(w)$ into equation (5), we determine the optimal wholesale price w^{SS} . Subsequently, w^{SS} is incorporated into the reaction functions $p_m(w)$ and $p_m(w)$, producing the optimal selling prices p_m^{SS} and p_m^{SS} .

The equilibrium wholesale and retail prices are derived using backward induction, as shown in equations (8)–(10).

$$\mathbf{w}^{SS} = \frac{1}{2}(3 + c + \mathbf{s}_1 - \frac{2(3 + \mathbf{s}_1 - \mathbf{s}_2)}{2 + \alpha + \theta - \alpha\theta}) \tag{8}$$

$$p_{m_1}^{SS} = 2 + s_1 - \frac{6 + 2s_1 + s_2 - \frac{3}{2}(3 + c + s_1 - \frac{2(3 + s_1 - s_2)}{2 + \alpha + \theta - \alpha\theta})}{4 + \alpha(-1 + \theta) - \theta}$$
(9)

$$p_{m_2}^{SS} = \frac{1}{2} (3 - c + s_1) + s_2 + \alpha - \frac{6 - 3c + 2s_1 + s_2}{4 + \alpha(-1 + \theta) - \theta} + \theta - \alpha\theta$$
 (10)

The proof is given in Appendix A.

Proposition 1 (Proof in Appendix B)

Table 2 delineates the influence of brand power θ and brand spillover level α on the equilibrium wholesale price w, retail prices (p_m, p_m) , and selling quantities (q_m, q_m) .

As brand power θ and brand spillover level α rise, so do the wholesale price w^{SS} and retail prices (p_1^{SS} , p_2^{SS}). Contrary to expectations, the manufacturers entering a market with minimal brand differentiation do not necessarily result in price reductions driven by competition. Instead, prominent brand spillover effects lead to higher car prices. Manufacturers with stronger brands can absorb higher battery procurement costs and still command premium prices.

If $s_1 < s_1^0$, accompanied by diminished brand power θ and elevated brand spillover effects α , m_1 's sales $q_{m_1}^{ss}$ will decrease, whereas m_2 will experience an increase $q_{m_2}^{ss}$, facilitating easier market entry and enabling m_2 to compete on equal terms with m_1 . Conversely, if m_1 excels in after-sales service $(s_1 > s_1^0)$, the scenario shifts. Established car manufacturers, equipped with extensive physical networks for maintenance and repairs, present formidable barriers to entry for newcomers. This trend underscores the pivotal role of after-sales service in shaping competitive landscapes within the electric vehicle sector.

Proposition 2 (Proof in Appendix B)

Table 3 delineates how brand power θ and brand spillover level α affect the profits $(\pi_s^{SS}, \pi_m^{SS}, \pi_m^{SS}, \pi_m^{SS})$.

Table 3. The impact of θ and α on π_s^{ss} , π_m^{ss} and π_m^{ss}

	θ	α
$\pi_s^{ ext{SS}}$	$s_1 < s_{1}^2, N; s_1 > s_{1}^2, P.$	$s_1 < s_{1, N_1}^2, s_1 > s_{1, P_1}^2$
$\pi_{_{\mathrm{m}_{_{\mathrm{l}}}}}^{\mathrm{SS}}$	N	N
$\pi_{ ext{m}_2}^{ ext{SS}}$	$s_1 < s_{1}^3, p; s_1 > s_{1}^3, N.$	$s_1 < s_1^3, p; s_1 > s_1^3, N.$

As brand power θ and brand spillover effects α increase, the profits of battery supplier s π_s^{SS} does not consistently increase. If $s_1 < s_1^2$, entrant manufacturers can leverage the spillover effect to gain a competitive edge, potentially at the expense of incumbent profits and subsequently impacting the supplier's revenue through reduced demand or lower wholesale prices. Conversely, when $s_1 > s_1^2$, the trend reverses. Effective after-sales service by incumbents can insulate suppliers from the competitive pressures arising from brand spillover effects.

The increasing of brand power θ and brand spillover level α is always disadvantageous for incumbent manufacturer m_1 . As brand power and spillover increase, m_1 stands to gain higher marginal profits. However, the battery supplier s capitalizes on this situation by setting higher wholesale prices, effectively siphoning off the potential gains. This mechanism results in a reduction of m_1 's net profits.

When $s_1 < s_1^3$, brand power θ is low and brand spillover level α is high, the profits of entrant manufacturer $m_2 \pi^{ss}$ surge. Entrant manufacturer m_2 should leverage the effects of brand spillover actively. However, when m_1 offers superior after-sales service $s_1 > s_1^3$, the scenario flips. This trend highlights that the spillover effect is a double-edged sword: It can amplify the competitive advantage derived from high-quality after-sales service and simultaneously magnify the disadvantages of inferior service offerings.

Direct Sourcing—Technology Cooperation (Model ST)

In this segment, we investigate the optimal decision-making process for entrant manufacturer m_2 under a technology collaboration strategy, particularly when incumbent manufacturer m_1 directly purchases batteries for supply.

The decision sequence is as follows: First, battery supplier s and entrant manufacturer m_2 engage in negotiations to determine the licensing fee r_{m_2} ; concurrently, the supplier s sets the wholesale price w_1 for the batteries. Subsequently, manufacturers m_1 and m_2 simultaneously decide on their respective car retail prices p_m , p_m .

The profit functions are calculated using the formulas shown in equations (11)–(13).

$$\pi_{s}(\mathbf{w}_{m_{1}}, r_{m_{2}}) = (\mathbf{w}_{m_{1}} - c) \mathbf{q}_{m_{1}} + r_{m_{2}} \mathbf{q}_{m_{2}}$$
(11)

$$\pi_{m_{i}}(p_{m_{i}}) = (p_{m_{i}} - w_{m_{i}}) q_{m_{i}}$$
(12)

$$\pi_{\rm m}(p_{\rm m}, r_{\rm m}) = (p_{\rm m} - c - r_{\rm m})q_{\rm m}$$
 (13)

We introduce parameter λ_2 to measure the negotiation power of entrant manufacturer m_2 . Correspondingly, the negotiation power of the battery supplier s will be $1 - \lambda_2$. The licensing fee negotiation process for the ST model is calculated as shown in equation (14).

$$\max_{\mathbf{r}_{m_1}} \pi_{\mathbf{s}_{m_2}}(\mathbf{r}_{m_2}) = \max_{\mathbf{r}_{m_1}} (\mathbf{r}_{m_2} \mathbf{q}_{m_2})^{1-\lambda_2} ((\mathbf{p}_{m_2} - c - r_{m_2}) \mathbf{q}_{m_2})^{\lambda_2}$$
(14)

	θ	λ_2
$W_{m_1}^{ST}$	$s_2 < s_{2}^0, p, s_2 > s_{2}^0, N$	N
r ST _{m2}	$s_2 < s_2^1, p; s_2 > s_2^1, N$	N
$p_{m_1}^{ST}$	$s_2 < s_2^2, p, s_2 > s_2^2, N$	N
$p_{m_2}^{ST}$	$s_2 < s_2^3, p, s_2 > s_2^3, N$	N
$q_{m_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{$	$s_2 < s_2^4, p; s_2 > s_2^4, N$	$\frac{\partial q_{_{\mathrm{m},}}^{_{\mathrm{ST}}}}{\partial \lambda_{_{2}}} = 0$
$q_{m_2}^{ST}$	$s_1 < s_{1}^4, p; s_1 > s_{1}^4, N.$	Р

Table 4. The effect of θ and λ_2 on \mathbf{w}_m^{ST} , \mathbf{r}_m^{ST} , \mathbf{p}_m^{ST} , \mathbf{p}_m^{ST} , \mathbf{q}_m^{ST} and \mathbf{q}_m^{ST}

According to the backward induction method, equations (12) and (13) are differentiated regarding p_{m_1} and p_{m_2} , resulting in the reaction functions $p_{m_1}(w_{m_1}, r_{m_2})$ and $p_{m_2}(w_{m_1}, r_{m_2})$. By inserting $p_{m_1}(w_{m_2}, r_{m_2})$ and $p_{m_2}(w_{m_1}, r_{m_2})$ into equations (11) and (14), we determine the optimal wholesale price $w_{m_1}^{ST}$ and licensing fee $v_{m_1}^{ST}$. Substituting $v_{m_1}^{ST}$ and $v_{m_2}^{ST}$ and $v_{m_1}^{ST}$ and $v_{m_2}^{ST}$ and v_{m

The equilibrium wholesale, licensing fee, and retail prices are derived using backward induction:

$$\mathbf{W}_{\mathbf{m_{1}}}^{\mathrm{ST}} \, = \, \frac{\left(1 + \mathbf{s_{1}}\right)(4 - \theta)(1 - \theta) + c(-2 + \theta)\left(-2 + \theta - \lambda_{2}\right) + \mathbf{s_{2}}(-2 + \theta)\lambda_{2} + \theta\left(-1 + \mathbf{s_{1}} + \theta\right)\lambda_{2}}{8 + \theta\left(-9 + 2\theta + \lambda_{2}\right)}$$

$$\Gamma_{m_{2}}^{ST} = \frac{\left(-8\,s_{2} - 6\theta + c(-1 + \theta)(-8 + 3\theta) + \theta\left(9\,s_{2} - s_{1}(-2 + \theta) + 8\theta - 2\theta\left(s_{2} + \theta\right)\right)\right)\left(-1 + \lambda_{2}\right)}{2(8 + \theta\left(-9 + 2\theta + \lambda_{2}\right)}$$

$$p_{m_{1}}^{ST} = \frac{\begin{pmatrix} -48 - 48\,s_{_{1}} + 8\,s_{_{2}} + 82\theta - 9\,s_{_{2}}\theta + 2\left(-20 + s_{_{2}}\right)\theta^{2} + 6\,\theta^{3} + 2(-3 + \theta)(-1 + \theta)\theta\lambda_{_{2}} \\ + s_{_{2}}\left(16 + \theta(-11 + 2\theta)\right)\lambda_{_{2}} + s_{_{1}}\theta\left(74 - 10\lambda_{_{2}} + \theta\left(-31 + 4\theta + 3\lambda_{_{2}}\right)\right) \\ - c\left(8\left(3 + 2\lambda_{_{2}}\right) + \theta\left(-23 - 13\lambda_{_{2}} + \theta\left(5 + 3\lambda_{_{2}}\right)\right)\right) \\ \hline 2\left(-4 + \theta\right)\left(8 + \theta\left(-9 + 2\theta + \lambda_{_{2}}\right)\right) \end{pmatrix}$$

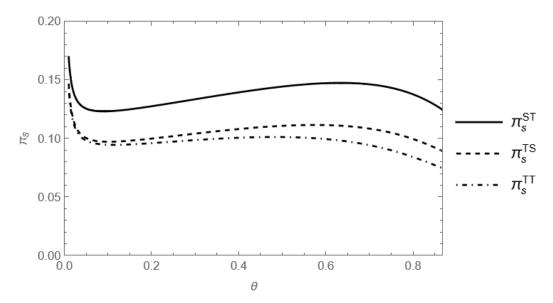
$$p_{m_2}^{ST} = \frac{ \frac{\left(8\,s_2 + s_1(-2 + \theta)\theta + 2(-3 + \theta)(-1 + \theta)\theta + s_2\theta(-9 + 2\theta))\,(-3 + \theta + \lambda_2)\right)}{\left(-c\left(8\left(1 + \lambda_2\right) + \theta\left(-3 - 7\lambda_2 + \theta(-3 + \theta + 2\lambda_2)\right)\right)}}{\left(-4 + \theta)\left(8 + \theta\left(-9 + 2\theta + \lambda_2\right)\right)}$$

The proof is given in Appendix A.

Proposition 3 (Proof in Appendix B)

Table 4 delineates the influence of brand power θ and negotiation power λ_2 on the equilibrium wholesale price w_m , licensing fee r_m , retail prices (p_m, p_m) , and sales volumes (q_m, q_m) .

Figure 5. The effect of θ on π_s (s₂=0.21)



The resilience of incumbent manufacturer m₁ is evident in its market position, which is unaffected by the ability of entrant manufacturer m, to secure power battery supply through technological collaboration. Practically, this manifests as a cooperative stance between entrant and incumbent manufacturers reliant on direct battery procurement, steering clear of direct market share

Beyond $q_{m_1}^{ST}$ merely increasing with the negotiation power λ_2 , the licensing fee $r_{m_1}^{ST}$, wholesale price $w_{m_1}^{ST}$, and retail prices $p_{m_1}^{ST}$, $p_{m_2}^{ST}$ exhibit a notable decline. This correlation is intuitively comprehensible: A stronger θ empowers m_2 to negotiate more favorable licensing terms with supplier s. Consequently, m_2 enjoys enhanced market competitiveness, marked by reduced retail prices and increased sales volume $q_{m_2}^{ST}$. In response, incumbent m1 must adjust its retail price $p_{m_1}^{ST}$ downwards to remain competitive. This intensification of price competition, spurred by technological collaboration between s and m_2 , benefits consumers, facilitating easier market penetration for technology-cooperative manufacturers.

Proposition 4 (Proof in Appendix B)

1.
$$\frac{\partial \pi_{s}^{ST}}{\partial \lambda_{2}} < 0, \frac{\partial \pi_{m_{1}}^{ST}}{\partial \lambda_{2}} = 0, \frac{\partial \pi_{m_{2}}^{ST}}{\partial \lambda_{2}} > 0.$$

$$\begin{array}{lll} 1. & \frac{\partial \pi_{*}^{\text{ST}}}{\partial \lambda_{2}} < 0, \frac{\partial \pi_{m_{*}}^{\text{m_{*}}}}{\partial \lambda_{0}} = 0, \frac{\partial \pi_{m_{*}}^{\text{m_{*}}}}{\partial \lambda_{2}} > 0. \\ 2. & \text{If } s_{2} < s_{2}^{5}, \frac{\partial \pi_{m_{*}}^{\text{m_{*}}}}{\partial \theta_{0}} > 0; \text{if } s_{2} > s_{2}^{5}, \frac{\partial \pi_{m_{*}}^{\text{m_{*}}}}{\partial \theta_{0}} < 0. \\ 3. & \text{If } s_{1} < s_{1}^{5}, \frac{\partial \pi_{m_{*}}^{\text{m_{*}}}}{\partial \theta_{0}} > 0; \text{if } s_{1} > s_{1}^{5}, \frac{\partial \pi_{m_{*}}^{\text{m_{*}}}}{\partial \theta_{0}} < 0. \end{array}$$

3. If
$$s_1^{5} < s_1^{5}, \frac{\partial \pi_1^{n_1}}{\partial \theta} > 0$$
; if $s_1^{5} > s_1^{5}, \frac{\partial \pi_1^{n_1}}{\partial \theta} < 0$.

Figures 5 and 6 illustrate the effect of entrant manufacturer m₂'s brand power on the profitability of battery supplier s.

Proposition 4 elucidates the impact of brand power θ and the negotiation power λ_2 on the profits. The implications of this proposition are as follows:

The negotiation power of entrant manufacturer m, significantly influences the profit allocation within technology cooperation frameworks. Specifically, the profitability of battery supplier s (and conversely, entrant manufacturer m₂) exhibits a negative (positive) correlation with the enhancement of m₂'s negotiation capabilities. Remarkably, incumbent manufacturer m₁'s profits remain unaltered

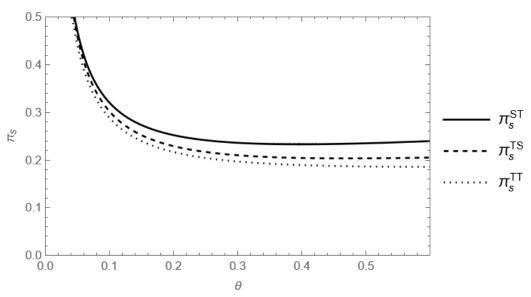


Figure 6. The effect of θ on π_s ($s_2 = 0.5$)

Note. Set parameters to $\alpha = 0.1$, c = 0.14, $s_1 = 0.04$, $\lambda_1 = 0.5$, $\lambda_2 = 0.2$

by these dynamics. Manufacturers do not engage in direct competition for market shares, and this lack of direct competition directly promotes the development of the electric vehicle market.

When $s_1 > s_1^5$, an escalation in brand power θ triggers a decline in entrant manufacturer m_2 's profitability $\pi_{m_1}^{ST}$. When $s_2 < s_2^5$, m_1 's profits $\pi_{m_1}^{ST}$ ascend with increasing brand power. Engaging in technology cooperation results in elevated brand power that may jeopardize the financial gains of m_2 . Hence, m_2 must vigilantly monitor m_1 's after-sales service quality and swiftly recalibrate its branding strategy accordingly.

Figures 5 and 6 reveal intricate patterns in the profit trajectory of battery supplier s as brand power θ fluctuates. As the brand power θ of m_2 increases, it does not necessarily favor supplier s. Higher brand differentiation benefits the supplier only when the after-sales service level of these entrants is subpar.

Technology Cooperation-Direct Sourcing (Model TS)

This section delves into the optimal decision-making process for entrant manufacturer m₂ operating under a direct sourcing strategy, juxtaposed against incumbent manufacturer m₁'s employment of a technology cooperation strategy.

The decision sequence is follows: Initially, the battery supplier s and incumbent manufacturer m_1 engage in negotiations to ascertain the licensing fee r_{m_1} . Concurrently, supplier s establishes the wholesale price w_{m_2} for the batteries. Subsequently, manufacturers m_1 and m_2 concurrently determine their respective car retail prices $p_{_{\rm m}}$, $p_{_{\rm m}}$.

The profit functions are calculated using the formulas shown in equations (15)–(17).

$$\pi_{s}(w_{m,}, r_{m_{1}}) = r_{m_{1}}q_{m_{1}} + (w_{m,} - c)q_{m_{1}}$$
(15)

$$\pi_{\mathbf{m}_{1}}(\mathbf{p}_{\mathbf{m}_{1}}, r_{\mathbf{m}_{1}}) = (\mathbf{p}_{\mathbf{m}_{1}} - c - r_{\mathbf{m}_{1}}) \mathbf{q}_{\mathbf{m}_{1}} \tag{16}$$

$$\pi_{m_2}(p_{m_2}) = (p_{m_2} - w_{m_1}) q_{m_2}$$
(17)

We introduce the parameter λ_1 to quantify the negotiation power of incumbent manufacturer m_1 . Correspondingly, the negotiation power of the battery supplier s is represented as $1 - \lambda_1$. The licensing fee negotiation process within the TS model unfolds, as shown in equation (18):

$$\max_{\mathbf{r}_{m_{i}}} \pi_{\mathbf{s}_{m_{i}}}(\mathbf{r}_{m_{i}}) = \max_{\mathbf{r}_{m_{i}}} (\mathbf{r}_{m_{i}} \mathbf{q}_{m_{i}})^{1-\lambda_{i}} ((\mathbf{p}_{m_{i}} - c - \mathbf{r}_{m_{i}}) \mathbf{q}_{m_{i}})^{\lambda_{i}}$$
(18)

According to the backward induction method, equations (16) and (17) are differentiated regarding p_{m_1} and p_{m_2} , resulting in the reaction functions $p_{m_1}(r_{m_1}, w_{m_2})$ and $p_{m_2}(r_{m_1}, w_{m_2})$. By inserting $p_{m_1}(r_{m_2}, w_{m_2})$ and $p_{m_2}(r_{m_1}, w_{m_2})$ into equations (15) and (18), we determine the optimal wholesale price $w_{m_2}^{TS}$ and licensing fee $r_{m_2}^{TS}$. Substituting $w_{m_2}^{TS}$ and $r_{m_1}^{TS}$ into $p_{m_1}(r_{m_1}, w_{m_2})$ and $p_{m_2}(r_{m_1}, w_{m_2})$, we obtain the optimal selling price $p_{m_2}^{TS}$ and $p_{m_2}^{TS}$.

The equilibrium wholesale, licensing fee and retail prices are derived using backward induction:

$$r_{m_{_{1}}}^{TS} = \frac{\left(-1+\lambda_{_{1}}\right)\left(-8-8\,s_{_{1}}+2\,s_{_{2}}+2c(-3+\theta)(-1+\theta)+11\theta-\theta\left(s_{_{2}}+3\theta+s_{_{1}}(-9+2\theta)\right)\right)}{2\left(8+\theta\left(-9+2\theta+\lambda_{_{1}}\right)\right)}$$

$$\mathbf{W}_{\mathbf{m_{2}}}^{\mathrm{TS}} = \frac{(-4+\theta)(-1+\theta)\left(\mathbf{s_{2}}+\theta\right) - c(-2+\theta)\left(2+\theta\left(-1+\lambda_{1}\right)\right) + \theta\left(-2+\mathbf{s_{2}}+\mathbf{s_{1}}(-2+\theta) + 2\theta\right)\lambda_{1}}{8+\theta\left(-9+2\theta+\lambda_{1}\right)}$$

$$\mathbf{p}_{\mathbf{m_{i}}}^{\mathrm{TS}} = \frac{\left(\left(8 - 2\,\mathbf{s_{2}} - 11\theta + \theta(\mathbf{s_{2}} + 3\theta) + \mathbf{s_{1}}(8 + \theta(-9 + 2\theta))\right)(-3 + \theta + \lambda_{1})\right)}{\left(-c\left(14 + 6\lambda_{1} + \theta(-14 + 3\theta + (-4 + \theta)\lambda_{1})\right)} \right)}{\left(-4 + \theta\right)\left(8 + \theta(-9 + 2\theta + \lambda_{1})\right)}$$

$$p_{m_{2}}^{TS} = \frac{\begin{pmatrix} -48\,s_{2} + (-1 + \theta)\theta \left(40 + 4\,\theta^{2} + 5\theta \left(-5 + \lambda_{1}\right) - 16\,\lambda_{1}\right) + s_{2}\,\theta \left(74 - 10\,\lambda_{1} + \theta \left(-31 + 4\theta + 3\,\lambda_{1}\right)\right) \\ + s_{1}\,\theta \left(8 + 16\,\lambda_{1} + \theta \left(-9 - 11\,\lambda_{1} + 2\theta \left(1 + \lambda_{1}\right)\right)\right) - 2c\left(8 + \theta \left(-3 + 7\,\lambda_{1} + \theta \left(-3 + \theta + \left(-5 + \theta\right)\right)\lambda_{1}\right)\right)}{2\left(-4 + \theta\right)\left(8 + \theta \left(-9 + 2\theta + \lambda_{1}\right)\right)}$$

The proof is given in Appendix A.

Proposition 5 (Proof in Appendix B)

Table 5 delineates the influence of brand power θ and negotiation power λ_1 on the equilibrium wholesale price w_{m_2} , licensing fee r_{m_1} , retail prices p_{m_1} and p_{m_2} , as well as the selling quantities q_{m_1} and q_{m_2} .

Proposition 5 delineates the nuanced effects of brand power and negotiation dynamics on the economic landscape of incumbent manufacturer m₁ and entrant manufacturer m₂.

The licensing fee $r_{m_1}^{TS}$ levied on incumbent manufacturer m_1 exhibits a downward trend in response to the ascent of entrant manufacturer m_2 's brand power θ . This observation contrasts markedly with the findings outlined in proposition 3, underscoring the differential impact on licensing fees across manufacturers. This discrepancy arises owing to the entrenched alliance and operational longevity of m_1 , engendering a greater trust in its profitability potential among battery suppliers compared with newcomer m_2 . Only when m_2 shows cases compelling competitive edges—such as superior

	•	
У	heta	λ_1
$r_{m_1}^{TS}$	N	N
$W_{m_2}^{TS}$	$s_2 < s_{2, P; s_2 > s_{2, N}^6}$	N
$p_{m_1}^{TS}$	$s_2 < s_{2}^7, p; s_2 > s_{2}^7, N$	N
$p_{m_2}^{TS}$	$s_2 < s_{2, P; s_2 > s_{2, N}^8}$	N
$q_{m_{_{_{1}}}}^{TS}$	$s_2 < s_{2}^9, p; s_2 > s_{2}^9, N$	Р
$q_{m_2}^{TS}$	$s_1 < s_{1}^6, p, s_1 > s_{1}^6, N.$	$\frac{\partial q_{m_i}^{TS}}{\partial \lambda_i} = 0$

Table 5. The effect of θ and λ_1 on $r_{m_i}^{TS}$, $w_{m_i}^{TS}$, $p_{m_i}^{TS}$, $p_{m_i}^{TS}$, $q_{m_i}^{TS}$ and $q_{m_i}^{TS}$

after-sales service—does the battery supplier s recalibrate its perception of m,'s profitability, leading to a reduction in the licensing fee.

Proposition 6 (Proof in Appendix B)

- $\begin{array}{ll} 1. & \frac{\partial \pi_1^{rs}}{\partial \lambda_1} < 0, \frac{\partial \pi_m^{rs}}{\partial \lambda_1} > 0, \frac{\partial \pi_{n_1}^{rs}}{\partial \lambda_1} = 0. \\ 2. & \text{If } s_2 < s_2^{10}, \frac{\partial \pi_{n_1}^{rs}}{\partial \theta} > 0. \text{ If } s_2 > s_2^{10}, \frac{\partial \pi_{n_1}^{rs}}{\partial \theta} < 0. \\ 3. & \text{If } s_1 < s_1^7, \frac{\partial \pi_{n_1}}{\partial \theta} > 0. \text{ If } s_1 > s_1^7, \frac{\partial \pi_{n_1}}{\partial \theta} < 0. \end{array}$

Figures 5 and 6 depict the influence of entrant manufacturer m_2 's brand power θ on the profitability of battery supplier s.

Proposition 6 elucidates that the negotiation power λ_1 of m₁ and the brand power θ of m₂ significantly influence the profitability of m₂, m₁, and battery supplier s. Specifically, proposition 6 reveals that the negotiation power λ_1 of incumbent manufacturer m₁ solely impacts the profit distribution between m₁ and s within the context of technology cooperation. Consequently, the profit of battery supplier s (m₁'s profit) exhibits a decrease (increase) as the negotiation power λ_1 of incumbent manufacturer m₁ escalates. Notably, this dynamic does not exert any influence on the profit of entrant manufacturer m₂.

Technology Cooperation—Technology Cooperation (Model TT)

This section delves into the optimal decision-making processes for entrant manufacturer m₂, operating under a technology collaboration strategy, in the strategic landscape where incumbent manufacturer m, also adopts a technology collaboration approach.

The decision sequence is as follows: The battery supplier s and the incumbent manufacturer m₁ engage in negotiations to establish the licensing fee r_m ; concurrently, s and the entrant manufacturer m_{γ} also negotiate to determine their respective licensing fee r_{m} . Subsequently, manufacturers m_{γ} and m_{s} concurrently determine their respective car retail prices $p_{m_{s}}$, $p_{m_{s}}$.

The profit functions are calculated using the formulas shown in equations (19)–(21).

$$\pi_{s}(r_{m_{1}}, r_{m_{2}}) = r_{m_{1}} q_{m_{1}} + r_{m_{2}} q_{m_{2}}$$
(19)

$$\pi_{m_i}(p_{m_i}, r_{m_i}) = (p_{m_i} - c - r_{m_i}) q_{m_i}$$
(20)

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$$\pi_{m_{1}}(p_{m_{1}}, r_{m_{1}} = (p_{m_{1}} - c - r_{m_{1}})q_{m_{1}}$$
(21)

The licensing fee negotiation process for the TT model is shown in equation (14) and (21). According to the backward induction method, equations (20) and (21) are differentiated regarding p and p_{m_1} , resulting in the reaction functions $p_{m_1}(r_{m_1}, r_{m_2})$ and $p_{m_2}(r_{m_1}, r_{m_2})$. By inserting $p_{m_1}(r_{m_1}, r_{m_2})$ and $p_{m_2}(r_{m_1}, r_{m_2})$ into equation (14) and (19), we determine the optimal licensing fee $r_{m_1}^{TT}$ and $r_{m_2}^{TT}$. Substituting $w_{m_2}^{TS}$ and $r_{m_1}^{TS}$ into $p_{m_1}(r_{m_1}, r_{m_2})$ and $p_{m_2}(r_{m_1}, r_{m_2})$, we obtain the optimal selling price $r_{m_1}^{TT}$ and $r_{m_2}^{TT}$. The equilibrium wholesale, licensing fee, and retail prices are derived using backward induction:

$$\mathbf{r}_{\mathbf{m_{i}}}^{\mathrm{TT}} = \ \left(1 - \lambda_{1}\right) \frac{\left(8 + 8 \, s_{1} - 2 \, s_{2} - 11\theta + \theta \left(s_{2} + 3\theta\right) + s_{2}(-2 + \theta) \lambda_{2} \right.}{\left. + (-1 + \theta)\theta \, \lambda_{2} - 2c(-1 + \theta)\left(-3 + \theta + \lambda_{2}\right) + s_{1}\theta \left(-9 + 2\theta + \lambda_{2}\right)\right)}{16 + \theta \left(-17 + 4\theta + \lambda_{1} + \lambda_{2} - \lambda_{1}\lambda_{2}\right))}$$

$$\mathbf{r}_{\mathbf{m}_{2}}^{\mathrm{TT}} = (1 - \lambda_{2}) \frac{\begin{pmatrix} 8 \, \mathbf{s}_{2} + \mathbf{s}_{1} (-2 + \theta) \theta (1 + \lambda_{1}) + 2 (-1 + \theta) \theta (-3 + \theta + \lambda_{1}) \\ + s_{2} \theta (-9 + 2\theta + \lambda_{1}) - c (-1 + \theta) (-8 + \theta (3 + \lambda_{1})) \end{pmatrix}}{16 + \theta (-17 + 4\theta + \lambda_{1} + \lambda_{2} - \lambda_{1} \lambda_{2})}$$

$$p_{m_1}^{TT} = \frac{\begin{pmatrix} -c(-2+\theta)\big(2\big(-7+\lambda_1\big(-3+\lambda_2\big)-3\,\lambda_2\big)+\theta\big(5+3\,\lambda_2+\lambda_1\big(3+\lambda_2\big)\big)\big)+2\big(-3+\theta+\lambda_1\big) \\ \big(s_2(-2+\theta)\big(1+\lambda_2\big)+(-1+\theta)\big(-8+\theta\big(3+\lambda_2\big)\big)+s_1\big(8+\theta\big(-9+2\theta+\lambda_2\big)\big)\big)}{(-4+\theta)\big(16+\theta\big(-17+4\theta+\lambda_1+\lambda_2-\lambda_1\lambda_2\big)\big)}$$

$$p_{m_2}^{TT} = \frac{\begin{pmatrix} 2 \big(8\,s_2 + s_1(2 - \theta)\theta(1 + \lambda_1)\,\lambda_1 + 2(1 - \theta)\theta(3 - \theta - \lambda_1) + s_2\theta(9 - 2\theta - \lambda_1) \big) \big(3 - \theta - \lambda_2) \\ -c(-2 + \theta) \big(-8(1 + \lambda_2) + \theta(-3 - 5\,\lambda_1 + 2\theta(1 + \lambda_1) + 5\,\lambda_2 + 3\,\lambda_1\lambda_2) \big) \end{pmatrix}}{(-4 + \theta) \big(16 + \theta(-17 + 4\theta + \lambda_1 + \lambda_2 - \lambda_1\lambda_2) \big)}$$

The proof is given in Appendix A.

Proposition 7 (Proof in Appendix B)

Table 6 illustrates the effects of brand power θ , the negotiation powers λ_1 , and λ_2 on the licensing fees r_m and r_m , retail prices p_m and p_m , and sales volumes q_m and q_m .

Interestingly, the selling quantity (q_m^{TT}, q_m^{TT}) exhibits a positive correlation with an individual manufacturer's negotiation power, while it shows an inverse relationship with the negotiation power of its competitor. This observation contrasts with the results presented in propositions 3 and 5. Compared with Models ST and TS, market competition is much more intensified in Model TT, making low prices the top choice for consumers. Consequently, manufacturers should reduce selling prices to enhance competitiveness.

Proposition 8 (Proof in Appendix B)

$$1. \quad \frac{\partial \pi_{\cdot,\cdot}^{\text{\tiny{TT}}}}{\partial \lambda_{1}^{1}} > 0, \frac{\partial \pi_{\cdot,\cdot}^{\text{\tiny{TT}}}}{\partial \lambda_{1}^{1}} > 0, \frac{\partial \pi_{\cdot,\cdot}^{\text{\tiny{TT}}}}{\partial \lambda_{1}^{1}} < 0. \frac{\partial \pi_{\cdot,\cdot}^{\text{\tiny{TT}}}}{\partial \lambda_{2}^{2}} < 0, \frac{\partial \pi_{\cdot,\cdot}^{\text{\tiny{TT}}}}{\partial \lambda_{2}^{2}} < 0, \frac{\partial \pi_{\cdot,\cdot}^{\text{\tiny{TT}}}}{\partial \lambda_{2}^{2}} > 0.$$

у	θ	$\lambda_{_{1}}$	λ_2
r _{m1} ^{TT}	$s_2 < s_2^{11}, p; s_2 > s_2^{11}, N$	N	N
r ^{TT} _{m2}	$s_2 < s_2^{12}, p; s_2 > s_2^{12}, N$	N	N
$p_{m_i}^{TT}$	$s_2 < s_2^{13}, p; s_2 > s_2^{13}, N$	N	N
$p_{m_2}^{TT}$	$s_2 < s_2^{14}, p; s_2 > s_2^{14}, N$	N	N
$q_{m_i}^{TT}$	$s_2 < s_2^{15}, p; s_2 > s_2^{15}, N$	Р	N
$q_{m_2}^{TT}$	$s_1 < s_1^8, p; s_1 > s_1^8, N.$	N	P

Table 6. The effect of θ , λ_1 , and λ_2 on r_m^{TT} , r_m^{TT} , p_m^{TT} , p_m^{TT} , q_m^{TT} , and q_m^{TT}

$$2. \quad \text{If } s_{_{2}} < s_{_{2}}^{_{16}}, \frac{\partial \pi_{_{m}}^{^{\Pi}}}{\partial \theta} > 0. \text{ If } s_{_{2}} > s_{_{2}}^{_{16}}, \frac{\partial \pi_{_{m}}^{^{\Pi}}}{\partial \theta} < 0. \text{ If } s_{_{1}} < s_{_{1}}^{9}, \frac{\partial \pi_{_{m}}^{^{\Pi}}}{\partial \theta} > 0. \text{ If } s_{_{1}} > s_{_{1}}^{9}, \frac{\partial \pi_{_{m}}^{^{\Pi}}}{\partial \theta} < 0.$$

Figures 5 and 6 show the impact of entrant manufacturer m2's brand power on the profits of battery supplier s. Proposition 8 illustrates the effects of brand power θ , as well as the negotiation powers λ_1 and, λ_2 on the profits $(\pi_s^{TT}, \pi_m^{TT}, \pi_m^{TT})$.

Higher negotiation power translates into better positioning for manufacturers in price competition, thus boosting their profits through increased sales volumes. As m₂'s negotiation power λ_2 escalates, the gain in sales volume for s through m, does not offset the loss in sales volume for m, resulting in a net reduction in s's profits. These dynamics highlight the strategic importance of negotiation power and brand power in determining the distribution of profits among battery suppliers and manufacturers within a competitive landscape, underscoring the complexity of market interactions and the need for strategic considerations regarding technology cooperation and negotiation tactics.

NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

In this section we analyze the optimal battery supply strategy for an entrant manufacturer and the influence on the profits of incumbent manufacturer m, and battery supplier s.

Proposition 9

In the scenario of the incumbent manufacturer directly purchases batteries, $\pi_{m_1}^{SS} > \pi_{m_2}^{ST}$ when $G(\theta, \theta)$

 $s_1, s_2 > 0, s_1 \neq s_2$ and $\alpha > \overline{\alpha}$. Otherwise, $\pi_{m_2}^{ST} > \pi_{m_2}^{SS}$.

According to the insights gleaned from proposition 9, the after-sales service capabilities of an entrant manufacturer, m,, play a critical role in dictating the optimal battery supply strategy. Specifically, if m, struggles to provide premium after-sales service, adopting a direct procurement strategy is advisable only under conditions of high brand spillover because entrant manufacturers such as m, can mitigate their service shortcomings by sourcing batteries from well-regarded brands. Conversely, in scenarios in which brand spillover is weak, consumers gravitate toward products from entrant manufacturers that excel in after-sales support.

In essence, the strategic partnership between m, and s serves as a countermeasure against the limitations posed by weak brand spillover and inadequate after-sales service capabilities. This collaborative approach ensures that m, can navigate the competitive landscape effectively, leveraging cost-efficiency and pricing strategies to carve out a successful niche in the market.

The proof is given in Appendix B.

Proposition 10

For this proposition, we get that $\pi_{m_1}^{SS} > \pi_{m_1}^{ST}$ when $s_2 < 1 + s_1 - \theta$ and $\alpha < \hat{\alpha}$ and that $\pi_{m_1}^{ST} < \pi_{m_1}^{SS}$ when $s_2 < 1 + s_1 - \theta$ and $\alpha > \hat{\alpha}$ or $s_2 > 1 + s_1 - \theta$.

Proposition 10 delineates that if entrant manufacturer m_2 fails to provide superior after-sales service, its competitive advantage over incumbent manufacturer m_1 is significantly reduced. With increasing brand spillover, incumbent manufacturer m_1 faces greater sales decline than entrant manufacturer m_2 in price competition within a technology collaboration framework. Accordingly, a critical threshold ($\hat{\alpha}$) exists; beneath this threshold, incumbent manufacturer m_1 garners augmented profits when entrant manufacturer m_2 adopts a direct sourcing strategy in lieu of a technology cooperation strategy. Conversely, beyond this threshold, m_1 realizes superior profits when m_2 engages in a technology cooperation strategy.

Should entrant manufacturer m_2 provide superior after-sales service, it accrues a pronounced competitive edge through the amplification of brand spillover effects. The brand spillover effect leads to sales losses for the incumbent manufacturer m_1 . Meanwhile, the entrant manufacturer m_2 gains advantages from lower wholesale prices through technology cooperation. Consequently, incumbent manufacturer m_1 achieves increased profitability when entrant manufacturer m_2 selects technology cooperation over direct sourcing.

The proof is given in Appendix B.

Proposition 11

For this proposition, we get that $\pi_s^{ST} > \pi_s^{SS}$ when $s_2 < s_2^{17}$, $\lambda_2 < \lambda_2^*$, $\alpha < \alpha^*$ or $s_2 > s_2^{17}$, $\lambda_2 < \lambda_2^*$. Otherwise $\pi_s^{SS} > \pi_s^{ST}$.

To visually grasp proposition 11, a numerical example is provided as shown in Figures 7 and 8. Building on previous scholars' studies, we assume $\theta = 0.1$, c = 0.14, $s_1 = 0.04$, and $\lambda_1 = 0.5$.

In the context where entrant manufacturer m_2 is unable to furnish satisfactory after-sales service (as depicted in Figure 7), as the brand spillover level α and the negotiation power λ_2 ascend, supplier s progressively accrues higher profits from m_2 when adhering to a direct sourcing strategy rather than accruing a higher licensing fee through technology collaboration. Conversely, in scenarios in which these conditions do not hold, supplier s stands to gain higher profits from entrant manufacturer m_2 through direct sourcing.

Should entrant manufacturer m_2 provide superior after-sales service $s_2 > s_2^{17}$ (illustrated in Figure 8), m_2 garners a pronounced competitive advantage. This enhanced service quality renders m_2 highly lucrative in the eyes of battery supplier s, thereby motivating s to reduce both the wholesale price of batteries and the licensing fee. When the negotiation power of m_2 is relatively weak, supplier s benefits from an elevated sales volume, resulting in higher profits through technology cooperation as opposed to direct sourcing. However, should m_2 possess significant negotiation power, the diminution in wholesale prices and licensing fee is detrimental to battery supplier s's profit.

The proof is given in Appendix B.

Proposition 12

For this proposition, we assume that $\pi_{m_2}^{TT} > \pi_{m_2}^{TS}$, $\pi_{m_1}^{TS} > \pi_{m_1}^{TT}$, and $\pi_s^{TS} > \pi_s^{TT}$.

Proposition 12 posits that in scenarios in which incumbent manufacturer m₁ opts for technology cooperation, entrant manufacturer m₂ should correspondingly adopt a technology cooperation strategy. Entrant manufacturer m₂ must pursue technology cooperation to compete effectively, negotiate for reduced licensing fees, and establish lower selling prices, thereby securing a viable presence in the market. The profits of incumbent manufacturer m₁ and battery suppliers diminish. For m₁, the adoption of a technology cooperation strategy by m₂ erodes its price-competitive edge, leading to reduced profits under the Model TT. This shift in competitive dynamics necessitates a recalibration of m₁'s strategy to maintain its market position. Battery supplier s experiences a decline in profits attributable to the reduction in licensing fees. This observation highlights the intricate interplay between negotiation

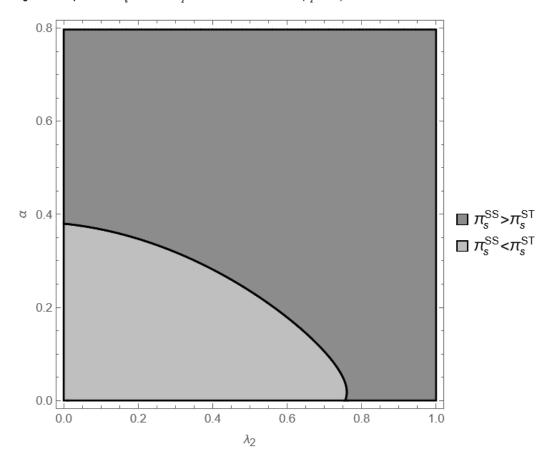


Figure 7. Comparison of π_s with α and λ_2 on model SS and model ST (s₂ = 0.3)

power, technology cooperation strategies, and the resultant profit dynamics among manufacturers and suppliers in the battery supply ecosystem.

Figure 9 illustrates the profits of battery supplier s, incumbent manufacturer m_1 , and entrant manufacturer m_2 under the Model TS and Model TT. We assume that $\theta=0.1$, c=0.14, $s_1=0.04$, and $\lambda_1=0.5$.

Win-Win-Win Dynamics: Collaborative Framework for S, M₁, and M₂

In accordance with proposition 12, when the incumbent manufacturer m_1 bases its battery supply strategy on technology cooperation, the absence of a similar cooperative arrangement between the entrant manufacturer m_2 and the battery supplier s precludes the emergence of a mutually beneficial, or "triple win" scenario among these three entities. This signifies that unless m_2 also engages in technology cooperation with s, the potential synergies and collective gains that could otherwise arise from a coordinated effort remain unrealized.

Figure 10(a) and Figure 10(b) depict the optimal battery supply strategies preferred by the entrant manufacturer m_2 , respectively, under the condition where the incumbent manufacturer m_1 opts for a direct sourcing strategy. Figure 11(a) and Figure 11(b) showcase the scenario of a triple win, wherein mutually beneficial outcomes are achieved by the battery supplier s, the incumbent manufacturer m_1 , and the entrant manufacturer m_2 . We set parameters to c = 0.14, $s_1 = 0.04$, $s_2 = 0.21$, $\lambda_1 = 0.5$,

Figure 8. Comparison of $\pi_{\rm s}$ with α and $\lambda_{\rm 2}$ on model SS and model ST (s $_{\rm 2}~$ = 0.5)

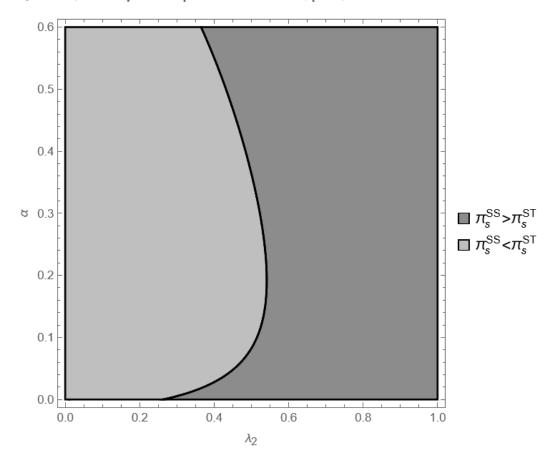
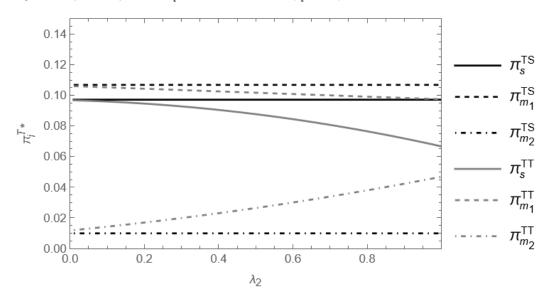


Figure 9. Comparison of profits with $\rm \Lambda_2$ on model TS and model TT (s $_{\rm 2}$ = 0.21)



and $\lambda_2 = 0.2$; we also assume that $\alpha < 0.5$ indicates a low level of brand spillover, with $\alpha = 0.1$, and that $\alpha > 0.5$ indicates a high level of brand spillover, with $\alpha = 0.59$.

Figure 10(a) demonstrates that with low brand spillover and weak brand power, entrant manufacturer m_2 faces challenges competing against incumbent m_1 . In this case, technical cooperation with battery supplier s is advantageous for both m_2 and s; they benefitted from price competition and licensing fees, respectively. As illustrated in Figure 11(a), high-quality after-sales service by m_2 facilitates a triple win through technology cooperation, allowing both manufacturers and s to thrive via market competition.

When brand spillover is high, it boosts m_2 's market appeal and entry. A triple win is possible under specific conditions (the dark gray area in Figure 11[b]) for m_2 , s, and m_1 . However, if brand spillover is too low, technology cooperation offers little competitive edge, intensifying price competition and hindering m_2 's ability to match m_1 . Direct sourcing then leads to higher prices, yet m_1 's competitive dominance remains, unaffected by brand spillover, culminating in a dual benefit when m_2 selects direct sourcing. Conversely, with high brand spillover and power, m_2 's competitive position strengthens, diminishing the spillover effect. Price competition in technology cooperation yields greater benefits than increased spillover, and m_1 gains from lower battery prices, achieving a triple win via technology cooperation with m_2 and s.

CONCLUSION

In this study, we analyzed the optimal battery supply strategies for an entrant manufacturer m₂ in an electric vehicle supply chain involving a battery supplier s and an incumbent manufacturer m₁.

Key Findings and Recommendations

First, we investigated the impact of brand metrics and negotiation power on strategy and profits. Increased brand power and spillover level boost battery and car prices under the model SS. However, these factors don't universally benefit m_2 . High after-sales service by m_2 or low service by m_1 shifts the advantage to m_2 , and brand metrics will benefit m_2 's profit. Technology cooperation by battery suppliers with multiple manufacturers can erode competitors' market shares and profit as their negotiation power rises.

Then, we examined the optimal battery supply strategy for m_2 . When m_1 directly sources batteries, m_2 should opt for direct sourcing if its after-sales service is low and brand spillover is significant. m_2 should pursue technology cooperation if its after-sales service is high. If m_1 employs technology cooperation, m_2 should also adopt this strategy.

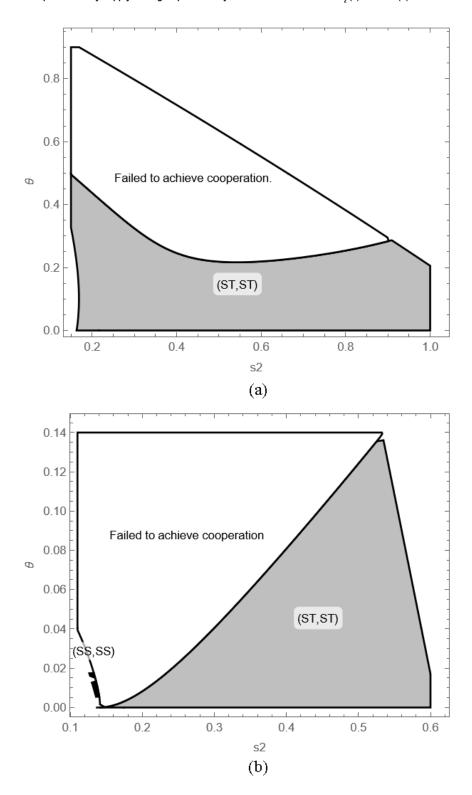
Moreover, we analyzed the triple-win scenarios achieved under varying conditions. We found that weak brand spillover and power, coupled with high after-sales service, prompted a triple win via technology cooperation. Then, high brand power and after-sales service in m₂ also enabled a triple win through technology cooperation. In addition, high brand spillover and low brand power (after-sales service) in m₂ favored a triple win through direct sourcing. However, no triple win occurred when m₁ adopted technology cooperation and m₂ failed to collaborate with battery supplier.

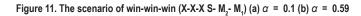
Management Insights

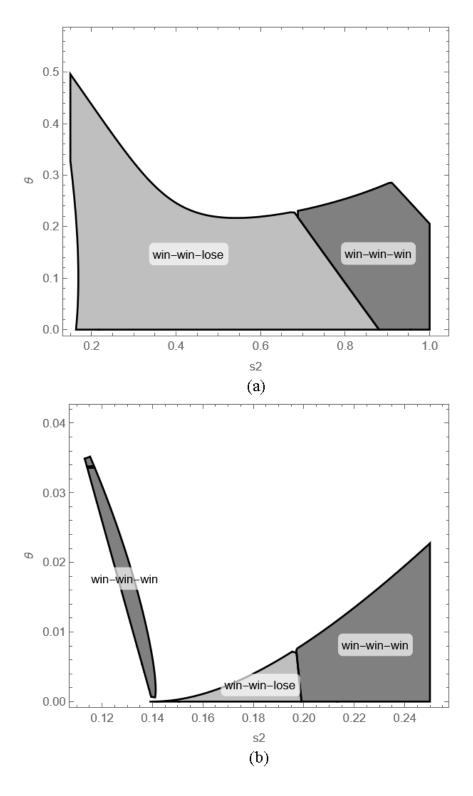
For entrant manufacturers without a technological tie-up with battery supplier, those with distinct brands and low after-sales should secure a procurement deal with a credible battery supplier to leverage brand spillover, expand market reach, and enhance profits. An illustrative example is Xiaomi's strategic move to procure batteries from BYD, leveraging the latter's brand reputation to bolster consumer confidence.

Manufacturers with less brand distinction or high after-sales service should co-produce batteries with a battery supplier. Incumbent manufacturers should aid entrant manufacturers in swiftly

Figure 10. The optimal battery supply strategies preferred by the entrant manufacturer M_2 (a) α = 0.1 (b) α = 0.59







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developing a robust after-sales service system. When entrant manufacturers provide high-quality after-sales services, incumbent manufacturers stand to benefit financially.

If m₁ has a technology partnership with a battery supplier, all entrants should collaborate with a battery supplier, although this may strain existing manufacturer-supplier relationships. Incumbent manufacturers should enhance their negotiation power in technology collaboration. This strategy aims to capture larger market shares while lowering costs. Alternatively, they could consider changing suppliers to preserve exclusive partnerships. Battery suppliers, in turn, should bolster their negotiation power in technology collaboration to limit new entrants' access to cooperative opportunities.

These insights guide entrant manufacturers in crafting effective battery supply strategies based on their brand differentiation, after-sales service levels, and the incumbent's actions.

Limitations and Future Research

It should be noted that in this paper we considered only technology cooperation and direct sourcing for EVM as a battery supply strategy. In practice, electric vehicle manufacturers may also adopt diverse procurement strategies. Exploring these varied procurement strategies of EVMs is an important issue for further research. Additionally, we focused on the brand spillover effect on entrant manufacturers. It might be interesting to investigate how the brand power of battery supplier impacts them.

DATA AVAILABILITY STATEMENTS

All data analyzed during this study are included in this published article and its supplementary information files.

CONFLICT OF INTERESTS

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APPENDIX A

Direct Sourcing—Direct Sourcing (Model SS)

Utilizing inverse induction enables an equilibrium solution to be derived. First, the reaction functions for the retail prices to the incumbent manufacturer and the entrant manufacturer are as follows:

$$p_1(w) = 2 + s_1 - \frac{6 + 2s_1 + s_2 - 3w}{4 + \alpha(-1 + \theta) - \theta}$$

$$p_2(w) = 3 + s_1 + s_2 - w + \alpha - \frac{2(6 + 2s_1 + s_2 - 3w)}{4 + \alpha(-1 + \theta) - \theta} + \theta - \alpha\theta$$

Next, substitute values for $p_1(w)$ and $p_2(w)$ into equation (12). Because $\frac{\partial^2 \pi^{SS}}{\partial w^{SS}} < 0$, π^{SS}_s is concave with respect to w^{SS} ; thus, there exists a unique w^{SS} that maximizes π^{SS}_s . Setting $\frac{\partial \pi^{SS}}{\partial w^{SS}} = 0$ yields the optimal solution for w^{SS} : $w^{SS} = \frac{1}{2} \left(3 + c + s_1 - \frac{2(3 + s_1 - s_2)}{2 + \alpha + \theta - \alpha \theta}\right)$. Lastly, substituting w^{SS} into $p_1(w)$ and p_2^{SS} reveals the optimal solutions for $p_2(w)$ and p_2^{SS} as follows:

$$p_1^{SS} \ = \ 2 + s_1^{} - \tfrac{6 + 2\,s_1^{} + s_2^{} - \frac{3}{2}\left(3 + c + s_1^{} - \frac{2\left(3 + s_1^{} - s_2\right)}{2 + a + \theta - a\theta\right)}\right)}{4 + \alpha(-1 + \theta) - \theta}$$

$$p_2^{SS} = \frac{1}{2}(3 - c + s_1) + s_2 + \alpha - \frac{6 - 3c + 2s_1 + s_2}{4 + \alpha(-1 + \theta) - \theta} + \theta - \alpha\theta$$

The demands for the incumbent manufacturer and the entering manufacturer are as follows:

$$q_1^{SS} = \frac{1}{6} \left(\frac{6 - 3c + 2s_1 + s_2}{4 + \alpha(-1 + \theta) - \theta} + \frac{2(s_1 - s_2)}{(-1 + \alpha)(-1 + \theta)} + \frac{3 + s_1 - s_2}{2 + \alpha + \theta - \alpha \theta} \right)$$

The profits for the battery supplier, incumbent manufacturer, and entering manufacturer are as follows:

$$\pi_{s}^{SS} = \frac{\left(2s_{2} - c(2+\alpha) + c(-1+\alpha)\theta + (3+s_{1})(\alpha+\theta-\alpha\theta)\right)^{2}}{4(-2+\alpha(-1+\theta)-\theta)(\alpha(-1+\theta)-\theta)(4+\alpha(-1+\theta)-\theta)}$$

$$\pi_{m_{_{1}}}^{SS} = \frac{\left(\frac{-8 - 8 s_{_{1}} + 6 s_{_{2}} + 7 \alpha + s_{_{1}} \alpha + \alpha^{2} + s_{_{1}} \alpha^{2} - c(-1 + \alpha)(-2 + \alpha(-1 + \theta) - \theta)(-1 + \theta)}{(-(-1 + \alpha)(7 + s_{_{1}} + 2(1 + s_{_{1}})\alpha)\theta + (1 + s_{_{1}})(-1 + \alpha)^{2}\theta^{2}} + \frac{(4(-1 + \alpha)(4 + \alpha(-1 + \theta) - \theta)^{2}(-1 + \theta)(2 + \alpha + \theta - \alpha\theta)^{2})}{(4(-1 + \alpha)(4 + \alpha(-1 + \theta) - \theta)^{2}(-1 + \theta)(2 + \alpha + \theta - \alpha\theta)^{2})} \right)}$$

$$\boldsymbol{\pi_{m_2}^{SS}} \; = \; - \frac{\left(\left(3\,s_1 + (-1 + \alpha)^2(-1 + \theta)^2\right)(\alpha(-1 + \theta) - \theta) + c(-1 + \alpha)(-2 + \alpha(-1 + \theta) - \theta)(-1 + \theta)\right)^2}{\left(+s_2(2 + 2\alpha(-1 + \theta)^2 - \alpha^2(-1 + \theta)^2 - (-2 + \theta)\theta)\right)^2}{(-1 + \alpha)(\alpha(-1 + \theta) - \theta)(4 + \alpha(-1 + \theta) - \theta)^2(-1 + \theta)(2 + \alpha + \theta - \alpha\theta)^2}\right)^2}$$

Direct Sourcing—Technology Cooperation (Model ST)

By utilizing inverse induction, an equilibrium solution can be derived. First, the reaction functions for the retail prices to the incumbent manufacturer and the entrant manufacturer are as follows:

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$$p_1(w_1, r_2) = -\frac{2+c+r_2+2s_1-s_2+2w_1-2\theta-s_1\theta}{-4+\theta}$$

$$p_2(w_1, r_2) = -\frac{2c + 2r_2 + 2s_2 + \theta - s_1\theta - s_2\theta + w_1\theta - \theta^2}{-4 + \theta}$$

Next, substitute values for $p_1(w_1, r_2)$ and $p_2(w_1, r_2)$ into equation (18) and equation (21) yields $\pi_s(w_1, r_2) = (w_1 - c)q_1 + r_2q_2$ and

$$\ln \pi_{\text{sm.}}(\mathbf{r}_2) = (1 - \lambda_2) \ln(\mathbf{r}_2 \mathbf{q}_2) + \lambda_2 \ln((p_2(w_1, r_2) - c - r_2) \mathbf{q}_2)$$

$$\tfrac{1}{r_{\text{sm}}(r_2)} \tfrac{d\pi_{\text{sm}}(r_2)}{dr_2} = (1-\lambda_2) \tfrac{1}{r_2q_2} \tfrac{dr_2q_2}{dr_2} + \lambda_2 \tfrac{1}{(p_2(w_1,r_2)-c-r_2)q_2} \tfrac{d(p_2(w_1,r_2)-c-r_2)q_2}{dr_2}$$

$$\frac{d\pi_{\text{sm}_1}(\mathbf{r}_2)}{d\mathbf{r}_2} = \pi_{\text{sm}_1}(\mathbf{r}_2)((1-\lambda_2)\frac{1}{\mathbf{r}_2}\frac{d\mathbf{r}_2}{d\mathbf{r}_2} + \lambda_2\frac{1}{(p_2(\mathbf{w}_1,\mathbf{r}_2)-c-\mathbf{r}_2)\mathbf{q}_2}\frac{d(p_2(\mathbf{w}_1,\mathbf{r}_2)-c-\mathbf{r}_2)\mathbf{q}_2}{d\mathbf{r}_2})$$

Solving the simultaneous equations of $\frac{d\pi_{sm_i}(r_2)}{dr_1} = 0$ and $\frac{\partial \pi_i(w_1, r_2)}{\partial w_1} = 0$ gives:

$$\mathbf{W}_{1}^{\mathrm{ST}} \, = \, \frac{ \left(1 + \mathbf{s}_{1} \right) (4 - \theta) (1 - \theta) + c (-2 + \theta) \left(-2 + \theta - \lambda_{2} \right) + \mathbf{s}_{2} (-2 + \theta) \lambda_{2} + \theta \left(-1 + s_{1} + \theta \right) \lambda_{2} }{8 + \theta \left(-9 + 2\theta + \lambda_{2} \right)}$$

$$\mathbf{r}_{2}^{\text{ST}} = \frac{\left(-8\,\mathbf{s}_{2} - 6\theta + c(-1 + \theta)(-8 + 3\theta) + \theta\left(9\,\mathbf{s}_{2} - \mathbf{s}_{1}(-2 + \theta) + 8\theta - 2\theta\left(\mathbf{s}_{2} + \theta\right)\right)\right)\left(-1 + \lambda_{2}\right)}{2(8 + \theta(-9 + 2\theta + \lambda_{2})}$$

Finally, substituting w_1^{ST} and r_2^{ST} into $p_1^{ST}(w_1, r_2)$ and $p_2^{ST}(w_1, r_2)$ yields the optimal solutions for p_1^{ST} and p_2^{ST} as follows:

$$p_{1}^{ST} = \frac{\begin{pmatrix} -48 - 48\,s_{1} + 8\,s_{2} + 82\theta - 9\,s_{2}\theta + 2\left(-20 + s_{2}\right)\theta^{2} + 6\,\theta^{3} + 2(-3 + \theta)(-1 + \theta)\theta\lambda_{2} \\ + s_{2}(16 + \theta(-11 + 2\theta))\lambda_{2} + s_{1}\theta\left(74 - 10\lambda_{2} + \theta\left(-31 + 4\theta + 3\lambda_{2}\right)\right) \\ - c\left(8\left(3 + 2\lambda_{2}\right) + \theta\left(-23 - 13\lambda_{2} + \theta\left(5 + 3\lambda_{2}\right)\right)\right)}{2\left(-4 + \theta\right)\left(8 + \theta\left(-9 + 2\theta + \lambda_{2}\right)\right)}$$

$$p_2^{ST} = \frac{\binom{(8s_2 + s_1(-2 + \theta)\theta + 2(-3 + \theta)(-1 + \theta)\theta + s_2\theta(-9 + 2\theta))(-3 + \theta + \lambda_2)}{\binom{-c(8(1 + \lambda_2) + \theta(-3 - 7\lambda_2 + \theta(-3 + \theta + 2\lambda_2)))}{(-4 + \theta)(8 + \theta(-9 + 2\theta + \lambda_2))}}$$

The demands for the incumbent manufacturer and the entering manufacturer are as follows:

$$q_1^{ST} = -\frac{-2 + c + s_2 + s_1(-2 + \theta) + 2\theta - c\theta}{2(-4 + \theta)(-1 + \theta)}$$

$$q_2^{\rm ST} \ = \ -\frac{(-2+\theta) \left(-8c+8s_2+\left(6+11c-2s_1-9s_2\right)\theta+\left(-8-3c+s_1+2s_2\right)\theta^2+2\theta^3\right) \left(1+\lambda_2\right)}{2(-4+\theta)(-1+\theta)\theta \left(8+\theta \left(-9+2\theta+\lambda_2\right)\right)}$$

The profits for the battery supplier, incumbent manufacturer, and entering manufacturer are as follows:

$$\pi_{s}^{ST} = \frac{\begin{pmatrix} \frac{1}{\theta}(-2+\theta)\left(-8c+8s_{2}+\left(6+11c-2s_{1}-9s_{2}\right)\theta+\left(-8-3c+s1+2s_{2}\right)\theta^{2}+2\theta^{3}\right)^{2}(-1+\lambda_{2})}{\left(1+\lambda_{2}\right)-2\left(-2+c+s_{2}+s1(-2+\theta)+2\theta-c\theta\right)\left(\left(1-c+s_{1}\right)(-4+\theta)(-1+\theta)+\left(s_{2}(-2+\theta)\right)\right)}{4\left(-4+\theta\right)\left(-1+\theta\right)\left(8+\theta\left(-9+2\theta+\lambda_{2}\right)\right)^{2}}$$

$$\pi_{m_1}^{ST} = -\frac{(-2+c+s_2+s_1(-2+\theta)+2\theta-c\theta)^2}{4(-4+\theta)^2(-1+\theta)}$$

$$\boldsymbol{\pi}_{m_{_{2}}}^{ST} \ = \ -\frac{(-2+\theta)^{2} \left(-8c+8\,s_{_{2}}+\left(6+11c-2\,s_{_{1}}-9\,s_{_{2}}\right)\theta+\left(-8-3c+s_{_{1}}+2\,s_{_{2}}\right)\theta^{2}+2\,\theta^{3}\right)^{2} \left(1+\lambda_{_{2}}\right)^{2}}{4\left(-4+\theta\right)^{2} \left(-1+\theta\right)\theta\left(8+\theta\left(-9+2\theta+\lambda_{_{2}}\right)\right)^{2}}$$

Technology Cooperation—Direct Sourcing (Model TS)

Utilizing inverse induction enables an equilibrium solution to be derived. First, the reaction functions for the retail prices to the incumbent manufacturer and the entrant manufacturer are as follows:

$$p_1(w_2, r_1) = 2 + s_1 + \frac{6 - 2c - 2r_1 + 2s_1 + s_2 - w_2}{-4 + \theta}$$

$$p_2(w_2, r_1) = \frac{-2(s_2 + w_2) + (-1 - c - r_1 + s_1 + s_2)\theta + \theta^2}{-4 + \theta}$$

Next, substitute values for $p_1(w_2, r_1)$ and $p_2(w_2, r_1)$ into equation (19) and equation (22) yields $\pi_s(w_2, r_1) = r_1 q_1 + (w_2 - c) q_2$ and $\ln \pi_{sm.}(r_1) = (1 - \lambda_1) \ln(r_1 q_1) + \lambda_1 \ln((p_1(w_2, r_1) - c - r_1) q_1)$

$$\frac{1}{\pi_{-}(\mathbf{r}_{1})} \frac{d\pi_{m_{-}}(\mathbf{r}_{1})}{d\mathbf{r}_{1}} = (1 - \lambda_{1}) \frac{1}{\mathbf{r}_{1}} \frac{d\mathbf{r}_{1}\mathbf{q}_{1}}{d\mathbf{r}_{1}} + \lambda_{1} \frac{1}{(\mathbf{p}_{1}(w_{2}, \mathbf{r}_{1}) - c - \mathbf{r}_{1})\mathbf{q}_{1}} \frac{d(\mathbf{p}_{1}(w_{2}, \mathbf{r}_{1}) - c - \mathbf{r}_{1})\mathbf{q}_{1}}{d\mathbf{r}_{1}}$$

$$\frac{d\pi_{\text{sm}}(\mathbf{r}_1)}{d\mathbf{r}_2} = \pi_{\text{sm}}(\mathbf{r}_1)((1-\lambda_1)\frac{1}{\mathbf{r}_1\mathbf{q}_1}\frac{d\mathbf{r}_1\mathbf{q}_1}{d\mathbf{r}_1} + \lambda_1\frac{1}{(\mathbf{p}_1(\mathbf{w}_2,\mathbf{r}_1)-c-\mathbf{r}_1)\mathbf{q}_1}\frac{d(\mathbf{p}_1(\mathbf{w}_2,\mathbf{r}_1)-c-\mathbf{r}_1)\mathbf{q}_1}{d\mathbf{r}_1})$$

Solving the simultaneous equations of $\frac{d\pi_{un}(r_i)}{dr_i} = 0$ and $\frac{\partial \pi_i(w_2, r_i)}{\partial w_i} = 0$ gives:

$$\mathbf{w}_{1}^{\mathrm{TS}} \, = \, \frac{(-4+\theta)(-1+\theta)(s2+\theta) - c(-2+\theta)\left(2+\theta\left(-1+\lambda_{_{1}}\right)\right) + \theta\left(-2+s_{_{2}}+s_{_{1}}(-2+\theta) + 2\theta\right)\lambda_{_{1}}}{8+\theta\left(-9+2\theta+\lambda_{_{1}}\right)}$$

$$\Gamma_2^{\text{ST}} = \frac{\left(-8 - 8\,s_1 + 2\,s_2 + 2c(-3 + \theta)(-1 + \theta) + 11\theta - \theta\left(s_2 + 3\theta + s_1(-9 + 2\theta)\right)\right)\left(-1 + \lambda_1\right)}{2(8 + \theta(-9 + 2\theta + \lambda_1))}$$

Finally, substituting $\mathbf{w}_1^{\mathrm{TS}}$ and $\mathbf{r}_2^{\mathrm{TS}}$ into $p_1^{\mathrm{TS}}(w_2, r_1)$ and $p_2^{\mathrm{TS}}(w_2, r_1)$ yields the optimal solutions for p_1^{TS} and p_2^{TS} as follows:

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$$\begin{aligned} \mathbf{p}_{1}^{\text{ST}} &= \\ \left(8 - 2s_{2} - 11\theta + \theta(s_{2} + 3\theta) + s_{1}(8 + \theta(-9 + 2\theta))\right)(-3 + \theta + \lambda_{1}) - c(14 + 6\lambda_{1} + \theta(-14 + 3\theta + (-4 + \theta)\lambda_{1}))/(-4 + \theta)(8 + \theta(-9 + 2\theta + \lambda_{1})) \end{aligned}$$

$$\begin{split} \mathbf{p}_{2}^{\text{ST}} &= \\ & \left(-48\,s_{2} + (-1 + \theta)\theta \left(40 + 4\,\theta^{2} + 5\theta \left(-5 + \lambda_{1} \right) - 16\,\lambda_{1} \right) + s_{2}\theta \left(74 - 10\,\lambda_{1} + \theta \left(-31 + 4\theta + 3\,\lambda_{1} \right) \right) + s_{1}\theta \left(8 + 16\,\lambda_{1} + \theta \left(-9 - 11\,\lambda_{1} + 2\theta \left(1 + \lambda_{1} \right) \right) \right) - 2c\left(8 + \theta \left(-3 + 7\,\lambda_{1} + \theta \left(-3 + \theta + \left(-5 + \theta \right)\lambda_{1} \right) \right) \right) \\ & \right) / 2(-4 + \theta)\left(8 + \theta \left(-9 + 2\theta + \lambda_{1} \right) \right) \end{split}$$

The demands for the incumbent manufacturer and the entering manufacturer are as follows:

$$q_{1}^{\text{ST}} \, = \, - \frac{(-2 + \theta) \left(-8 - 8 \, s_{_{1}} + 2 \, s_{_{2}} + 2 \, c (-3 + \theta) (-1 + \theta) + 11 \theta - \theta \left(s_{_{2}} + 3 \theta + s_{_{1}} (-9 + 2 \theta)\right)\right) \left(1 + \lambda_{_{1}}\right)}{2 (-4 + \theta) (-1 + \theta) \theta \left(8 + \theta \left(-9 + 2 \theta + \lambda_{_{1}}\right)\right)}$$

$$q_2^{\text{ST}} = \frac{2s_2 + 2c(-1+\theta) + \theta - \theta(s_1 + s_2 + \theta)}{2(-4+\theta)(-1+\theta)\theta}$$

The profits for the battery supplier, incumbent manufacturer, and entering manufacturer are as follows:

$$\begin{split} \pi_{s}^{\text{TS}} &= \\ &(-2+\theta)\left(8+8s_{1}-2s_{2}-2c(-3+\theta)(-1+\theta)-11\theta+\theta\left(s_{2}+3\theta+s_{1}(-9+2\theta)\right)\right)^{2} \\ &(-1+\lambda_{1})\left(1+\lambda_{1}\right)-\frac{1}{\theta}2\left(s_{2}(-2+\theta)-2c(-1+\theta)+\theta\left(-1+s_{1}+\theta\right)\right)((-4+\theta)(-1+\theta)\left(-c+s_{2}+\theta\right)+\theta(-2+c+s_{2}+s_{1}(-2+\theta)+2\theta-c\theta)\lambda_{1}\right)\left(8+\theta\left(-9+2\theta+\lambda_{1}\right)\right)/(4(-4+\theta)(-1+\theta)\left(8+\theta\left(-9+2\theta+\lambda_{1}\right)\right)^{2}) \end{split}$$

$$\pi_{\mathbf{m_{i}}}^{\mathrm{TS}} = -\frac{(-2+\theta)^{2} \left(8-6c+8 \, s_{1}-2 \, s_{2}+\left(-11+8 c-9 \, s_{1}+s_{2}\right) \theta+\left(3-2 c+2 \, s_{1}\right) \theta^{2}\right)^{2} \left(1+\lambda_{1}\right)^{2}}{4 \left(-4+\theta\right)^{2} (-1+\theta) \left(8+\theta \left(-9+2 \theta+\lambda_{1}\right)\right)^{2}}$$

$$\pi_{\mathrm{m}_{2}}^{\mathrm{TS}} = -\frac{\left(s_{2}(-2+\theta)-2c(-1+\theta)+\theta(-1+s_{1}+\theta)\right)^{2}}{4(-4+\theta)^{2}(-1+\theta)\theta}$$

Technology Cooperation—Technology Cooperation (Model TT)

Utilizing inverse induction enables an equilibrium solution to be derived. First, the reaction functions for the retail prices to the incumbent manufacturer and the entrant manufacturer are as follows:

$$p_1(r_1, r_2) = 2 + s_1 + \frac{6 - 3c - 2r_1 - r_2 + 2s_1 + s_2}{-4 + \theta}$$

$$p_2(r_1, r_2) = \frac{-2(c + r_2 + s_2) + (-1 - c - r_1 + s_1 + s_2)\theta + \theta^2}{-4 + \theta}$$

Next, substitute values for $p_1(r_1, r_2)$ and $p_2(r_1, r_2)$ into equation (14) and equation (18) yields

$$\ln \pi_{\text{sm.}}(\mathbf{r}_{_{1}}) = (1 - \lambda_{_{1}}) \ln(\mathbf{r}_{_{1}}\mathbf{q}_{_{1}}) + \lambda_{_{1}} \ln((\mathbf{p}_{_{1}}(\mathbf{r}_{_{1}},\mathbf{r}_{_{2}}) - c - \mathbf{r}_{_{1}})\mathbf{q}_{_{1}})$$

and

$$\ln \pi_{\rm sm}({\bf r}_2) = (1 - \lambda_2) \ln ({\bf r}_2 {\bf q}_2) + \lambda_2 \ln ((p_2(r_1, r_2) - c - r_2) {\bf q}_2)$$

$$\frac{1}{\pi_{\text{un}}(\mathbf{r}_{1})} \frac{d\pi_{\text{un}}(\mathbf{r}_{1})}{d\mathbf{r}_{2}} = (1 - \lambda_{1}) \frac{1}{\mathbf{r}_{1}} \frac{d\mathbf{r}_{1}\mathbf{q}_{1}}{d\mathbf{r}_{1}} + \lambda_{1} \frac{1}{(\mathbf{p}_{1}(\mathbf{r}_{1},\mathbf{r}_{2}) - c - \mathbf{r}_{1})\mathbf{q}_{1}} \frac{d(\mathbf{p}_{1}(\mathbf{r}_{1},\mathbf{r}_{2}) - c - \mathbf{r}_{1})\mathbf{q}_{1}}{d\mathbf{r}_{1}}$$

$$\frac{1}{\pi_{\text{sm}}(r_2)} \frac{d\pi_{\text{sm}}(r_2)}{dr_2} = (1 - \lambda_2) \frac{1}{r_2 q_2} \frac{dr_2 q_2}{dr_2} + \lambda_2 \frac{1}{(p_2(r_1, r_2) - c - r_2) q_2} \frac{d(p_2(r_1, r_2) - c - r_2) q_2}{dr_2}$$

$$\frac{d\pi_{\text{cm}}(\mathbf{r}_1)}{d\mathbf{r}_1} = \pi_{\text{sm}}(\mathbf{r}_1)((1-\lambda_1)\frac{1}{\mathbf{r}_1\mathbf{q}_1}\frac{d\mathbf{r}_1\mathbf{q}_1}{d\mathbf{r}_1} + \lambda_1\frac{1}{(\mathbf{p}_1(\mathbf{r}_1,\mathbf{r}_1)-c-\mathbf{r}_1)\mathbf{q}_1}\frac{d(\mathbf{p}_1(\mathbf{r}_1,\mathbf{r}_2)-c-\mathbf{r}_1)\mathbf{q}_1}{d\mathbf{r}_1}$$

$$\frac{d\pi_{\text{sm}}(r_2)}{dr_1} = \pi_{\text{sm}}(r_2)((1-\lambda_2)\frac{1}{r_2q_2}\frac{dr_2q_2}{dr_1} + \lambda_2\frac{1}{(p,(r_1,r_2)-c-r_2)q_1}\frac{d(p_2(r_1,r_2)-c-r_2)q_2}{dr_1})$$

Solving the simultaneous equations of $\frac{d\pi_{m_i}(r_i)}{dr_i} = 0$ and $\frac{d\pi_{m_i}(r_2)}{dr_2} = 0$ gives:

$$\begin{split} \mathbf{r}_{1}^{\text{TT}} &= \\ &- \left(-1 + \lambda_{1} \right) (8 + 8 \, s_{1} - 2 \, s_{2} - 11\theta + \theta \left(s_{2} + 3\theta \right) + s_{2} (-2 + \theta) \, \lambda_{2} + (-1 + \theta) \theta \, \lambda_{2} - 2c (-1 + \theta) \\ &(-3 + \theta + \lambda_{2}) + s_{1} \theta \left(-9 + 2\theta + \lambda_{2} \right)) / 16 + \theta \left(-17 + 4\theta + \lambda_{1} + \lambda_{2} - \lambda_{1} \lambda_{2} \right) \end{split}$$

$$\begin{aligned} \mathbf{r}_{2}^{\text{TT}} &= \\ &- (8 \, s_2 \, s_1 (-2 + \theta) \theta \left(1 + \lambda_1\right) + 2 (-1 + \theta) \theta \left(-3 + \theta + \lambda_1\right) + s_2 \theta \left(-9 + 2\theta + \lambda_1\right) - c (-1 + \theta) (-8 + \theta) \\ &\left(3 + \lambda_1\right) \left(-1 + \lambda_2\right) / (16 + \theta \left(-17 + 4\theta + \lambda_1 + \lambda_2 - \lambda_1 \lambda_2\right)) \end{aligned}$$

Finally, substituting \mathbf{r}_1^{TT} and \mathbf{r}_2^{TT} into $p_1^{\text{TT}}(r_1, r_2)$ and $p_2^{\text{TT}}(r_1, r_2)$ yields the optimal solutions for p_1^{TT} and p_2^{TT} as follows:

$$\begin{split} \mathbf{p}_{1}^{\text{TT}} &= \\ &(-c(-2+\theta)\left(2\left(-7+\lambda_{1}\left(-3+\lambda_{2}\right)-3\,\lambda_{2}\right)+\theta\left(5+3\,\lambda_{2}+\lambda_{1}\left(3+\lambda_{2}\right)\right)\right)+2\\ &\left(-3+\theta+\lambda_{1}\right)\left(s_{2}(-2+\theta)\left(1+\lambda_{2}\right)+\left(1-\theta\right)\left(8-\theta\left(3+\lambda_{2}\right)\right)+s_{1}\left(8-\theta\left(9-2\theta-\lambda_{2}\right)\right)\right))/(-4+\theta)\\ &(16+\theta(-17+4\theta+\lambda_{1}+\lambda_{2}-\lambda_{1}\lambda_{2})) \end{split}$$

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$$\begin{array}{l} \mathbf{p}_{2}^{\text{TT}} = \\ (2 \left(8 \, \mathbf{s}_{2} + \mathbf{s}_{1} (2 - \theta) \theta (1 + \lambda_{1}) \, \lambda_{1} + 2 (1 - \theta) \theta (3 - \theta - \lambda_{1}) + \mathbf{s}_{2} \, \theta (9 - 2 \theta - \lambda_{1}) \right) (3 - \theta - \lambda_{2}) \,) + c \\ (2 - \theta) \left(-8 \left(1 + \lambda_{2} \right) + \theta \left(-3 - 5 \, \lambda_{1} + 2 \theta \left(1 + \lambda_{1} \right) + 5 \, \lambda_{2} + 3 \, \lambda_{1} \, \lambda_{2} \right) \right) / (-4 + \theta) \left(16 + \theta - 17 + 4 \theta + \lambda_{1} + \lambda_{2} - \lambda_{1} \, \lambda_{2} \right) \end{array}$$

The demands for the incumbent manufacturer and the entering manufacturer are as follows:

$$\begin{split} q_1^{TT} &= \\ &-((-2+\theta)\left(1+\lambda_1\right)(8+8s_1-2s_2-11\theta+\theta(s2+3\theta)+s_2(-2+\theta)\lambda_2+(-1+\theta)\theta\lambda_2-2c_2(-1+\theta)\left(-3+\theta+\lambda_2\right)+s_1\theta\left(-9+2\theta+\lambda_2\right))/(-4+\theta)(-1+\theta)\left(16+\theta\left(-17+4\theta+\lambda_1+\lambda_2-\lambda_1\lambda_2\right)\right)) \end{split}$$

$$\begin{split} q_2^{\text{TT}} &= \\ (2 - \theta)8\,\mathbf{s}_2 - \mathbf{s}_1(2 - \theta)\theta\big(1 + \lambda_1\big)\,\lambda_1 + 2(1 - \theta)\theta\big(3 - \theta - \lambda_1\big) - \mathbf{s}_2\theta\big(9 - 2\theta - \lambda_1\big) - c(1 - \theta)(-8 + \theta)(3 + \lambda_1\big)\,\big(1 + \lambda_2\big)/(-4 + \theta)(-1 + \theta)\big(16 + \theta\big(-17 + 4\theta + \lambda_1 + \lambda_2 - \lambda_1\lambda_2\big)\big) \end{split}$$

The profits for the battery supplier, incumbent manufacturer, and entering manufacturer are as follows:

$$\begin{split} \pi_{s}^{TT} &= \\ &(2-\theta) \left(\left(8 \operatorname{s}_{2} - \operatorname{s}_{1} (2-\theta) \theta (1+\lambda_{1}) \lambda_{1} + 2 (1-\theta) \theta (3-\theta-\lambda_{1}) - \operatorname{s}_{2} \theta (9-2\theta-\lambda_{1}) + c (1-\theta) (8-\theta(3+\lambda_{1})) \right)^{2} (1-\lambda_{2}) (1+\lambda_{2}) + \left(8 + 8 \operatorname{s}_{1} - 2 \operatorname{s}_{2} - 11 \theta + \theta (s2+3\theta) + \operatorname{s}_{2} (-2+\theta) \lambda_{2} - (1-\theta) (\theta \lambda_{2} - 2c(1+\theta)(3-\theta-\lambda_{2}) + \operatorname{s}_{1} \theta (-9+2\theta+\lambda_{2}) \right)^{2} (-1+\lambda_{1}) (1+\lambda_{1})/(4-\theta)(1-\theta) \left(16 + \theta (-17+4\theta+\lambda_{1}+\lambda_{2}-\lambda_{1}\lambda_{2}) \right)^{2} \end{split}$$

$$\begin{split} \pi_{\mathbf{m_{1}}}^{\mathrm{TS}} &= \\ &-(-2+\theta)^{2} \left(8+8 \, s_{1}-2 \, s_{2}-11 \theta+\theta (s2+3 \theta)+s_{2} (-2+\theta) \, \lambda_{2}+(-1+\theta) \theta \, \lambda_{2}-2 c (-1+\theta) (-3+\theta+\lambda_{2})+s_{1} \theta \left(-9+2 \theta+\lambda_{2}\right)\right)^{2} \left(1+\lambda_{1}\right)^{2} / (-4+\theta)^{2} (-1+\theta) \left(16+\theta \left(-17+4 \theta+\lambda_{1}+\lambda_{2}-\lambda_{1} \lambda_{2}\right)\right)^{2} \end{split}$$

$$\begin{split} \pi_{m_2}^{TS} &= \\ -(-2+\theta)^2 (8\,s_2 - s_1(2-\theta)\theta(1+\lambda_1)\,\lambda_1 + 2(1-\theta)\theta(3-\theta-\lambda_1) - s_2\theta(9-2\theta-\lambda_1) - c(1-\theta) \\ \left(-8+\theta(3+\lambda_1)\right)^2 (1+\lambda_2)^2 / (-4+\theta)^2 (-1+\theta)\left(16+\theta(-17+4\theta+\lambda_1+\lambda_2-\lambda_1\lambda_2)\right)^2 \theta \end{split}$$

APPENDIX B

PROOF OF PROPOSITION 1

For this proposition we assume that

$$\frac{\partial q_{ss}^{ss}}{\partial \theta} = \frac{1}{6} \left(-\frac{(6-3c+2s_1+s_2)(-1+\alpha)}{(4+\alpha(-1+\theta)-\theta)^2} - \frac{2(s_1-s_2)}{(-1+\alpha)(-1+\theta)^2} + \frac{(3+s_1-s_2)(-1+\alpha)}{(2+\alpha+\theta-\alpha\theta)^2} \right), \frac{\partial^2 q_{ss}^{ss}}{\partial \theta s_s^{s}} > 0$$

Setting
$$\frac{\partial q_1^{ss}}{\partial \theta} = 0$$
 yields

$$\begin{split} s_1^0 &= \\ &- (18s2(-2-2\alpha(1-\theta)^2+\alpha^2(1-\theta)^2-(2-\theta)\theta)-c(1-\alpha)^2(-1+\theta)^2(2+\alpha+\theta-\alpha\theta)^2 + \\ &(1-\alpha)^2(1-\theta)^2(8-\alpha^2(1-\theta)^2+2\alpha(1-\theta)(8+\theta)+\theta(16+\theta)))/(40+\alpha^4(-1+\theta)^4-2\alpha^3) \\ &(-1+\theta)^3(1+2\theta)+3\alpha^2(-1+\theta)^2(-7+2\theta(1+\theta))-2\alpha(-1+\theta)^2(-16+\theta(5+2\theta))+\theta(32+\theta)(-21+\theta(2+\theta)))) \end{split}$$

Thus, when $s_1 \in (0, s_1^0), \frac{\partial q_1^{ss}}{\partial \theta} < 0$; when $s_1 \in (s_1^0, 1), \frac{\partial q_1^{ss}}{\partial \theta} > 0$. The proof process of $\frac{\partial q_1^{ss}}{\partial \theta}, \frac{\partial q_2^{ss}}{\partial \theta}$ and $\frac{\partial q_2^{ss}}{\partial \theta}$ is similar to that of proposition 1, and

$$\begin{split} s_1^1 &= \\ &-(2c\,(-1+\alpha)^2(2+\alpha(-1+\theta)-\theta)\,(-1+\theta)^2(2+\alpha+\theta-\alpha\theta)^2 + (-1+\alpha)^2(-1+\theta)^2 \\ &(\alpha+\theta-\alpha\theta)^2(10-2\alpha(-1+\theta)^2+\alpha^2(-1+\theta)^2 + (-2+\theta)\theta) - s2(16-2\,\alpha^5(1-\theta)^5 + 2\,\alpha^2 \\ &(1-\theta)^3(1+2\theta)(11-5\theta) + \alpha^4(1-\theta)^4(-9+10\theta) - 4\,\alpha^3(-1+\theta)^3(1+\theta(-9+5\theta)) + \theta(24+\theta)^2 \\ &(22+(4-\theta)\theta(1-2\theta))) + 2\alpha(1-\theta)\big(12+\theta(22+\theta(6+\theta(-18+5\theta)))\big))/9\,(\alpha+\theta-\alpha\theta)^2 \\ &(-2-2\alpha(-1+\theta)^2+\alpha^2(-1+\theta)^2 + (-2+\theta)\theta) \end{split}$$

The proof process of propositions 2–8 is similar to that of proposition 1.

PROOF OF PROPOSITION 2

The values of s_1^2 and s_1^3 are as follows:

$$s_1^2 = -3 + c - \frac{2(c - s2)}{\alpha + \theta - \alpha\theta} - \frac{8(c - s2)(-1 + \alpha)(-1 + \theta)}{8 + (\alpha + \theta - \alpha\theta)^2}$$

$$s^{3} =$$

$$\frac{1}{528}(\underbrace{\$11}_{(-135}(-135+s_2(22-8\alpha)-4\alpha+c(26+8\alpha))}_{\theta+\frac{1}{\alpha+\theta-\alpha\theta}} + \underbrace{(27(156-40c+40s_2+183\alpha-74c\alpha+74s_2\alpha+(183-74c+74s_2)(1-\alpha))}_{\theta+\frac{1}{\alpha+\theta-\alpha\theta}} + \underbrace{(27(156-40c+40s_2+183\alpha-74c\alpha+74s_2\alpha+(183-74c+74s_2)(1-\alpha))}_{\theta+\frac{1}{\alpha+\theta-\alpha\theta}} + \underbrace{(27(156-40c+40s_2+183\alpha-74c\alpha+74s_2\alpha+(183-74c+74s_2)(1-\alpha))}_{\theta+\frac{1}{\alpha+\theta-\alpha\theta}} + \underbrace{(27(156-40c+40s_2+183\alpha-74c\alpha+74s_2)(1-\alpha)}_{\theta+\frac{1}{\alpha+\theta-\alpha\theta}} + \underbrace{(27(156-40c+40s_2+183\alpha-74c\alpha+74c\alpha+74s_2)}_{\theta+\frac{1}{\alpha+\theta-\alpha\theta}} + \underbrace{(27(156-40c+40s_2+183\alpha-74c\alpha+74c\alpha+74s_2)}_{\theta+\frac{1}{\alpha+\theta-\alpha\theta}} + \underbrace{(27(156-40c+40s_2+183\alpha-74c\alpha+74c\alpha+74s_2)}_{\theta+\frac{1}{\alpha+\theta-\alpha\theta}}$$

The values of s_2^0 , s_2^1 , s_2^2 , s_2^3 , s_2^4 and s_1^4 are as follows:

$$\begin{split} s_2^0 &= \\ (4 + s_1(4 - \theta^2)(1 - \lambda_2) + 12\lambda_2 - \theta(\theta + 16\lambda_2 - \theta(6 - \lambda_2)\lambda_2) - c(4 - \theta^2(1 - 3\lambda_2) - 8\theta\lambda_2 + 2(3 - \lambda_2)\lambda_2)(2\lambda_2(-5 - (-4 + \theta)\theta + \lambda_2))/(2\lambda_2(-5 - (-4 + \theta)\theta + \lambda_2))) \end{split}$$

$$s_2^1 \; = \; \frac{s_1 \left(16 - \theta \left(16 - \theta \left(5 - \lambda_2\right)\right)\right) - 4(2 - \theta) \left(6 - \theta \left(13 - \theta \left(7 - \theta - \lambda_2\right)\right)\right) - c(8\left(2 + \lambda_2\right) - \theta(16 - \theta \left(5 - 3\lambda_2\right)))}{2(-4 + \theta^2)\lambda_2}$$

$$\begin{split} s_2^2 &= \\ &(-128 \left(4+3 \lambda_2\right)+c (320-496 \theta+315 \theta^2-92 \theta^3+10 \theta^4-2 (3-\theta) (32-\theta (40-\theta (17-3 \theta)) \lambda_2-(64+(-32+\theta) \theta) \lambda v^2)+2 \theta (464+304 \lambda_2+\theta-327-13 \left(14-\lambda_2\right) \lambda_2-\theta^2 \left(11+\left(6-\lambda_2\right) \lambda_2\right)+\theta (100+52 \lambda_2-8 \lambda_2^2))-2 s_1 \left(64 \left(2-\lambda_2\right)-\theta \left(176-48 \lambda_2-\theta \left(101-\left(6-\lambda_2\right) \lambda_2-\theta^2 \left(3+\lambda_2\right)+4 \theta (7+\lambda_2)\right)\right)\right))/((8+\theta (-9+2 \theta))^2+2 (160+\theta (-200+\theta (97+2 (-11+\theta) \theta)))) \lambda_2-(-8+\theta) (-8+3 \theta) \lambda_2^2) \end{split}$$

$$\begin{split} s_2^3 &= \\ &(64s_2\lambda_2 - 192\left(-3 + s_1 + 10\theta + \lambda_2\right) + 2\theta(\theta\left(1179 + \theta\left(-708 + \theta(221 + 2(-17 + \theta)\theta\right)\right)) \\ &+ 2(-4 + \theta)(-32 + \theta(26 - (10 - \theta)\theta))\lambda_2 + \theta(13 + (-8 + \theta)\theta)\lambda_2^{-2}) - s_1\theta(64 \\ &(-5 + \lambda_2) + \theta(214 - 64\theta - 2\left(12 - \lambda_2\right)\lambda_2 + \theta^2(7 + \lambda_2))) + c(-20\theta^3\left(5 + \lambda_2\right) + \theta^4\left(11 + 3\lambda_2\right) + 16\theta \\ &(-29 + \left(-8 + \lambda_2\right)\lambda_2\right) - 32\left(-8 + \left(-3 + \lambda_2\right)\lambda_2\right) + \theta^2\left(327 + \lambda_2\left(64 + \lambda_2\right)\right)))/\left((8 - \theta(9 - 2\theta))^2 + 2(2 - \theta)(40 - \theta(28 - (8 - \theta)\theta))\lambda_2 - (32 - (16 - \theta)\theta)\lambda_2^{-2}\right) \end{split}$$

$$s_2^4 = \frac{-2(-1+\theta)^2 + c(-1+\theta)^2 - s1(6+(-4+\theta)\theta)}{-5+2\theta}$$

$$\begin{split} s_1^4 &= \\ & \left(-s_2(8 - \theta(9 - 2\theta))^2(8 - \theta(20 - \theta(11 - 2\theta))) + s_2\theta \left(128 + \theta(344 - \theta(324 - \theta(135 + 2(13 - \theta)\theta)) \right) \right) \lambda_2 + 2(1 - \theta)^2\theta^2 \left(8(-13 + 3\lambda_2) + \theta\left(140 - 12\lambda_2 + \theta(-77 - 2(-10 + \theta)\theta + \lambda_2) \right) \right) + c(-1 + \theta)^2 \\ & \left(-512 + \theta\left(-128(-11 + \lambda_2) + \theta\left(8(-167 + 13\lambda_2) + \theta\left(604 - 28\lambda_2 + 3\theta(-45 + 4\theta + \lambda_2) \right) \right) \right) \right) \right) / \left((-2 + \theta)\theta^2 \left(88 - 8\lambda_2 + \theta\left(8(-21 + 2\lambda_2) + \theta(-6(-19 + \lambda_2) + \theta(-35 + 4\theta + \lambda_2)) \right) \right) \right) \end{split}$$

PROOF OF PROPOSITION 4

The values of s_2^5 and s_1^5 re as follows:

$$s_2^5 = -2 + c + \frac{8 - 4c + 4s_1}{6 - 3\theta} + \frac{1}{3}(-2 + c - s_1)\theta$$

The values of s_2^6 , s_2^7 , s_2^8 , s_2^9 and s_1^6 are as follows:

$$\begin{split} s_2^6 &= \\ &(4c\left(1+3\,\lambda_1\right)-16\left(-2+\lambda_1+s_1\lambda_1\right)-c\theta\left(\theta+16\,\lambda_1+\theta\left(-6+\lambda_1\right)\lambda_1\right)+\theta(-80+16\left(2+s_1\right)\lambda_1+\theta\left(61+2\,\theta^2+2\theta\left(-9+\lambda_1\right)+\lambda_1\left(-19-5\,s_1+\left(2+s_1\right)\lambda_1\right)\right))/\left((-4+\theta^2)\left(-1+\lambda_1\right)\right) \end{split}$$

$$\begin{split} s_2^7 &= \\ (\theta(464 - \theta(327 - (100 - 11\theta)\theta)) + (4 - \theta)\theta(32 - \theta(8 - 3\theta))\lambda_1 + (32 - \theta(16 + \theta))\lambda_1^2 - 32 \\ (8 + 3\lambda_1) + 2c(84 - \theta(142 - \theta(95 - \theta(28 - 3\theta))) + 40\lambda_1 - \theta(56 - \theta(29 - (8 - \theta)\theta))\lambda_1 - 6 \\ (2 - \theta)\lambda_1^2) + s_1(-(8 + \theta(9 - 2\theta))^2 + 2(2 - \theta)(40 - \theta(28 - (8 - \theta)\theta))\lambda_1 + (32 - (16 - \theta)\theta)\lambda_1^2))/(2\theta^4 + 4\theta^3(-5 + \lambda_1) + \theta^2(77 + (-30 + \lambda_1)\lambda_1) - 4\theta(35 + (-20 + \lambda_1)\lambda_1) + 8(13 + (-10 + \lambda_1)\lambda_1)) \end{split}$$

$$\begin{split} s_2^8 &= \\ &(256\big(5-2\lambda_1\big)+2\,s_1(-2\,(8-\theta(9-2\theta))^2+(2-\theta)\big(128-\theta(112-\theta(38-5\theta))\big)\lambda_1+\theta^2(14-(8-\theta)\theta)\lambda_1^2)+\theta(64\big(-65+21\lambda_1\big)+\theta(4964-\theta(2904-\theta(891-8(17-\theta)\theta))-1352\lambda_1+8\theta\\ &(76-(15-\theta)\theta)\lambda_1+(68-5(8-\theta)\theta)\lambda_1^2))+2c(64\big(4+3\lambda_1\big)+\theta\big(-16\big(29+19\lambda_1\big)+\theta\big(327-13(-14+\lambda_1\big)\lambda_1+\theta^2\big(11-\big(-6+\lambda_1\big)\lambda_1\big)+4\theta\big(-25+\lambda_1\big(-13+2\lambda_1\big)\big)\big)\big)))/\big(2\big(-64\big(-2+\lambda_1\big)+\theta\big(-176+48\lambda_1+\theta\big(101+\big(-6+\lambda_1\big)\lambda_1\big)+\theta^2\big(3+\lambda_1\big)-4\theta\big(7+\lambda_1\big)\big)\big)\big)))). \end{split}$$

$$\begin{split} s_2^9 &= \\ & \left(-s_1 (6(4-\theta)\theta) \left(8 + \theta (-9 + 2\theta) \right)^2 + s_1 \left(64 + \theta (-160 + 3\theta (42 - (12 - \theta)\theta)) \right) \lambda_1 + 2(1 - \theta)^2 \left(32 + 2(12 + 2\theta) \theta \right) + 2(12 + 2\theta)^2 \left(12 + 2(12 + 2\theta) \theta \right) + 2(12 + 2(12 + 2\theta)^2 \left(12 + 2(12 + 2\theta) \theta \right) + 2(12 + 2(12 + 2\theta)^2 \left(12 + 2(12 + 2\theta) \theta \right) + 2(12 + 2(12 + 2\theta)^2 \left(12 + 2(12 + 2\theta) \theta \right) + 2(12 + 2(12 + 2\theta)^2 \left(12 + 2(12 + 2\theta) \theta \right) + 2(12 + 2(12 + 2\theta)^2 \left(12 + 2(12 + 2\theta) \theta \right) + 2(12 + 2(12 + 2\theta)^2 \left(12 + 2(12 + 2\theta) \theta \right) + 2(12 + 2(12 + 2\theta)^2 \left(12 + 2(12 + 2\theta) \theta \right) + 2(12 + 2(12 + 2\theta)^2 \left(12 + 2(12 + 2\theta) \theta \right) + 2(12 + 2(12 + 2\theta)^2 \left(12 + 2(12 + 2\theta) \theta \right) +$$

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$$s_1^6 = \frac{1}{\theta^2 (-5 + 2\theta)} \left(8 s_2 + 4 c (-2 + \theta) (-1 + \theta)^2 - \theta \left((-1 + \theta)^2 \theta + s_2 (20 + \theta (-11 + 2\theta)) \right) \right)$$

PROOF OF PROPOSITION 6

The values of s_2^{10} and s_1^{7} are as follows:

$$\begin{split} s_2^{10} &= \\ & \left(-s_1(4 - (2 - \theta)\theta)(8 - \theta(9 - 2\theta))^2 + s_1(128 + \theta(-256 + \theta(132 + \theta(6 + \theta(-15 + 2\theta)))))\lambda 1 + \\ & \left(1 - \theta \right) \left(128\lambda_1 + \theta(64 - 128\lambda_1 + \theta(-4(31 + \lambda_1) + \theta(114 + 6\theta^2 + 22\lambda_1 - 3\theta(15 + \lambda_1)))) \right) + 2c \\ & \left(-1 + \theta \right) \left(-16 + 48\lambda_1 + \theta(64 - 48\lambda_1 + \theta(-80 + \theta(53 + 2\theta^2 + 7\lambda_1 - \theta(17 + \lambda_1)))) \right) \right) / ((-2 + \theta)) \\ & \left(16(7 - \lambda_1) + \theta(24(-9 + \lambda_1) + \theta(-6(-25 + \lambda_1) + \theta(-49 + 6\theta + \lambda_1))) \right)) \end{split}$$

$$s_1^7 = \frac{-((1-\theta)\theta(4-7\theta)) - 2c(1-\theta)(4-\theta(3-2\theta)) + s2(8-\theta(18-(9-2\theta)\theta))}{\theta(-4+\theta(-1+2\theta))}$$

PROOF OF PROPOSITION 7

The values of s_2^{11} , s_2^{12} , s_2^{13} , s_2^{14} , s_2^{15} and s_1^{8} are as follows:

$$\begin{split} s_2^{11} &= \\ &(-8\,s_1\,\lambda_1 + 32\theta\,\lambda_2 + 8\,\big(s_1 + \lambda_1 + s_1\,\lambda_1\big)\,\lambda_2 - 8\,\big(5 + s_1 - 4\theta + \lambda_1 + 3\,\lambda_2\big) + \theta^2(-7 + 2\,s_1\big(1 + \lambda_1\big) \\ &(1 - \lambda_2\big) - \big(10 - \lambda_2\big)\,\lambda_2 + \lambda_1\big(1 - \lambda_2\big)\,\big(3 + \lambda_2\big)\,\big) + 2c(13 + \lambda_1\big(-3 + \lambda_2\big)\,\big(-1 + \lambda_2\big) - \big(-4 + \lambda_2\big)\,\lambda_2 - 8\theta \\ &(1 + \lambda_2\big) + \theta^2\big(1 + \lambda_1\big(-1 + \lambda_2\big) + 3\,\lambda_2\big)\big)\big)/\big(2\big(9 + 2(-4 + \theta)\theta + \lambda_1\big(-1 + \lambda_2\big) - \lambda_2\big)\big(1 + \lambda_2\big)\big) \end{split}$$

$$\begin{split} s_{2}^{12} &= \\ &(32 \left(3 - \lambda_{1}\right) + 2\theta \left(32 \left(-4 + \lambda_{1}\right) + \theta \left(104 - 34\theta + 4\theta^{2} - 17\lambda_{1} + 2\theta\lambda_{1} + \lambda_{1}^{2} - \left(1 - \lambda_{1}\right) \right. \\ &\left. \left. \left(4 - 2\theta - \lambda_{1}\right)\lambda_{2}\right)\right) - s_{1} \left(1 + \lambda_{1}\right)\left(32 - \theta \left(32 - \theta \left(9 - \lambda_{1} - \left(1 - \lambda_{1}\right)\lambda_{2}\right)\right)\right) + c \left(-32\theta \left(1 + \lambda_{1}\right) + 8 \right. \\ &\left. \left(5 + 3\lambda_{1} + \lambda_{2} - \lambda_{1}\lambda_{2}\right) + \theta^{2} \left(7 - 3\lambda_{2} + \lambda_{1} \left(10 - \lambda_{1} + \left(2 + \lambda_{1}\right)\lambda_{2}\right)\right)\right) / \left(2 \left(-4 + \theta^{2}\right) \left(-1 + \lambda_{1}\right) \left(1 + \lambda_{2}\right)\right) \end{split}$$

$$\begin{split} s_2^{13} &= \\ & (-976\theta + 615\,\theta^2 - 176\,\theta^3 + 19\,\theta^4 - 192\theta\,\lambda_1 + 168\,\theta^2\,\lambda_1 - 64\,\theta^3\,\lambda_1 + 9\,\theta^4\,\lambda_1 - 32\,\lambda_1^{\ 2} + 16\theta\,\lambda_1^{\ 2} + \theta^2\,\lambda_1^{\ 2} + 2\left(\theta(-280 + \theta(195 - \theta(60 - 7\theta))) - 3\left(2 - \theta\right)^2(8 - \theta^2)\,\lambda_1 + \left(4 - \theta\right)^2\,\lambda_1^{\ 2}\right)\lambda_2 - \theta^2\big(13 - (8 - \theta))\,\theta - 3\,\lambda_1\big)\big(1 - \lambda_1\big)\,\lambda_2^{\ 2} + 32\big(19 + 2\,\lambda_1 + 9\,\lambda_2\big) + s_1\big(1 + \lambda_1\big)(-16\theta\big(21 - \lambda_1\big(1 - \lambda_2\big) - 5\,\lambda_2\big) + 32\big(7 - \lambda_1\big(1 - \lambda_2\big) - 3\,\lambda_2\big) + 2\,\theta^4\big(3 + \lambda_2\big) - 8\,\theta^3\big(7 + \lambda_2\big) + \theta^2\big(201 - \lambda_1\big(-1 + \lambda_2\big)^2 + \big(-10 + \lambda_2\big)\,\lambda_2\big)\big) + 2c\big(4\big(17 - \lambda_1\big(3 - \lambda_2\big) - 3\,\lambda_2\big)\big(-3 - \lambda_1\big(1 - \lambda_2\big) - \lambda_2\big) + 24\,\theta^3\big(2 + \lambda_1 + \lambda_2\big) + 2\theta\big(151 + 4\,\lambda_1\big(11 - \lambda_2\big) + 2\,\lambda_1\big(1 - \lambda_2\big) - \lambda_1^2\big(3 - \lambda_2\big)\big(1 - \lambda_2\big) + \big(44 - 3\,\lambda_2\big)\,\lambda_2\big) - \theta^4\big(5 + 3\,\lambda_2 + \lambda_1\big(3 + \lambda_2\big)\big) - \theta^2\big(177 + 71\,\lambda_2 + \lambda_1\big(71 - \big(-32 + \lambda_1\big(1 - \lambda_2\big) + \lambda_2\big)\,\lambda_2\big)\big)\big)\big)/\big(\big(1 + \lambda_2\big)(-4\,\theta^4 - 8\,\theta^3\big(-5 + \lambda_1\big) - 8\big(23 + 2\,\lambda_1\big)\big) - \theta^2\big(177 + 2\,\lambda_1\big) + 2\,\lambda_1^2\big(1 - \lambda_2\big) + 2\,\lambda_1^2\big(1 - \lambda_2\big) + 2\,\lambda_2^2\big(1 - \lambda_2\big) + 2\,\lambda_1^2\big(1 - \lambda_2\big) + 2\,\lambda_1^2\big(1 - \lambda_2\big) + 2\,\lambda_1^2\big(1 - \lambda_2\big) + 2\,\lambda_2^2\big(1 - \lambda_2\big) + 2\,\lambda_1^2\big(1 - \lambda_2\big) + 2\,\lambda_2^2\big(1 - \lambda_2\big) + 2\,\lambda_1^2\big(1 - \lambda_2\big) + 2\,\lambda_2^2\big(1 - \lambda$$

$$\begin{array}{l} \left(-16+\lambda_{_{1}}\right)\lambda_{_{1}}\right) + 8\left(3-\lambda_{_{1}}\right)\left(1-\lambda_{_{1}}\right)\lambda_{_{2}} + \theta^{2}\left(-153+\lambda_{_{1}}\left(58+\lambda_{_{1}}\left(-1+\lambda_{_{2}}\right)-2\lambda_{_{2}}\right)+\lambda_{_{2}}\right) + 4\theta \\ \left(67-3\lambda_{_{2}}+\lambda 1\left(\lambda 1+4\left(-9+\lambda_{_{2}}\right)-\lambda_{_{1}}\lambda_{_{2}}\right)\right))) \end{array}$$

$$\begin{split} s_2^{14} &= \\ &(384 \left(-3 + 10\theta + \lambda_1 + \lambda_2\right) + 2(\theta(\theta\left(-2307 + \theta(1360 - \theta(423 - 66\theta + 4\theta^2))\right) - 2(4 - \theta)(64 - \theta)) \\ &(44 - (14 - \theta)\theta))\lambda_1 - \theta(13 - (8 - \theta)\theta)\lambda_1^2) + (2(4 - \theta)\theta(-64 + \theta(44 - (14 - \theta)\theta)) - (8 - (4 - \theta)\theta)) \\ &(48 - \theta(12 - \theta(9 - 2\theta)))\lambda_1 + (-4 + \theta)^2\theta^2\lambda_1^2)\lambda_2 + \theta^2(13 + (-8 + \theta)\theta - 3\lambda_1)\left(-1 + \lambda_1\right)\lambda_2^2) + c(\theta^4(-19 - 14\lambda_1 + \lambda_1^2 - (3 + \lambda_1)^2\lambda_2) + 16\theta(61 + \lambda_1(7 - \lambda_2)(5 - \lambda_2) + (12 - \lambda_2)\lambda_2) - 32(19 + \lambda_1(3 - \lambda_2)^2 + (2 - \lambda_2)\lambda_2) + 8\theta^3(22 + \lambda_1^2(-1 + \lambda2) + 8\lambda_2 + 3\lambda_1(5 + \lambda_2)) + \theta^2(-615 - \lambda2) \\ &(168 + \lambda 2_2) + \lambda_1^2(-1 + \lambda_2)(-13 + 3\lambda_2) - 2\lambda_1(195 + (-12 + \lambda_2)\lambda_2))) + s_1(1 + \lambda_1)(-128) \\ &(-3 + \lambda_2) + \theta(128(-5 + \lambda_2) + \theta(414 - 2\lambda_1(-7 + \lambda_2)(-1 + \lambda_2) + 2(-16 + \lambda_2)\lambda_2 - 8\theta(15 + \lambda_1) \\ &(-1 + \lambda2) + \lambda_2) + \theta^2(13 + \lambda_1(-1 + \lambda_2) + 3\lambda_2)))))/((1 + \lambda_2)(\theta(336 - 80\lambda_1 - \theta(201 - (10 - \lambda_1))\lambda_1 - 2\theta^2(3 + \lambda_1) + 8\theta(7 + \lambda_1))) + (\theta(16 + \theta(1 - \lambda_1))(1 - \lambda_1) - 32\lambda_1)\lambda_2 - 32(7 - 3\lambda_1 - \lambda_2))) \end{split}$$

$$\begin{split} s_2^{15} &= \\ \left(s_1(\theta \left(160 \left(13 - \lambda_1\right) + \theta \left(-2510 + 126 \lambda_1 + \theta \left(1588 - 36 \lambda_1 - \theta \left(559 - 8(13 - \theta)\theta - 3 \lambda_1\right)\right)\right)\right) - 64 \\ \left(11 - \lambda_1 + \lambda_2\right) - 4 \left(\theta \left(8 - \theta \left(53 - 2\theta \left(23 - (8 - \theta)\theta\right)\right)\right) + (4 - 5\theta + \theta^2)^2 \lambda_1\right) \lambda_2 - \theta^2 (6 - (4 - \theta)\theta) \\ \left(1 - \lambda_1\right) \lambda_2^{-2}\right) + 2c \left(1 - \theta\right)^2 (\theta \left(\theta \left(153 - 4(10 - \theta)\theta - \lambda_1\right) - 4(67 - 3\lambda_1)\right) + 2(\theta \left(72 - \theta \left(29 - 4\theta\right)\right) + (4 - \theta)^2 \lambda_1\right) \lambda_2 - (8 + (4 - \theta)\theta) \left(1 - \lambda_1\right) \lambda_2^{-2} + 8 \left(23 - 3 \lambda_1 - 16 \lambda_2\right)\right) + 2 \left(1 - \theta\right)^2 \left(-2\theta^4 \left(3 + \lambda_2\right) + 8 \\ \theta^3 \left(7 + \lambda_2\right) + 16\theta \left(21 - \lambda_1 - \left(5 - \lambda_1\right) \lambda_2\right) - 32 \left(7 - \lambda_1 - \left(3 - \lambda_1\right) \lambda_2\right) + \theta^2 \left(-201 + \lambda_1 - 2\left(-5 + \lambda_1\right) \lambda_2 - \left(1 - \lambda_1\right) \lambda_2^{-2}\right)\right) / \left((-2 + \theta) \left(1 + \lambda_2\right) \left(16\theta \left(-20 + \lambda_1\right) + \theta^2 \left(6\left(37 - \lambda_1\right) - \theta \left(69 - 8\theta - \lambda_1\right)\right) + \theta \\ \left(16 + (6 - \theta)\theta\right) \left(1 - \lambda_1\right) \lambda_2 + 8 \lambda_1 \lambda_2 + 8 \left(21 - \lambda_1 - \lambda_2\right)\right)) \end{split}$$

$$\begin{split} s_1^8 &= \\ &(\theta \big(32 - \theta \big(7 - 3 \, \lambda_1 \big) \big) + 8 \, \lambda_1 \, \lambda_2 + 2 \theta \big(16 - \theta \big(5 + \lambda_1 \big) \big) \, \lambda_2 + \theta^2 \big(1 - \lambda_1 \big) \, \lambda_2^{\ 2} - 8 \big(5 + \lambda_1 + 3 \, \lambda_2 \big) - 2 \\ &s_2 \big(1 + \lambda_2 \big) \big(9 - 2 \big(4 - \theta \big) \theta - \lambda_1 - \big(1 - \lambda_1 \big) \, \lambda_2 \big) + 2 c \big(13 + \lambda_1 \big(3 - \lambda_2 \big) \big(1 - \lambda_2 \big) + \big(4 - \lambda_2 \big) \, \lambda_2 - 8 \theta \\ &\big(1 + \lambda_2 \big) + \theta^2 \big(1 - \lambda_1 \big(1 - \lambda_2 \big) + 3 \, \lambda_2 \big) \big) \big) / \big(2 \big(4 - \theta^2 \big) \big(1 + \lambda_1 \big) \big(1 - \lambda_2 \big) \big) \end{split}$$

The values of s_2^{16} and s_1^{9} re as follows:

$$\begin{split} s_2^{16} &= \\ &((\theta \Big(128\lambda_1 + \theta \Big(4(63+\lambda_1\Big) - \theta \Big(250+22\lambda_1 - 3\theta \big(31-4\theta+\lambda_1\big)\Big)\Big)\Big) + 2(64\lambda_1 + \theta (64\big(6-\lambda_1\big) - \theta \big(4(65-\lambda_1\big) - (6-\theta)\theta \big(11+2\theta+\lambda_1\big)\Big)))\lambda_2 + \theta^2 \big(12-(10-\theta)\theta \big) \big(1-\lambda_1\big)\lambda_2^{-2} - 128\big(1+\lambda_1+3\lambda_2\big)\big) \\ &(-1+\theta) - 2c(1-\theta)(\theta \Big(80-48\lambda_1 - \theta \big(160-\theta \big(113+4\theta^2+7\lambda_1 - \theta \big(35+\lambda_1\big)\big)\Big) - (\theta \big(288-\theta \big(228-\theta (82-11\theta)\big)) + (4-\theta)^2 \big(4-\theta \big(2+\theta\big))\lambda_1\big)\lambda_2 - \big(16-\theta \big(16-(4-\theta)\theta\big)\big) \big(1-\lambda_1\big)\lambda_2^{-2} + 16 \\ &(1+3\lambda_1+8\lambda_2\big)\big) + s_1(-8\theta^6-128\big(3-\lambda_1\big(1-\lambda_2\big) + \lambda_2\big) + 128\theta\big(9-2\lambda_1\big(1-\lambda_2\big) + \lambda_2\big) + 2\theta^5 \\ &(45+\lambda_1-\big(5+\lambda_1\big)\lambda_2\big) - 4\theta^2 \big(369-\lambda_1\big(33-\lambda_2\big)\big(1-\lambda_2\big) - \big(18-\lambda_2\big)\lambda_2\big) + 2\theta^3 \big(517-\big(70-\lambda_2\big)\lambda_2 + \lambda_1\big(1-\lambda_2\big)\big(3+\lambda_2\big)\big) + \theta^4 \big(-417-\big(-66+\lambda_2\big)\lambda_2 + \lambda_1\big(-1+\lambda_2\big)\big(15+\lambda_2\big)\big)\big) / ((-2+\theta)) \end{split}$$

$$(208 - 408\theta + 294\theta^2 - 97\theta^3 + 12\theta^4 - 16\lambda_1 + 24\theta\lambda_1 - 6\theta^2\lambda_1 + \theta^3\lambda_1 + 12(2-\theta)^4\lambda_2 - \left(16-\theta(24-(6-\theta)\theta)\right)\left(1-\lambda_1\right)\lambda_2^2))$$

$$\begin{split} s_1^9 &= \\ (s_2(1024 + \theta(-16\theta^6 + 8\theta^5(27 - 2\lambda_1) - 2\theta^4(618 + (-67 + \lambda_1)\lambda_1) + 8\theta(963 + (-52 + \lambda_1)\lambda_1) + \theta^3(3877 - \lambda_1(430 - 9\lambda_1)) - 2\theta^2(3581 - \lambda_1(326 - 9\lambda_1)) - 192\lambda_1\lambda_2 - \theta \\ (1 - \lambda_1)(8(55 - \lambda_1) + \theta(\theta(93 - 2\theta(4 - \lambda_1) - 9\lambda_1) - 6(55 - 3\lambda_1)))\lambda_2 - 64(69 - \lambda_1 - 3\lambda_2))) - c \\ (1 - \theta)(1024 + \theta(\theta(4136 + 8\theta^4(3 + \lambda_1) - 8\lambda_1(68 + \lambda_1) - 4\theta^3(65 + 11\lambda_1) + \theta^2(1183 + 3\lambda_1(10 + \lambda_1)) + 2\theta(-1463 + \lambda_1(118 + \lambda_1))) - (192\lambda_1 + \theta(1 - \lambda_1)(8(33 + \lambda_1) - \theta(94 - 7\theta + 2\lambda_1 + 3\theta\lambda_1)))\lambda_2 - 64(49 - 5\lambda_1 - 3\lambda_2))) - 2(1 - \theta)\theta(128(3 - \lambda_1) + 2\theta^2(713 - \lambda_1(72 + \lambda_1(1 - \lambda_2) + \lambda_2) - \theta^3(795 + \lambda_1(2 + 3\lambda_1(1 - \lambda_2) + 38\lambda_2) - 35\lambda_2) - 2\theta^5(11 + 3\lambda_1 - (1 - \lambda_1)\lambda_2) + \theta^4(3(71 + 9\lambda_1) - 17(1 - \lambda_1)\lambda_2) - 8\theta(3(51 + \lambda_2) - \lambda_1(32 + \lambda1 + 4\lambda_2 - \lambda_1\lambda_2))))/((-2 + \theta)\theta(1 + \lambda_1) + \lambda_1 + \lambda_2)))) \end{split}$$

For this proposition we assume that $\frac{\partial (\pi_{n_i}^{SI} - \pi_{n_i}^{SI})}{\partial a} = \frac{\partial \pi_n^{SI}}{\partial a}$. If

$$\begin{split} G(\theta,\mathbf{s}_{1},\mathbf{s}_{2}) &= \frac{1}{528}((11\left(-135+s_{2}(22-8\alpha)-4\alpha+c(26+8\alpha)\right)+\frac{128\left(4-3c+3\,s_{2}\right)}{2+\alpha(-1+\theta)-\theta}-44\left(1-2c+2\,s_{2}\right)(1-\alpha)\theta+\frac{352(c-s_{2})}{\alpha+\theta-\alpha\theta}+(27(156-40c+40\,s_{2}+183\alpha-74c\alpha+74\,s_{2}\alpha+(183-74c+74\,s_{2})(1-\alpha)\theta)))/(4+\alpha+4\,\alpha^{2}(1-\theta)^{2}+\theta+\alpha(7-8\theta)\theta+4\,\theta^{2}))-s_{1}>0\\ \frac{\partial(\pi_{m_{1}}^{SS}-\pi_{m_{1}}^{ST})}{\partial\alpha}>0 \end{split}$$

$$|\pi_m^{SS} - \pi_m^{ST}|_{\alpha=0} < 0$$

- (i). When $s_1 \neq s_2$, $\lim_{\alpha \to 1} \pi_{m_2}^{SS} \pi_{m_2}^{ST} > 0$. There exists a unique $\alpha = \widetilde{\alpha} \in [0,1]$ such that $\pi_{m_2}^{SS} \pi_{m_2}^{ST} = 0$. Thus, when $G(\theta, s_1, s_2) > 0$, $s_1 \neq s_2$ and $\alpha > \widetilde{\alpha}$, $\pi_{m_3}^{SS} > \pi_{m_2}^{ST}$. When $G(\theta, s_1, s_2) > 0$, $s_1 \neq s_2$ and
- $\begin{array}{l} \alpha < \widetilde{\alpha}, \pi_{m_{_{2}}}^{ST} > \pi_{m_{_{2}}}^{SS}. \\ \text{(ii) When } s_{_{1}} = s_{_{2}}, \lim_{\alpha \to 1} \pi_{m_{_{2}}}^{SS} \pi_{_{m_{_{2}}}}^{ST} < 0. \text{ Thus, when } G(\theta, s_{_{1}}, s_{_{2}}) > 0 \text{ and } s_{_{1}} = s_{_{2}}, \pi_{m_{_{2}}}^{ST} > \pi_{m_{_{2}}}^{SS}. \\ \text{(iii) When } G(\theta, s_{_{1}}, s_{_{2}}) < 0, \pi_{m_{_{2}}}^{ST} > \pi_{m_{_{2}}}^{SS}. \end{array}$

In summary, when $G(\theta, s_1, s_2) > 0$, $s_1 \neq s_2$ and, $\pi_{m_1}^{SS} > \pi_{m_2}^{ST}$. Otherwise, $\pi_{m_2}^{ST} > \pi_{m_2}^{SS}$.

For this proposition we assume that $\frac{\partial(\pi_{m_i}^{rs} - \pi_{m_i}^{rs})}{\partial \alpha} = \frac{\partial \pi_{m_i}^{rs}}{\partial \alpha} < 0$

$$\pi_{m_1}^{SS} - \pi_{m_1}^{ST} \Big|_{\alpha=0} = \frac{\left(1 + s_1 - s_2 - \theta\right) \left(-12 \left(1 + s_1\right) + 8 s_2 + \left(9 + s_1 + s_2\right) \theta + \left(3 + 2 s_1\right) \theta^2 - 2 c \left(-2 + \theta + \theta^2\right)\right)}{4 \left(-4 + \theta\right) \left(-1 + \theta\right) \left(2 + \theta\right)^2}$$

- $\begin{array}{ll} \text{(i)} & \text{When s}_2 > 1 + \text{s}_1 \theta, \, \pi^{SS}_{m_i} \pi^{ST}_{m_i} \big|_{\alpha = 0} < 0. \text{ So } \pi^{SS}_{m_i} \pi^{ST}_{m_i} < 0. \\ \text{(ii)} & \text{When s}_2 > 1 + \text{s}_1 \theta, \pi^{SS}_{m_i} \pi^{ST}_{m_i} \big|_{\alpha = 0} > 0. \text{ Thus there exists a unique } \alpha = \widehat{\alpha} \in [0, 1] \text{ and when } \\ & \alpha < \widehat{\alpha}, \, \pi^{SS}_{m_i} > \pi^{ST}_{m_i}; \text{ when } \alpha > \widehat{\alpha}, \, \pi^{ST}_{m_i} > \pi^{SS}_{m_i}. \end{array}$

In summary, when $s_2 < 1 + s_1 - \theta$ and $\alpha < \hat{\alpha}, \pi_{m_1}^{SS} > \pi_{m_1}^{ST}$. When $s_2 > 1 + s_1 - \theta$ or $s_2 < 1 + s_1 - \theta$ and $\alpha > \hat{\alpha}, \pi_{m_1}^{ST} > \pi_{m_1}^{SS}$.

PROOF OF PROPOSITION 11

When $(\pi_s^{SS} - \pi_s^{ST})|_{x=0} = 0$, $s_2 = s_2^{18}$ ($s_2 = s_2^{19} < 0$, deletion), and

$$s_2^{18} \ = \ 1 \ + \ s_1 \ - \ \theta \ + \ \tfrac{\sqrt{(1-c+s_i)^2(-4+\theta)^2(-2+\theta)(-1+\theta)\theta(2+\theta)(8+\theta(-9+2\theta))^2}}{(4-\theta)(8+\theta(-9+2\theta))^2}$$

$$s_2^{19} \ = \ 1 + s_1 - \theta - \tfrac{\sqrt{(1-c+s_1)^2(-4+\theta)^2(-2+\theta)(-1+\theta)\theta(2+\theta)(8+\theta(-9+2\theta))^2}}{(4-\theta)(8+\theta(-9+2\theta))^2}$$

 $\begin{array}{l} s_{_{2}} < s_{_{2}}^{18} \, \text{and} \, (\pi_{_{s}}^{SS} - \pi_{_{s}}^{ST})\big|_{\lambda_{_{2}=0}\alpha=0} > 0, \, \text{when} \, 1 + s_{_{1}} - \theta - s_{_{2}} > 0; \\ s_{_{2}} > s_{_{2}}^{18} \, \text{and} \, (\pi_{_{s}}^{SS} - \pi_{_{s}}^{ST})\big|_{\lambda_{_{2}=0}\alpha=0} > 0 \\ \text{when} \, 1 + s_{_{1}} - \theta - s_{_{2}} < 0. \\ \text{Because} \, \frac{\partial (\pi_{_{s}}^{SS} - \pi_{_{s}}^{ST})\big|_{s=0}}{\partial \lambda_{_{2}}} < 0, \, \text{there exists a unique} \, \lambda_{_{2}} = \lambda_{_{2}}^{*} \in [0,1] \, \text{yields} \, \pi_{_{s}}^{SS} - \pi_{_{s}}^{ST}\big|_{\alpha=0} = 0. \, \pi_{_{s}}^{SS} - \pi_{_{s}}^{ST} \\ \big|_{\alpha=0} > 0 \, \text{when} \, \lambda_{_{2}} < \lambda_{_{2}}^{*}. \, \frac{\partial (\pi_{_{s}}^{SS} - \pi_{_{s}}^{ST})\big|_{\lambda_{_{2}=0}\alpha=0}}{\partial \alpha} < 0 \, \text{when} \, s_{_{2}} < s_{_{2}}^{17}, \, \text{and} \end{array}$

$$s_2^{17} = 3 - c + s_1 + \frac{2c}{\alpha + \theta - \alpha\theta} + \frac{8c(-1 + \alpha)(-1 + \theta)}{8 + (\alpha + \theta - \alpha\theta)^2} / (\frac{2}{\alpha + \theta - \alpha\theta} + \frac{8(-1 + \alpha)(-1 + \theta)}{8 + (\alpha + \theta - \alpha\theta)^2})$$

There exists a unique $\alpha=\alpha^*\in[0,1]$ yields $\pi_s^{SS}-\pi_s^{ST}=0$. $\pi_s^{SS}<\pi_s^{ST}$ when $\alpha<\alpha^*$; $\pi_s^{SS}>\pi_s^{ST}$ when $\alpha>\alpha^*$. $\pi_s^{SS}-\pi_s^{ST}|_{\alpha=0}<0$ when $\lambda_2>\lambda_2^*$. Because $\frac{\partial(\pi_s^{SS}-\pi_s^{SS})|_{\lambda_s}}{\partial\alpha}<0$ when $s_2< s_2^{17}$, $\pi_s^{SS}<\pi_s^{ST}$. $\frac{\partial(\pi_s^{SS}-\pi_s^{ST})|_{\lambda_s}}{\partial\alpha}>0$ when $s_2>s_2^{17}$. Because $(\pi_s^{SS}-\pi_s^{ST})|_{\lambda_{2s0}\alpha=0}>0$, $\pi_s^{SS}<\pi_s^{ST}$. In summary, when $s_2< s_2^{17}$, $\lambda_2<\lambda_2^*$ and $\alpha<\alpha^*$ or $s_2>s_2^{17}$ ad $\lambda_2<\lambda_2^*$. Otherwise, $\pi_s^{SS}>\pi_s^{ST}$

