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Z-numbers Based Hybrid MCDM Approach for Energy Resources Ranking and Selection

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ABSTRACT

Energy resource selection is a multi-criteria decision-making task, and in most cases, it is characterized by uncertainties, information incompleteness, and vagueness. In such circumstances, a group decision making and experts` opinion study are appropriate approaches for the problem solution. These approaches allow to a considerable extent to compensate the deficiency of the relevant and reliable data on alternatives and decision-makers' priorities. In this paper fuzzy information and Z-numbers based MCDM models have been studied and used as a group decision-making tool for energy resources ranking and selection. In order to make a well-founded and reliable decision and to get insights how methodological backgrounds of models used are influencing results of evaluations, comparative study of applications of the outranking, distance-based and pairwise comparisons methods for the same subject area is carried out and merits and demerits of each approach from a decision-makers' point of view are evaluated. The availability of solutions based on various approaches allows to decision-maker to select the more justified and consistent solution. A numerical example of energy resources ranking and selection for Azerbaijan illustrates the efficiency of the integrated approach and comparative study.

Keywords: Multi-criteria Decision, Fuzzy, Z-number, Energy Resources
JEL Classifications: D81, P48, Q35, Q42

1. INTRODUCTION

Conscious decision making is an intrinsic part of human beings' daily activities. At the earlier stages of the development decision-making process was purely intuition and rationale based and less formalized, and decision problems from a formulation standpoint were relatively simple, based on a single criterion and a few alternatives. As societies and economies are developing, the decision-making problems are becoming less structured, more complicated, multi-dimensional, and multi-criterial and people responsible for decision making need appropriate and justified information and tools.

At present this is a realm of the Multi-criteria Decision Making (MCDM). Depending on the approach, in general, we can trace back the development of MCDM for centuries, but formal mathematical foundations of the MCDM methodology were laid down in seminal papers (Kuhn and Tucker, 1951; Charnes et al., 1955; Kimball and Howard, 1959) in the middle of the 20th century and these results stimulated successive achievements in the field of MCDM during the following decades.

Kuhn and Tucker (1951) presented optimality conditions for the solution of the problems with vector objectives known at present as Karush-Kuhn-Tucker condition. In Charnes et al. (1955) for the first-time ideas of a powerful method for the multi-criteria problem solution, later named the goal programming, was introduced. Kimball and Howard (1959) presented sequential decision processes based on rewards and policy improvement.

Since then various methods for the solution of the MCDM problems were developed and successfully applied in various fields (Zavadskas et al., 2014; Greco et al., 2016) and contributions of the energy economics and energy policy development related studies (Kahraman et al., 2015; Kumar et al., 2017; Kaya et al., 2018) in this subject area are unquestionable.

Multi-criteria decision-making techniques are based on various theoretical foundations but the most widely used MCDM tools are pairwise comparison, outranking, and distance-based methods and these techniques have different methodological foundations.

Preference Ranking Organization Method for Enrichment (PROMETHEE) is an outranking method well suited for use of human judgment and since its introduction (Brans, 1982) has been extensively and successfully applied in various fields of research as an efficient tool for solving multi-criteria decision analysis and decision-making tasks. Well-known fuzzy extensions of the PROMETHEE (Goumas and Lygerou, 2000; Geldermann et al., 2000) later were extended and modified in several other research publications, including Z-number based versions (Tavakkoli-Moghaddam et al., 2015; Adalı et al., 2016; Gul et al., 2018; Qiao et al., 2019).

A Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) first introduced in (Hwang and Yoon, 1981) is based on the selection of the alternative with the shortest distance from the best and farthest distance from the worst solution. This method has been successfully used in various areas, requiring multi-criteria decision making. A fuzzy version of the TOPSIS introduced in (Chen, 2000) and other publications (Madi et al., 2015) laid down a methodological foundation for applications of the Z-number based versions of the TOPSIS (Yaakob and Gegov, 2015; Krohling et al., 2016).

Based on pairwise comparisons Analytical Hierarchy Process (Saaty, 1977) is one of the most widely used MCDM methods. In AHP decision-makers judgment is playing a decisive role and fuzzy extensions proposed in (Buckley, 1985; van Laarhoven and Pedrycz, 1983) significantly increased a descriptive power of the approach and solutions relevance. Extensive reviews (Singh, 2016; Kaya et al., 2019) are illustrating the applicability and effectiveness of the fuzzy AHP in MCDM in general and in energy resources selection in particular. Z-numbers based AHP applied in various areas. Zhang (2017) applied this approach for risks ranking in the underground construction. Karthika and Sudha (2018) illustrated an application of the Z-numbers based FAHP for dengue fever risks assessment in various states of India. Yildiz and Kahraman (2020) applied Z-numbers based approach for social development evaluation and illustrated the applicability of the approach by example.

Energy policy development, resources selection and planning require analysis and evaluation of the finite set of available alternatives with respect to a given set of criteria in order to select the most appropriate solution in terms of multiple criteria. Distinctive features of this decision-making process are subjectivity, information uncertainty, impreciseness, and incompleteness. In such circumstances as efficient problem-solving tools, fuzzy information and Z-number based multi-criteria decision-making methods (MCDM) are used (Kahraman et al., 2015; Kaya et al., 2018; Krohling et al., 2016; Chatterjee and Kar, 2018; Kaya et al., 2019).

Other features of this decision-making process are that country-level energy policy development and energy resources selection decisions have long-run effects and adjustments and changes of these decisions are very difficult and costly. Moreover, any changes usually have a negative spillover effect on other parties involved in the decisions' implementation process. For increasing soundness and reliability of the energy resources selection decision in view of the above-mentioned features in this paper hybrid approach to energy resources ranking and selection task has been applied. The approach is based on comparative study of the solutions provided by various methods and group decision making.

In this paper, we are applying classical representatives of the above-mentioned methods for solution of the energy resources ranking and selection problem in uncertain environments, namely the AHP, PROMETHEE, and TOPSIS methods. Comparative study and integration of the various approaches are used as an efficient tool for improvement of the quality and consistency of decisions.

The remaining part of the paper is set out as follows: In section 2 preliminaries of the fuzzy and Z-numbers, Z-numbers reliability and restriction conversion, Z-numbers based MCDM problem general statement and basics of the criteria weighting are briefly reviewed. Methodological foundations of the applied methods are presented in section 3. Section 4 describes a country-level application of the hybrid approach for energy resources ranking and selection. The conclusion discusses the specifics of the hybrid MCDM application for energy resources selection and problem solution results.

2. PRELIMINARIES

2.1. Fuzzy and Z-numbers

A Z-number (Zadeh, 2011), Z, has two fuzzy components, $Z = (\tilde{A}, \tilde{B})$. The first component, $\tilde{A}$, is a restriction constraint) on the values which a real-valued uncertain variable, X, can take. The second component, $\tilde{B}$, is a measure of reliability (certainty) of the first component.

In applications, various types of membership functions for a fuzzy number have been used: triangular, trapezoidal, Gaussian, sigmoidal, L-R and many others. The most widely used membership functions are triangular and trapezoidal membership functions (Buckley, 1985; van Laarhoven and Pedrycz, 1983; Chang, 1996). In this paper, we are using triangular membership functions.

The support M of the triangular fuzzy number (l, m, u) is $\{x \in R \; I \; l<x<u\}$ and its membership function $\mu_M(x)$: $R \rightarrow [0,1]$ is equal to:

$$\mu_M(x) = \left\{ \begin{array}{ll} \frac{x}{m-l} - \frac{l}{m-l}, & x \in [l,m], \\ \frac{x}{m-u} - \frac{u}{m-u}, & x \in [m,u], \\ 0 & otherwise \end{array} \right\} \quad (1)$$

Basic fuzzy calculation operations (Chang, 1996) used in MCDM are presented below:

$$(l_1,m_1,u_1) \oplus (l_2,m_2,u_2) = (l_1+l_2,m_1+m_2,u_1+u_2) \tag{2}$$

$$(l_1,m_1,u_1) \ominus (l_2,m_2,u_2) = (l_1-u_2,m_1-m_2,u_2-l_2) \tag{3}$$

$$(l_1,m_1,u_1) \odot (l_2,m_2,u_2) = (l_1l_2,m_1m_2,u_1u_2) \tag{4}$$

$$(l,m,u)^{-1} \approx (1/u,\ 1/m,\ 1/l) \tag{5}$$

As it is shown in (Bhanu and Velammal, 2015) and in many other publications, direct computations with Z-numbers, especially in large-scale problems, are complicated, sensitive to the probability density functions and do not in all cases ensure the successful solution of the task. In applications, an approach based on converting the Z-number to a classical fuzzy number (Kang et al., 2012) can be used.

Ideas of the approach can be briefly outlined in the following way:

a. Accordingly, the approach, at the first step reliability of the Z-number B should be converted into a crisp number:

$$\alpha = \frac{\int x\mu_{\tilde{B}}(x)\,dx}{\int \mu_{\tilde{B}}(x)\,dx} \tag{6}$$

where $\int$ is an algebraic integration.

b. In the second step, the weight of reliability should be added to the restriction $\tilde{A}$. The weighted Z-number is d

$$\tilde{Z}^{\alpha} = \{\langle x,\ \mu_{\tilde{A}^{\alpha}}(x)\rangle \mid \mu_{\tilde{A}^{\alpha}}(x) = \alpha\mu_{\tilde{A}}(x), x\in[0,1]\} \tag{7}$$

c. Finally, the irregular fuzzy number (weighted restriction) should be converted to a regular fuzzy number

$$\tilde{Z}' = \{\langle x, \mu_{\tilde{A}'}(x)\rangle \mid \mu_{\tilde{A}'}(x) = \mu_{\tilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right), x\in[0,1]\} \tag{8}$$

$\tilde{Z}'$ and $\tilde{Z}^{\alpha}$ are equal with respect to Fuzzy Expectation.

2.2. Z-numbers Based MCDM Problem

In economics and business, we basically encounter MCDM problems with a finite number of alternatives. In the most cases scale, scope and complexity of the problem, uncertainty, information incompleteness, conflict between criteria like "profit" and "cost", lack of ideal solution and mixes of the quantitative and qualitative information require application of the approaches based on group expertise and human judgment. Fuzzy information and Z-numbers are powerful tools for modelling a decision-making process in uncertain environments and the following analysis of the MCDM is based on the application of these tools.

For a group MCDM approach, based on fuzzy information and Z-numbers, let $A = [a_i]$, $i = \overline{1,m}$ is a set of alternatives; $C = [c_j]$, $j = \overline{1,n}$ is a set of criteria; $D_k = \left|Z_{ij}^k\right|_{mxn} = \left|(\tilde{A}_{ij}^k, \tilde{B}_{ij}^k\right|_{mxn}$ is a k-th expert's decision matrix; $D = \left|Z_{ij}\right|_{mxn} = \left|(\tilde{A}_{ij}, \tilde{B}_{ij}\right|_{mxn}$ is an aggregated decision matrix; Z_{ij}^k denotes an evaluation of the alternative i with respect to criteria j by k-th expert; $W = \left|w_j^k\right|_{1xn} = \left|(\tilde{A}_j^k, \tilde{B}_j^k\right|_{1xn}$ is a weight matrix based on an evaluation of the criteria importance by k-th expert and $W = \left|w_j\right|_{1xn}$ is an aggregated weights matrix that represents an aggregated evaluation of the criteria importance by the group of experts (decision-makers). Decision maker, based on information provided by experts, results of aggregation and normalization, selects an alternative a_s $a_s^{\oplus}$ that is most appropriate for a given set of criteria $C = [c_j]$.

If decision-maker is not confident in the solution developed based on group evaluations, additional measures should be taken in order to improve the quality of the solution and to increase the confidence level. One option is to compare criteria weights assigned by individual experts with aggregated weights of the criteria. In the case of a significant discrepancies between individual and aggregated evaluations, feedback based on aggregate weights can be provided to experts with higher discrepancies. Based on feedback expert re-evaluates weights and provides justification of the evaluation. The more complicated case is the situation when discrepancy of individual weights with respect to aggregated weights are insignificant but there are serious differences between solution based on aggregated decision matrix and solutions based on individual decision matrix. This option will require re-evaluations and adjustments of the individual and aggregated decision matrix.

There are various approaches applicable for the MCDM in fuzzy and Z-numbers environment. In this paper, we are studying the applicability and efficiency of the tools with various methodological backgrounds, like PROMETHEE, TOPSIS, and AHP for the solution of the similar problems.

2.3. Weights in MCDM

Weights assigned to objectives allow to regulate priorities of decision-makers. Mainly used approaches to weighting are subjective and objective weights assignments. A combination of weights could be used as an additional tool for increasing decision reliability and consistency. Wang and Lee (2009) presents a modification of the fuzzy VICOR based on aggregated use of the subjective and objective weights. Objective weights are determined by the entropy measure. Entropy measure is used in other publications as well (Chatterjee and Kar, 2018; Suh et al., 2019).

Calculations of the weights based on Shannon's entropy include the following procedures:

a. Normalization of the aggregated decision matrix

$$\left[Z_{ij}\right]_{mxn} = \left[Z_{ij} / \sum_{i=1}^{m} Z_{ij}\right]_{mxn} \tag{9}$$

b. Calculation of the entropy measure of each index

$$e_j = -k\sum_{i=1}^{m} p_{ij}\ \ln(p_{ij}) \text{ where constant } k = (\ln(m))^{-1} \tag{10}$$

c. Calculation of the divergence of each criterion

$$div_j = 1 - e_j \tag{11}$$

d. Objective weights calculations

$$W_j^o = {div_j} \Big/ {\sum_j div_j} \tag{12}$$

$$W_j^{comb} = \alpha W_j^s + (1-\alpha) W_j^o, \text{where } \alpha \in [0;1] \tag{13}$$

Parameter α allows to decision maker regulate individual preference by means of "weighting weights" for subjective or objective approaches.

3. METHODOLOGICAL BACKGROUND OF THE APPLIED MODELS

3.1. Fuzzy Information and Z-number Based PROMETHEE

PROMETHEE based group MCDM requires sequential performance of the following steps:

Step 1: Generation of the subject area-relevant criteria and alternatives.
Step 2: Design of the questionnaire for experts` opinion study.
Step 3: Experts group composition.
Step 4: Opinion study.
Step 5: Individual Z-numbers based decision matrix composition.
Step 6: Converting individual Z-numbers based decision matrix into fuzzy matrix according to (6)-(8).
Step 7: Aggregated weights matrix composition.

Experts opinion-based approaches mainly are using subjective weights, but combination of the objective and subjective weights could be used.

Step 8: Weighted fuzzy decision matrix composition.
Step 9: A preference function (Brans and Vincke, 1985) selection and calculation for each pair of alternatives. In this study we are using Type 4 function:

$$P_j = \begin{Bmatrix} 0, \ if\ 0 < d_{a_{i,i'}} \le q(indifference) \\ 0.5, \ \ if\ q < d_{a_{i,i'}} \le p \\ 1, \ if\ d_{a_{i,i'}} > p\ (strict\,indifference) \end{Bmatrix} \tag{14}$$

In (14), q is the largest negligible deviation for a decision-maker, p is the smallest deviation generating a full preference. The indifference threshold is the largest deviation which is considered as negligible by the decision-maker, while the preference threshold is the smallest deviation which is considered as sufficient to generate a full preference.

Step 10: Pairwise comparison of the all alternatives over all the criteria and construction of the difference matrix based on aggregated weighted decision matrix. The difference for each pair of alternatives and for each criterion is calculated according to formula (3).
Step 11: Preference matrix composition and global preference index calculation for each alternative pairwise comparison.
Step 12: Calculation of the leaving $\Phi^+(a)$, entering $\Phi^-(a)$, and net $\Phi(a)$ flows:

$$\Phi^+(a) = \frac{1}{m-1}\sum[\pi(a,b)] \tag{15}$$

$$\Phi^-(a) = \frac{1}{m-1}\sum[\pi(b,a)] \tag{16}$$

$$\Phi(a) = \Phi^+(a) - \Phi^-(a) \tag{17}$$

Step 13: Ranking alternatives according to the net flows. Alternative with highest $\Phi(a)$ is the best performing, and alternative with lowest $\Phi(a)$ is the worst one.

3.2. Fuzzy Information and Z-number Based TOPSIS

In Z-number based TOPSIS experts opinion study and decision matrix composition are basically similar to the preliminary steps of the Z-number based PROMETHEE.

Main idea of the TOPSIS method is outlined in the following steps:

Step 1: Categorizing criteria as a benefit and a cost criterion.
Step 2: Normalization of the aggregated fuzzy decision matrix.

$$\tilde{z}_{ij}^{\circ} = \left(\frac{l_{ij}}{u_j^+}, \frac{m_{ij}}{u_j^+}, \frac{u_{ij}}{u_j^+}\right), \ u_j^+ = max_i u_{ij}\left(Benefit\,criteria\right) \tag{18}$$

$$\tilde{z}_{ij}^{\circ} = \left(\frac{l_j^-}{u_{ij}}, \frac{l_j^-}{m_{ij}}, \frac{l_j^-}{l_{ij}}\right), \ l_j^- = min_i l_{ij}\left(Cost\,criteria\right) \lim_{x\to\infty} \tag{19}$$

Step 3: Calculation of the weighted normalized decision matrix $\tilde{D}_w^{\circ} = \tilde{\vartheta}_{ij}$

$$\tilde{\vartheta}_{ij} = \tilde{z}_{ij}^{\circ} \times \tilde{w}_j \tag{20}$$

Step 4: Determination of the fuzzy positive ideal solution A^+ and fuzzy negative ideal solution A^-.
Step 5: Calculation of the distances of each solution from fuzzy ideal positive and negative solutions:

$$d_i^+ = \sum\nolimits_{j=1}^{n} d_{\vartheta}(\tilde{\vartheta}_{ij}, \tilde{\vartheta}_j^+) \tag{21}$$

$$d_i^- = \sum\nolimits_{j=1}^{n} d_{\vartheta}(\tilde{\vartheta}_{ij}, \tilde{\vartheta}_j^-) \tag{22}$$

Distance between two fuzzy triangular numbers $\tilde{\varphi}_1 = (l_1, m_1, u_1)$ and $\tilde{\varphi}_2 = (l_2, m_2, u_2)$ is equal to

$$d(\tilde{\varphi}_1, \tilde{\varphi}_2) = \sqrt{\frac{1}{3}((l_1 - l_2)^2 + (m_1 - m)^2 + (u_1 - u_2)^2)} \tag{23}$$

Step 6: Calculation of the relative closeness δ_i for each alternative:

$$\delta_i = \frac{d_i^-}{(d_i^+ + d_i^-)} \tag{24}$$

Step 7: Alternative ranking in accordance with the relative closeness δ_i, the best alternative has higher closeness coefficient relative to a positive ideal solution.

3.3. Fuzzy and Z-information Based Analytical Hierarchy Process (FZ AHP)

Taking into account AHP features, Z-numbers based AHP includes the following steps.

Step 1: Problem statement and identification of the criteria, sub-criteria, and key factors.

Step 2: Problem hierarchical structure development.

Step 3: Z-numbers ($Z = (\tilde{A}, \tilde{B})$) based description of the classical nine points AHP scale.

Step 4: Transforming a reliability $\tilde{B}$ into a crisp number and adding a weight of the reliability to the restriction $\tilde{A}$.

Step 5: Transforming the irregular fuzzy number to regular fuzzy number.

Step 6: Experts group composition and questionnaire design.

Step 7: Matrix representations of the criteria and sub-criteria pairwise comparisons. In case of fuzzy information matrix $\tilde{A} = (\tilde{a}_{ij})nxn$ should be used.

Step 8: Criteria and sub-criteria prioritization.

Step 9: Inputting into pairwise comparison matrix pairwise judgments and reciprocals.

Step 10: Calculations of priorities.

For a group decision-making, fuzzy judgements data $\tilde{a}_{ij}^k$ are averaged according to formula (25):

$$\tilde{a}_{ij} = \frac{\sum_{k=1}^{K} \tilde{a}_{ij}^k}{K} \tag{25}$$

K is a number of decision-makers (experts).

Based on averaged preferences matrix $\tilde{A}$ is composed:

$$\tilde{A} = \begin{pmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \vdots & & \tilde{a}_{ij} & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{r2} & \cdots & 1 \end{pmatrix} \tag{26}$$

Geometric mean of each criteria (alternative) (Buckley, 1985) is used as a mean value of the fuzzy comparisons:

$$\tilde{r}_i = \left(\prod_{j=1}^{n} \tilde{a}_{ij}\right)^{1/n} \tag{27}$$

A fuzzy weight $\tilde{w}_i$ of criterion i is calculated by the formula (28):

$$\tilde{w}_i = \tilde{r}_i \odot \left(\tilde{r}_1 \oplus \tilde{r}_2 \oplus \ldots \oplus \tilde{r}_n\right)^{-1} \tag{28}$$

Step 11: Center of area (COA) is used for defuzzification of fuzzy weights:

$$X_{COA} = \frac{\int x\mu_A(x)\,dx}{\int \mu_A(x)\,dx} \tag{29}$$

For the triangular fuzzy numbers, it has a simple form:

$$W_i = (l_{w_i} + m_{w_i} + u_{w_i})/3 \tag{30}$$

Step 12: Normalization of the weights:

$$W_i^N = W_i / \sum_{i=1}^{n} w_i \tag{31}$$

Step 13: The best alternative selection according to higher priority.

4. APPLICATION FOR ENERGY RESOURCES SELECTION

As a case, we are analysing the energy resources ranking and selection task for Azerbaijan. Country's geographical location and energy resources endowment predetermines four alternatives for the energy resources that are of interest: solar energy, wind, hydro, and natural gas. Even though Azerbaijan is the oil exporting country, in the paper we are not analysing this option due to environmental issues and state electricity production long-term policy. Alternatives are analysed with respect to nine criteria: Government policy and regulation (GP&R); Social acceptance; Labour impact; Cost; Spill over effects; Technical efficiency; Technology reliability; Resource availability; Environmental impact.

As a source of information, we are using the experts' group opinion study. Linguistic terms for alternatives evaluation, criteria importance evaluation, and their Z-number based description are presented in Tables 1 and 2.

Taking into account that from decision-makers standpoint importance of the criteria differ, Z-number based weights are used for criteria importance evaluation as well. Table 3 represents the

Table 1: Linguistic terms for alternatives evaluation

Restriction linguistic term	Restriction fuzzy value	Reliability linguistic term	Reliability fuzzy value
Very poor (VP)	(0.0,0.0,0.2)	Very low (VL)	(0.0,0.0,0.2)
Poor (P)	(0.05,0.2,0.35)	Low (L)	(0.05,0.2,0.35)
Below average (BA)	(0.2,0.35,0.5)	Medium low (ML)	(0.2,0.35,0.5)
Average (A)	(0.35,0.5,0.65)	Medium (M)	(0.35,0.5,0.65)
Above average (AA)	(0.5,0.65,0.8)	Medium high (MH)	(0.5,0.65,0.8)
Good (G)	(0.65,0.8,0.95)	High (H)	(0.65,0.8,0.95)
Very good (VG)	(0.8, 1.0,1.0)	Very high (VH)	(0.8, 1.0,1.0)

Table 2: Linguistic terms for criteria importance evaluation

Criteria importance	Restriction fuzzy value	Reliability fuzzy value
Very low (VL)	(0.0,0.0,0.2)	(0.0,0.0,0.2)
Low (L)	(0.05,0.2,0.35)	(0.05,0.2,0.35)
Medium low (ML)	(0.2,0.35,0.5)	(0.2,0.35,0.5)
Medium (M)	(0.35,0.5,0.65)	(0.35,0.5,0.65)
Medium high (MH)	(0.5,0.65,0.8)	(0.5,0.65,0.8)
High (H)	(0.65,0.8,0.95)	(0.65,0.8,0.95)
Very high (VH)	(0.8, 1.0,1.0)	(0.8, 1.0,1.0)

Table 3: Criteria weights assigned by experts

Criteria	E_1	E_2	E_3
Government policy and regulation (C_{11})	(0.8, 1.0,1.0); (0.8, 1.0,1.0)	(0.8, 1.0,1.0); (0.65,0.8,0.95)	(0.65,0.8,0.95); (0.65,0.8,0.95)
Social acceptance (C_{12})	(0.35,0.5,0.65); (0.65,0.8,0.95)	(0.2,0.35,0.5); (0.65,0.8,0.95)	(0.2,0.35,0.5); (0.2,0.35,0.5)
Labor impact (C_{13})	(0.05,0.2,0.35); (0.35,0.5,0.65)	(0.05,0.2,0.35); (0.35,0.5,0.65)	(0.05,0.2,0.35); (0.65,0.8,0.95)
Cost Efficiency (C_{21})	(0.65,0.8,0.95); (0.65,0.8,0.95)	(0.8, 1.0,1.0); (0.8, 1.0,1.0)	(0.65,0.8,0.95); (0.35,0.5,0.65)
Spillover effects (C_{22})	(0.35,0.5,0.65); (0.05,0.2,0.35)	(0.05,0.2,0.35); (0.05,0.2,0.35)	(0.05,0.2,0.35); (0.35,0.5,0.65)
Technical efficiency (C_{31})	(0.35,0.5,0.65); (0.35,0.5,0.65)	(0.35,0.5,0.65); (0.65,0.8,0.95)	(0.35,0.5,0.65); (0.05,0.2,0.35)
Technology reliability (C_{32})	(0.2,0.35,0.5); (0.65,0.8,0.95)	(0.35,0.5,0.65); (0.35,0.5,0.65)	(0.2,0.35,0.5); (0.65,0.8,0.95)
Resource availability (C_{41})	(0.8, 1.0,1.0); (0.8, 1.0,1.0)	(0.8, 1.0,1.0); (0.65,0.8,0.95)	(0.65,0.8,0.95); (0.8, 1.0,1.0)
Environmental impact (C_{42})	(0.8, 1.0,1.0); (0.65,0.8,0.95)	(0.65,0.8,0.95); (0.8, 1.0,1.0);	(0.8, 1.0,1.0); (0.65,0.8,0.95)

Table 4: Z-numbers based evaluations of alternatives by experts with respect to various criteria

Expert	Alternative	C_{11}	C_{12}	C_{13}	C_{21}	C_{22}	C_{31}	C_{32}	C_{41}	C_{42}
E_1	Solar	VG,VH	G,H	AA,H	A,H	A,H	A,H	G,H	BA,H	VG,VH
	Wind	VG,VH	G,H	AA,H	AA,H	AA,H	AA,VH	G,VH	A,H	VG,VH
	Hydro	P,H	P,H	BA,M	G,H	BA,H	G,H	VG,H	VP,H	G,H
	NG	G,H	A,MH	BA,M	P.H	BA,H	G,H	VG,H	H,H	BA,H
E_2	Solar	VG,VH	G,H	AA,H	A,H	AA,H	A,H	G,H	BA,H	VG,VH
	Wind	VG,VH	G,VH	G,H	AA,H	G,H	G,VH	G,VH	A,H	VG,VH
	Hydro	BA,H	P,H	BA,M	G,H	BA,H	G,H	G,H	VP,H	G,H
	NG	A,M	A,MH	BA,M	VP.H	BA,H	G,H	VG,H	G,H	BA,H
E_3	Solar	G,H	G,H	AA,H	AA,H	A,H	A,H	G,H	BA,H	VG,VH
	Wind	VG,VH	G,H	AA,H	G,H	AA,H	G,VH	G,VH	AA,H	VG,VH
	Hydro	A,H	P,H	BA,M	G,H	BA,H	G,H	VG,H	VP,H	G,H
	NG	G,H	A,MH	BA,M	BA.H	BA,H	G,H	VG,H	H,H	BA,H

Table 5: Aggregated weighted normalized decision matrix (regular fuzzy numbers based)

Criteria	Solar	Wind	Hydro	Natural gas
C_{11}	(0.75,0.933,0.983)	(0.8,1,1)	(0.172,0.301,0.431)	(0.509,0.694,0.833)
C_{12}	(0.249,0.307,0.365)	(0.249,0.307,0.365)	(0.019,0.077,0.134)	(0.121,0.173,0.225)
C_{13}	(0.097,0.126,0.155)	(0.106,0.135,0.164)	(0.031,0.053,0.076)	(0.031,0.053,0.076)
C_{21}	(0.119,0.152,0.208)	(0.098,0.119,0.152)	(0.098,0.119,0.152)	(0.238,0.455,1)
C_{22}	(0.072,0.098,0.125)	(0.098,0.125,0.152)	(0.098,0.125,0.152)	(0.036,0.063,0.089)
C_{31}	(0.168,0.239,0.311)	(0.287,0.359,0.431)	(0.287,0.359,0.431)	(0.311,0.383,0.455)
C_{32}	(0.234,0.288,0.342)	(0.253,0.311,0.369)	(0.234,0.288,0.342)	(0.288,0.36,0.36)
C_{41}	(0.19,0.333,0.476)	(0.381,0.523,0.666)	(0,0,0.19)	(0.618,0.761,0.904)
C_{42}	(0.065,0.065,0.081)	(0.065,0.065,0.081)	(0.07,0.07,0.087)	(0.139,0.199,0.349)

criteria weights assigned by experts. As it was mentioned earlier, we are using both subjective and combination of the subjective and objective weighs for criteria weighting.

Z-numbers based evaluations of the alternatives with respect to the various criteria provided by experts are shown in Table 4.

Based on information provided in Tables 2 and 3, Z-numbers based weights $W_k = \left[w_j^k\right]_{1xn} = \left[(\tilde{A}_j^k, \tilde{B}_j^k)\right]_{1xn}$ assigned by experts are aggregated into weights matrix $W = \left[\tilde{w}_j\right]_{1xn} = \left[(\tilde{A}_j, \tilde{B}_j)\right]_{1xn}$. In accordance with equation (6), reliability components $\tilde{B}_j^k$ of the weights converted into crisp numbers and added to restrictions $(\tilde{A}_j^k)$ and, finally, irregular fuzzy weights are converted according to equation (8) into regular fuzzy weights. Aggregated fuzzy weights are to be used for the composition of the weighted decision matrix.

Decision making is based on analysis of the limited set of alternatives A_i $(i=\overline{1,m}$ with respect to set of criteria C_j $(j=\overline{1,n})$ for selection of the alternative with higher aggregated value. As a rule, criteria have different importance for decision-makers and in order to take into consideration these differences, they assign to each criteria C_j $(j=\overline{1,n})$ unique weights w_j $(j=\overline{1,n})$.

Table 6: Alternatives ranking (PROMETHEE)

Alternatives	Subjective weights				Combined weights			
	Φ+	Φ−	Φ	Rank	Φ+	Φ−	Φ	Rank
Solar	2.83	2	0.83	2	3.5	3.167	0.333	2
Wind	3.67	0.5	3.17	1	5	0	5	1
Hydro	1.17	3.67	−2.5	4	1.185	4.167	−2.981	4
NG	2.33	3.83	−1.5	3	2	5	−3	3

In case of group decision-making, according to procedures of the outranking and distance based MCDM methods, weights W_k $(k=\overline{1,K})$ of criteria and decision matrixes $D_k = \left[z_{ij}^k\right]_{mxn}$ provided by each expert should be aggregated. If $W_k = \left[w_j^k\right]_{1xn} = \left[(\tilde{A}_j^k, \tilde{B}_j^k)\right]_{1xn}$ is Z-numbers based matrix that reflects an evaluation of the criteria importance by k-th expert $(k=\overline{1,K})$, then $W = \left[\tilde{w}_j\right]_{1xn}$ is an aggregated weights matrix that reflects an aggregated evaluation of the criteria importance by the group of experts (decision-makers), and, if

Table 7: Distances to positive ideal and negative ideal solutions (combined weights)

Criteria	d(SA+)	d(WA+)	d(HA+)	d(NGA+)	d(SA−)	d(WA−)	d(HA−)	d(NGA−)
C_{11}	0.08626	0.066539	0.407826	0.200437	0.894961	0.938534	0.31948	0.692094
C_{12}	0.117793	0.117793	0.461875	0.3107	0.851645	0.851645	0.246785	0.488586
C_{13}	0.159453	0.131683	0.395	0.395	0.777793	0.835643	0.344727	0.344727
C_{21}	0.485638	0.506559	0.506559	0.312315	0.163871	0.124844	0.124844	0.649039
C_{22}	0.220085	0.131521	0.131521	0.34965	0.662934	0.836073	0.836073	0.436264
C_{31}	0.283324	0.142391	0.142391	0.11756	0.542085	0.800241	0.800241	0.852247
C_{32}	0.144446	0.117454	0.144446	0.074206	0.789252	0.852522	0.789252	0.914821
C_{41}	0.371498	0.253213	0.539766	0.116564	0.391666	0.595163	0.121965	0.85484
C_{42}	0.373411	0.373411	0.365607	0.23788	0.201685	0.201685	0.217853	0.703627

Table 8: Alternatives ranking (TOPSIS)

Alternatives	Subjective weights				Combined weights			
	d_i^+	d_i^-	δ_i	Rank	d_i^+	d_i^-	δ_i	Rank
Solar	1.718	5.78	0.771	2	1.748	5.769	0.767	2
Wind	0.847	6.589	0.886	1	1.287	6.590	0.837	1
Hydro	2.506	4.364	0.635	4	2.566	4.331	0.628	4
NG	2.389	5.393	0.693	3	2.917	5.134	0.638	3

Table 9: Z-numbers based pairwise comparison of criteria

Criteria	Environmental	Economical	Social	T&Mgt
Environmental	(1,1,1)	(1,2,3); (0.5,0.75,1)	(2,3,4); (0.75,1,1)	(2,3,4); (0.75,1,1)
Economical		(1,1,1)	(1,2,3); (0.75,1,1)	(2,3,4); (0.5,0.75,1)
Social			(1,1,1)	(1,2,3); (0.5, 0.75,1)
T&Mgt				(1,1,1)

Table 10: Z-numbers based pairwise comparison of the energy resources to economic criterion

Alternatives	Solar	Wind	Hydro	Natural gas (NG)
Solar	1		(1,2,3); (0.25,0.5,0.75)	(2,3,4); (0.5,0.75,1)
Wind	(3,4,5); (0.5,0.75,1)	1	(2,3,4); (0.5,0.75,1)	(3,4,5); (0.5,0.75,1)
Hydro			1	(2,3,4); (0.25,0.5,0.75)
NG				1

$D_k = z_{ij}^k{}_{\text{mxn}} = (\tilde{A}_{ij}^k, \tilde{B}_{ij}^k{}_{\text{mxn}}$, then $D = z_{ij}{}_{\text{mxn}} = (\tilde{A}_{ij}, \tilde{B}_{ij})_{\text{mxn}}$ is an aggregated decision matrix.

Chain of common for the POMETHEE and TOPSIS operations:

Z-number based group evaluations → Transformation of the Z-values to fuzzy values → Aggregation → Normalization → Defuzzification.

As it was underlined in 2.3, the weighting is a powerful tool that allows to regulate priorities of decision-maker and indirectly to influence best solution search process and results. Variations of the weights in addition to changes of the particular indicators could change a ranking of the alternatives as a whole. Therefore, it would be useful to study how changes in weights are influencing solutions.

Table 11: Z-numbers based pairwise comparison of social sub-criteria

Sub-criteria	GP&R	Labor impact	Acceptance	Weights, W_i
GP&R	(1,1,1)	(2,3,4); (0.5,0.75,1)	(4,5,6); (0.75,1,1)	0.5977
Labor Impact		(1,1,1)	(3,4,5); (0.75,1,1)	0.3021
Acceptance			(1,1,1)	0.1001

A resultant decision matrix represented in Table 5 is based on: Z-numbers based assessments of the alternatives with respect to objectives by a group of experts; Z-numbers reliability transformation and addition to restriction part of the Z-numbers; Z-numbers representation as regular fuzzy numbers; weighting of the matrix elements; normalization of the matrix. These transformations simplify computations with Z-numbers and eliminate difficulties caused by the size of the decision matrix. Since we are using subjective and combined (subjective/objective) weights, we have to compose two different decision matrixes.

In this paper, we separately studied the influence of subjective and combined (subjective + objective) weights on energy resources alternatives ranking via PROMETHEE and TOPSIS techniques. In our calculations, we assumed that in case of combination subjective and objective weights are equally important and in equation (13) α =0.5. Results are presented in Tables 6-11. As one can note, the numerical values of indicators are different, but the results of rankings are the same.

In Table 12 results of the energy resources ranking in accordance with Z-numbers based AHP are presented. The best option is wind and hydro with lower weight is the worst option. According to the AHP method ranking of the energy resources, the margin of the second and third best options, namely Solar and Hydro, is <1%.

Despite methodological differences, the results of applications Z-numbers based PROMETHEE, TOPSIS and AHP methods

Table 12: Energy resources Z-numbers based ranking

Objective	Criterion category	Weight	Criterion	Weight	Solar	Wind	Hydro	NG
Energy Resources Ranking	Social	0.1651	GP&R	0.0987	0.0243	0.0522	0.0114	0.0108
			Social acceptance	0.0165	0.0069	0.0061	0.0026	0.0053
			Labor impact	0.0499	0.0111	0.0301	0.0052	0.0035
	Economics	0.3251	Cost Efficiency	0.2566	0.0575	0.1317	0.0499	0.0176
			Spillover effects	0.0685	0.0171	0.0320	0.0118	0.0076
	Technological	0.1306	Technical efficiency	0.0959	0.0020	0.0039	0.0010	0.0066
			Technology reliability	0.0347	0.0098	0.0034	0.0036	0.0135
	Environmental	0.3791	Resource availability	0.2992	0.0579	0.0979	0.0212	0.1222
			Environmental impact	0.0799	0.0377	0.0263	0.0118	0.0040
Total weights of alternatives					0.2360	0.3985	0.1302	0.2276
Normalized weights of alternatives					0.2378	0.4016	0.1312	0.2294
Rank					2	1	4	3

for energy resources ranking provided similar ordering of the alternatives: wind → solar → NG → hydro.

The methods are efficiently modelling decision-making processes based on fuzzy and Z-numbers and are suitable for individual and group opinions-based decisions. Decision making and modelling procedures are well structured, and these features allow decision-makers if necessary, to change and modify alternatives, criteria, and priorities. Stepwise and well-defined sequential computations are significantly simplifying the solution process. At the same time, based on models' development, adjustments, and user experience, it is necessary to emphasize that experts` and decision-makers' judgments and opinions are significantly influencing results of the subjective information based multi-criteria decision problems solutions. It is well known that problem solution result can be no better than information on whish this solution is based.

Applying for calculation of the number of judgments procedures used in (Junior et al., 2014; Ghaleb et al, 2020), the number of judgments for a problem statement with m criteria and n alternatives are calculated as followings:

$$J_{m,n}^{\mathrm{T}|P} = m + mn = m(n+1) \tag{32}$$

$$J_{m,n}^{AHP} = m(m-1)/2 + m(n(n-1/2)) \tag{33}$$

In our case m=9 and n=4, so $J_{m,n}^{T|P} = 45$ and $J_{m,n}^{AHP} = 90$. In these calculations, we are assuming that classical and fuzzy decision making judgmental processes are similar and agilities are equal. But each Z-number has two different fuzzy components, $Z = (\tilde{A}, \tilde{B})$. The first component, $\tilde{A}$, is a restriction and the second component, $\tilde{B}$, is a measure of the reliability of the first component. In fact, each bi-component Z-number requires two different judgments, one for the restriction and second, for reliability. Moreover, the number of experts participating in the evaluation process are also increasing the number of judgments required for decision composition and decision making. These simple examples once again show that in fuzzy and Z-numbers based MCDM particular attention should be paid to opinion study and judgmental processes.

5. CONCLUSION

Fuzzy information and Z-numbers based models allow to make up for the deficiency of quantitative data in the decision-making process, to formalize impreciseness, uncertainties, and information incompleteness inherent to policy development and planning problems. In view of the fact that country-level energy policy decisions have long-run effects and changes of these decisions are costly and difficult, in order to increase a justification of the decision it was decided to use a hybrid approach based on the combination of the various MCDM. For a country level energy resources ranking and selection task Z-number based outranking (PROMETHEE), distance (TOPSIS) and pairwise comparison (AHP) models were developed and resources ranking tasks are solved. In PROMETHEE and TOPSIS models subjective weights and combination of the subjective and objective weights are used for criteria importance rating. Differences in weights assignment approaches influenced interim calculations results like entering, leaving, net flows, distance to ideal solutions, and closeness but ranking order in our examples has stayed the same. Preliminary phases of the outranking and distance-based methods are similar up to the decision matrix composition step. AHP model`s theoretical background is different and from the beginning requires formalization of the hierarchical structure, pairwise comparisons and weightings, and final ranking is also based on alternative weights.

All three approaches used provide comparable and similar from ranking standpoint results. According to solutions, the wind is the best option for further development of the country's energy sector and the worst one is hydro. At the same time according to the AHP method, the ranking margin of the solar energy resource in comparison to NG is very small. In spite of the general advantages of renewables, in case of Azerbaijan domestic production of the NG, environmentally-friendly technological advancement and declining oil and gas prices are necessitating a need for the more scrupulous study and analysis of the role of NG as a source of energy for the country.

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